A New GNSS Relative Positioning Algorithm Based on Alternative Use of the Positions of Reference Receivers

Hirokazu Hasegawa, Goshi Okuda, Yukihiro Kubo, and Sueo Sugimoto

Department of Electrical and Electronic Engineering, Ritsumeikan University

1-1-1 Noji-Higashi, Kusatsu City, Shiga 525-8577 Japan

Tel: +81-(0)77-561-2673, FAX: +81-(0)77-561-2663

E-mail: sugimoto@se.ritsume.ac.jp

ABSTRACT

In this paper, we present a new relative positioning algorithm based on GNSS Regression measurement models (GR models) by using multiple antennas based on alternative use of the positions of reference receivers. We show the algorithms of estimating all unknown antenna positions by applying the double difference (DD). Then we derive updating algorithms by using true positions of the reference receivers. Finally, we carried out the experiments using real GNSS data and show the positioning results for the the proposed method as well as the conventional method.

1 INTRODUCTION

It is well known that relative positioning can give accurate results of positioning as a relative position by using at least two receivers. For estimating a relative position, you need GNSS measurements including the true position of the reference receivers on the premise that receivers observe GNSS signals of the same satellites. If the baseline length between the reference receiver (known) and the rover receiver (unknown) is not long, GNSS measurements from satellites observed by receivers include similar observation errors; therefore, you can cancel observation errors by using GNSS measurements including the true reference position and calculate the accuracy position.

In this paper we present the new relative positioning algorithm based on alternative use of the positions of reference receivers. To estimate all receiver positions including reference receiver, GNSS measurements are applied the DD without true position of reference. Then, rover position are estimated again by using constraint condition of true position of the reference receiver. In other words, this method updates the rover position already estimated with the constraint condition of true reference position. This paper is aimed at verifying the performance of this method.

2 GNSS REGRESSION MODELS AND DD MEASUREMENT EQUATIONS

First of all, similarly to [1]-[7], we formulate all observed positioning data consisting of the L1 carrier-phase and pseu-

doranges based on C/A code, by using the GNSS regression models. The natural extensions of GNSS regression models for multiple frequencies of GPS, Galileo, Compass/BeiDou, GLONASS, US-GPS and QZSS modernization are also similarly formulated. Namely, we consider the following fundamental measurements of the pseudoranges $\rho_{CA,u}^p(t)$ based on the C/A code and L1 band carrier-phases $\varphi_{L1,u}^p(t)$ (equivalently, $\Phi_{L1,u}^p(t)$ as the unit of length), respectively, as follows [8]-[15]:

$$\begin{aligned}\rho_{CA,u}^p(t) &= r_u^p(t, t - \tau_u^p) + \delta I_u^p(t) + \delta T_u^p(t) \\ &\quad + c[\delta t_u(t) - \delta t^p(t - \tau_u^p)] \\ &\quad + \delta b_{CA,u} - \delta b_{CA}^p + e_{CA,u}^p(t) \\ \Phi_{L1,u}^p(t) &= \lambda_1 \varphi_{L1,u}^p(t) \\ &= r_u^p(t, t - \tau_u^p) - \delta I_u^p(t) + \delta T_u^p(t) \\ &\quad + c[\delta t_u(t) - \delta t^p(t - \tau_u^p)] \\ &\quad + \delta b_{L1,u} - \delta b_{L1}^p + \lambda_1 N_u^p + \lambda_1 \varepsilon_{L1,u}^p(t)\end{aligned}\quad (1)$$

where λ_1 is the wavelength of the carrier wave and calculated by c/f_1 . c ($\cong 2.99792458 \times 10^8$ [m/s]) denotes the speed of light, and f_1 is the central frequency of the L1 carrier wave

$$f_1 = 2 \times 77 \times 10.23 \text{ [MHz]} = 1575.42 \text{ [MHz]},$$

In Eqs.(1)-(2), the so-called receiver biases, $\{\delta b_{CA,u}, \delta b_{L1,u}\}$, and the satellite biases, $\{\delta b_{CA}^p, \delta b_{L1}^p\}$, are contained in the usual observed positioning data consisting of pseudorange based on the C/A codes and the L1 carrier-phase [16]. $r_u^p(t, t - \tau_u^p)$ is the geometric distance between the receiver u at the time t and the satellite p at the time $t - \tau_u^p$ (τ_u^p denotes the travel time from the satellite p ($p = 1, \dots, n_s$) to the receiver u). Namely,

$$\begin{aligned}r_u^p(t) &\equiv r_u^p(t, t - \tau_u^p) \\ &= \left[(x_u(t) - x^p(t - \tau_u^p))^2 + (y_u(t) - y^p(t - \tau_u^p))^2 \right. \\ &\quad \left. + (z_u(t) - z^p(t - \tau_u^p))^2 \right]^{1/2} \\ &= \|u(t) - s^p(t - \tau_u^p)\|,\end{aligned}\quad (2)$$

where $u \equiv [x_u, y_u, z_u]^T$ and $s^p \equiv [x^p, y^p, z^p]^T$ are a user (unknown) and satellite positions, respectively. n_s denotes the number of the observable satellites. $\delta I_u^p(t)$ and $\delta T_u^p(t)$ in Eqs. (1)-(2) reflect the delay or the advance associated with the transmission of the L1 signal through the ionosphere and the troposphere, respectively. $\delta t_u(t)$ and $\delta t^p(t - \tau_u^p)$ are the

clock errors of the receiver u at the time t and the satellite p at the time $t - \tau_u^p$. N_u^p denotes integer ambiguity between the satellite p and the receiver u , and $e_{CA,u}^p(t)$, $\varepsilon_{L1,u}^p(t)$ denote measurement errors.

