Hybrid Method of Tone-Injection and Clipping-and-Filtering for Suppressing Nonlinear Distortion of OFDM Signal

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Abstract

In this paper, we propose hybrid methods of Tone-Injection (TI) and Clipping-and-Filtering (CF) for suppressing the non-linear distortion of OFDM. TI utilizes the genetic algorithm to search for an effective solution. The proposed methods are evaluated in terms of Peak-to-Average-Power Ratio (PAPR), Bit-Error-Rate (BER), and out-of-band radiation. The effectiveness of the proposed methods is demonstrated in the simulation.

1 Introduction

With the advances of ICT, higher speed and higher reliability communication has been demanded. Due to this background, communication systems based on orthogonal frequency division multiplexing (OFDM) have been extensively studied and developed[1]. Although OFDM is superior in spectral efficiency, one of its drawbacks is that the peak-to-average power ratio (PAPR) of the transmitted signal tends to be high. This causes the problems of out-of-band radiation and degradation in bit error rate (BER) characteristics. In order to overcome this problem, various techniques for peak power reduction have been proposed[2, 3, 4, 5, 6]. The clipping and filtering (CF) technique is one of them[6]. Although the CF technique is of simple and easy implementation, it distorts the modulated signal and tends to degrade the BER characteristics. On the other hand, although the tone injection (TI) technique needs much time to obtain a good reduction effect of PAPR, it does not cause distortion in the modulated signal[4, 5]. For TI, some complex symbols used for modulating subcarriers are moved from their original positions to other positions so as to suppress the peak power of OFDM signals. Therefore, TI has to solve a combinatorial optimization problem that determines the subset of symbols to be moved.

In this paper, in order to improve both BER and out-of-band radiation characteristics, it is considered that hybrid methods using TI and CF for suppressing the nonlinear distortion of OFDM signals. By means of the synergistic effect of CF and TI, it is attempted to improve PAPR, BER and out-of-band radiation characteristics. The considered hybrid methods employ the genetic algorithm (GA) to search for an effective solution of TI. Two types of the hybrid methods are proposed. Especially, one of them employs a new fitness function that evaluates the quality of the solution for GA. The proposed hybrid methods are compared with CF method, a conventional GA without CF and TI based on random search with CF. According to the simulation results, the hybrid method with a new fitness function is superior in terms of any of PAPR, BER and out-of-band radiation characteristics.

2 OFDM and PAPR Reduction

2.1 OFDM

In OFDM, multiple N subcarriers being orthogonal to each other are used for transmitting N binary codes. For the OFDM symbol time of T [sec], the subcarriers are spaced $\frac{1}{T}$ [Hz] apart from each other. Fig.1 shows the flow of OFDM from a transmitting end to a receiving end.

In the transmitter, at first, N binary codes $c_0, c_1, \cdots, c_{N-1}$ are converted into N complex symbols $X_0, X_1, \cdots, X_{N-1}$ by using a digital modulation scheme such as quadrature amplitude modulation (QAM). When using $M$-QAM, the binary code is $\log_2 M$-bit in length. And next, samples of OFDM signal $Y_0, Y_1, \cdots, Y_{N-1}$ are generated by IDFT (Inverse Discrete Fourier Trans-
where $r$ is the power of the input OFDM signal and $\sigma^2$ is the average power of input signal. The OFDM signal often has a very high peak power compared to its average power, because the signal is produced as a synthetic signal of a number of subcarriers. The degree of the peak power is evaluated by peak-to-average power ratio (PAPR) $\Phi$ as follows:

$$\Phi = \frac{\max_{0 \leq k < N} |Y_k|^2}{E\{|Y_n|^2\}},$$

(5)

where $E\{|Y_n|^2\}$ is the average power of the OFDM signal and $\max_{0 \leq k < N} |Y_k|^2$ is the peak power of the OFDM signal.

When the power of the input OFDM signal $r$ exceeds the threshold $A$, the power of the output signal is limited to the constant output $A$. This introduces clipping noise which causes distortion and out-of-radiation.

2.3 Clipping and Filtering (CF)

Clipping and Filtering (CF) effectively suppresses the out-of-band radiation due to the nonlinear distortion introduced by the HPA[6]. The CF process consists of two steps. The first step, clipping, limits the amplitude of the input signal to a certain level $A_{\text{max}}$. $CR = \frac{A_{\text{max}}}{A}$ is called the clipping ratio (CR), where $\sigma^2$ is the average power of the input signal. The second step, filtering, removes the out-of-band radiation induced by clipping. The drawback of CF is to degrade the BER characteristic.

3 Tone Injection and GA for TI

3.1 Tone Injection

Tone Injection (TI) suppresses the peak power of the OFDM signal by moving some complex symbols[4, 5]. Fig. 2 shows the constellation map of 64-QAM for TI. Black circles show usual complex symbols and white circles show alternative symbols. Each decimal number beside a circle shows the associated binary code. If black and white circles are associated with the same binary code, then either of them can be adopted for the code. Therefore, for each $c_i$ of every binary codes with two associated symbols, the TI method has to determine which of the usual and alternative symbols is used as $X_i$. When the objective is to minimize PAPR, the problem to be solved by the TI method is formulated as follows.
Problem 1 (PAPR minimization by TI): Given the binary codes $c_i$’s to be transmitted. Let $S$ be the number of codes having two associated complex symbols and let $c_{i_1}, c_{i_2}, \cdots, c_{i_S}$ be the codes, where $i_1 < i_2 < \cdots < i_S$. Let $I = (i_1, i_2, \cdots, i_S)$ be the $S$-dimensional binary vector, let $l_s = 0$ mean that $c_{i_s}$ uses the usual symbol (black circle), that is, *not move*, and let $l_s = 1$ mean that $c_{i_s}$ uses the alternative one (white one), that is, *move*. Then, find $I$ such that PAPR $\Phi$ is minimized.

The number of candidate solutions of Problem 1 is $2^S$. For example, when $M = 64$ and $N = 256$, the average of $S$ is 112 and the averaging number of candidate solutions is approximately $5.19 \times 10^{33}$. Since all the candidate solution cannot be examined in reasonable time, probabilistic search methods such as random search and GA are employed for solving the problem.

3.2 Genetic Algorithm for TI

This subsection explains the conventional GA for solving Problem 1. In order to obtain good sub-optimal solutions, GA evolves multiple solutions by using genetic operators such as selection, crossover and mutation. The algorithm is shown below.

**GA for Problem 1**

**Step 1:** Generate $K$ solutions $l_1, l_2, \cdots, l_K$. Initialize the generation number $g$ as $g = 1$.

**Step 2:** For each $k \in \{1,2,\cdots,K\}$,

(FC) calculate $k$-th solution’s fitness $f_k$.

The conventional GA uses the following fitness function.

$$f_k = \left( \frac{1}{\text{PAPR}_k} \right)^\alpha$$

Set $f_{\text{bat}} = \max_k f_k$ and $l_{\text{bat}} = l_{k_{\text{max}}} = \max_k l_{k_{\text{max}}}$.

**Step 3:** Select $K$ solutions $l'_1, l'_2, \cdots, l'_K$ from $K$ solutions $l_1, l_2, \cdots, l_K$, according to the fitness values and the adopted selection policy such as roulette selection and tournament one.

**Step 4:** For $k \in \{0,1,\ldots,K/2-1\}$, apply the crossover operation to a pair of $l'_{2k+1}$ and $l'_{2k+2}$ with the probability of $P_c$. When using uniform crossover, for $s \in \{1,2,\cdots,S\}$, $l'_{2k+1,s}$ and $l'_{2k+2,s}$ are swapped with the probability of $0.5$.

**Step 5:** For $k \in \{1,2,\ldots,K\}$, apply the mutation operation to a solution $l'_k$. When using uniform mutation, for $s \in \{1,2,\cdots,S\}$, a bit $l'_{k,s}$ is flipped with the probability of $P_m$, where $l'_k = (l'_{k,1}, l'_{k,2}, \cdots, l'_{k,S})$.

**Step 6:** Copy $l'_k$’s to $l'_k$’s. For each $k \in \{1,2,\cdots,K\}$,

(FC) calculate $k$-th solution’s fitness $f_k$.

If $f_{\text{bat}} < \max_k f_k$, then set $f_{\text{bat}} = \max_k f_k$ and $l_{\text{bat}} = l_{k_{\text{max}}} = \max_k l_{k_{\text{max}}}$.

**Step 7:** If $g < G_{\text{max}}$, then set $g \leftarrow g + 1$ and go to Step 3. Otherwise, terminate the algorithm and return $l_{\text{bat}}$ as the solution.

4 Hybrid Methods of TI and CF

Although the CF technique is of simple and easy implementation, it distorts the modulated signal and tends to degrade the BER characteristics. On the other hand, although the TI technique needs much time to obtain a good reduction effect of PAPR, it does not cause distortion in the modulated signal. To aim to obtain good synergistic effects of CF and TI, hybrid methods (HMs) of TI and CF are proposed. The process flow in the transmitter of HMs is shown in Fig.3. In the following, two types of HMs of TI and CF are presented.

4.1 HM with PAPR-Based Fitness Function

The first type of HM uses the same fitness function as the conventional GA described in the previous section. The difference between the HM and the conventional GA is (FC) the calculation of the fitness $f_k$ that the conventional one performs in Steps 2 and 6. The proposed HM takes into account the effect of CF in the fitness calculation. The proposed method uses the following algorithm for the fitness calculation of (FC).

**PAPR-Based Fitness Calculation of HM**

**Step FC-1:** Generate complex symbols $X_n$’s with TI for $l_k$ and the OFDM signal by Eq.(1).

**Step FC-2:** Apply CF to the OFDM signal.

**Step FC-3:** Calculate the fitness $f_k$ by using Eq.(6).

4.2 HM with Error-Based Fitness Function

The fitness function of Eq.(6) favors solutions with smaller PAPRs. However, such a fitness function seems to indirectly optimize the BER performance. In order to more directly optimize the BER performance, a fitness function based on the error between the original complex symbol $X_n$ and the restored symbol $\hat{X}_n$ calculated by using Eq.(2) is proposed. The proposed fitness function is as follows.

$$f_k = \left( \frac{1}{\pi} \sum_{n=0}^{N-1} (X_n - \hat{X}_n)(X_n^* - \hat{X}_n^*) \right)^\alpha$$ (7)
The restored symbol $\hat{X}_n$ is calculated from the OFDM signal obtained by applying the CF process and then passing through HPA. The detailed algorithm for calculating the fitness value is as follows.

**Error-Based Fitness Calculation of HM**

**Step FC-1:** Generate complex symbols $X_n$’s with TI for $I_k$ and the OFDM signal by Eq.(1).

**Step FC-2:** Apply CF to the OFDM signal and let $\tilde{Y}_n$’s be the samples of the OFDM signal obtained after CF.

**Step FC-3:** Calculate the restored complex symbols $\hat{X}_n$’s from $\tilde{Y}_n$’s by using Eq.(2).

**Step FC-4:** Calculate the fitness $f_k$ by using Eq.(7).

5 **Numerical Simulation**

In order to demonstrate the effectiveness of the proposed methods, the proposed HMs are compared with some conventional methods in terms of PAPR, BER and out-of-band radiation characteristics.

5.1 **Simulation Condition**

The following five types of methods are evaluated.

- **CF:** solo use of CF.
- **TI (GA) w/o CF:** conventional GA for TI described in 3.2.
- **TI (random) w/ CF:** random search for TI with CF.
- **HM w/ P-FC:** HM of TI and CF using PAPR-based Fitness Calculation (P-FC).
- **HM w/ E-FC:** HM of TI and CF using Error-based Fitness Calculation (E-FC).

The parameters used for the simulation is shown in Table 1. The parameters $\alpha$, $K_{ts}$, $P_c$ and $P_m$ are determined based on preliminary experiments. The random search randomly generates solutions, evaluates their PAPR and returns the solution with the best PAPR as the final solution. The number of trials for the random search is set so as to be equal to the number of solutions examined by the GA methods. For this setting, TI (GA) w/o CF can perform much faster than TI (random) w/ CF. Because the solutions examined by GA contain a lot of same solutions, each of which may be evaluated just once. In [5], redundant fitness calculations are omitted by using two types of fitness tables. Table 2 shows the computation time for random search, GA without fitness table and GAs with fitness tables. For HMs, the same technique can be used. However, additional computation time or hardware for acceleration is needed.

In the following, the above five methods are evaluated in terms of PAPR, BER and out-of-band radiation.

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<tr>
<th>Item</th>
<th>Method</th>
<th>Value</th>
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<td>Modulation method</td>
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<td>For CF</td>
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<td>For GA</td>
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5.2 **Result and Discussion**

The simulation result on PAPR is shown in Fig.4. For CF, two cases of CR=3[dB] and CR=5[dB] are shown. The vertical axis corresponds to the probability that PAPR exceeds PAPR$_0$ corresponding to the horizontal axis. The curve locating at lefter position means the method is better at PAPR reduction. The tendency of the result is that, 1) the solo use of CF needs low CR to reduce PAPR and its PAPR reduction performance is not good, 2) the proposed HM with E-FC is the best and the proposed HM with P-FC is the second best, and 3) for smaller PAPR$_0$ the conventional GA without CF is better than the random search with CF and for larger PAPR$_0$ the tendency is opposite.

The simulation result on BER is shown in Fig.5. The tendency of the result is that, 1) the three GA-based TI methods, including two HMs, are much better than the random search with CF, and 2) though the proposed HM with P-FC is not better than the conventional GA without CF, the proposed HM with E-FC outperforms the conventional method.

The simulation result on PSD (Power Spectral Density) is shown in Fig.6. From the result, the out-of-band radiation characteristics can be observed. In addition to the above five methods, the case without any addi-
tional technique such as TI and CF is also plotted as “Original”. The tendency of the result is that, 1) every five methods are better than the original case, 2) the proposed HM with E-FC is the best and the proposed HM with P-FC is the second best, and 3) the conventional GA without CF and the random search with CF are almost same and are better than CF.

From the results mentioned above, it can be concluded that the proposed HM with E-FC is superior to any other methods in terms of PAPR, BER and out-of-band radiation characteristics.

6 Conclusions

In this paper, two types of hybrid methods of TI and CF for suppressing nonlinear distortion of OFDM signal. Both methods employ GA for searching for an effective solution of TI. The first proposed one, HM with P-FC, tries to minimize PAPR, and the second one, HM with E-FC, tries to minimize the error of the restored complex symbols against the original one. The simulation results demonstrate that the HM with E-FC is superior to other methods to reduce PAPR, BER and out-of-band radiation.

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Fig. 6: PSD simulation result.

References