Eq. (3) contains the satellite orbital errors. The estimated satellite orbits are obtained from the navigation messages which are decoded from the transmitted L1 signal. Let us denote $\hat{s}^p$ as the estimated position of the satellite s^p at the time $t - \tau_u^p$.

We use the following relations of the derivatives

$$\begin{aligned} \frac{\partial r_u^p}{\partial x_u} &= \frac{(x_u - x^p)}{r_u^p}, & \frac{\partial r_u^p}{\partial y_u} &= \frac{(y_u - y^p)}{r_u^p}, \\ \frac{\partial r_u^p}{\partial z_u} &= \frac{(z_u - z^p)}{r_u^p}, & (p = 1, 2, \dots, n_s), \end{aligned} \quad (4)$$

and also

$$\begin{aligned} \frac{\partial r_u^p}{\partial x^p} &= -\frac{(x_u - x^p)}{r_u^p}, & \frac{\partial r_u^p}{\partial y^p} &= -\frac{(y_u - y^p)}{r_u^p}, \\ \frac{\partial r_u^p}{\partial z^p} &= -\frac{(z_u - z^p)}{r_u^p}, & (p = 1, 2, \dots, n_s). \end{aligned} \quad (5)$$

Then we have the relation:

$$\frac{\partial r_u^p}{\partial u} = -\frac{\partial r_u^p}{\partial s^p}. \quad (6)$$

Thus the 1st order Taylor series approximation of Eq. (3) around the previous estimated value $u = \hat{u}$ and $s^p = \hat{s}^p$ is given by

$$\begin{aligned} r_u^p &\cong r_{\hat{u}}^{\hat{p}} + (g_{\hat{u}}^{\hat{p}})^T [u - s^p - (\hat{u} - \hat{s}^p)] \\ &= ||\hat{u} - \hat{s}^p|| + \frac{(\hat{u} - \hat{s}^p)^T}{||\hat{u} - \hat{s}^p||} [u - s^p - (\hat{u} - \hat{s}^p)] \\ &= \frac{(\hat{u} - \hat{s}^p)^T}{||\hat{u} - \hat{s}^p||} (u - s^p) \end{aligned} \quad (7)$$

for $p = 1, 2, \dots, n_s$, where gradient vectors are as follows:

$$g_{\hat{u}}^{\hat{p}} \equiv \left[\frac{\partial r_u^p}{\partial u} \right]_{u=\hat{u}, s^p=\hat{s}^p} = \frac{(\hat{u} - \hat{s}^p)}{||\hat{u} - \hat{s}^p||} \quad (8)$$

The GR models utilize the extended Kalman filter based on the geometric distance linearized by the gradient vectors [6], and provide appropriate positioning performances. The DD-based PPP methods employ the same estimation method as that of previous PPP methods.

The following parameters contained in the usual observed positioning in Eq. (1-2) are canceled by DD-based GR models:

- $\delta t_{u_i}, \delta t^p$: Receiver and satellite clock errors,
- $I_{u_i}^p, T_{u_i}^p$: Ionospheric and tropospheric delays,
- $\delta b_{CA}^p, \delta b_{L1}^p$: Satellite hardware biases,
- $\delta b_{CA,u_i}, \delta b_{L1,u_i}$: Receiver hardware biases.

Let us assume that the estimated values: $\hat{s}^p, p = 1, \dots, n_s$, of the satellite positions: $s^p, p = 1, \dots, n_s$, are practically obtained by using ephemeris parameters broadcast by GPS

satellites, or precise ephemerides of all satellites provided by the International GNSS Service (IGS), as follows [1]-[6]:

$$\hat{s}^p = s^p + e_{s^p}, \quad p = 1, \dots, n_s, \quad (9)$$

where we assume e_{s^p} are Gaussian white noises.

Let us write the final results of the double difference measurement equations as follows [10]- [12].

$$\begin{aligned} &\tilde{\rho}_{CA,\hat{u}_j\hat{u}_i}^{\hat{q}\hat{p}} \\ &\equiv \rho_{CA,u_j u_i}^{qp} + (g_{\hat{u}_j\hat{u}_i}^{\hat{q}})^T \tilde{s}^q - (g_{\hat{u}_j\hat{u}_i}^{\hat{p}})^T \tilde{s}^p \\ &\cong (g_{\hat{u}_j\hat{u}_i}^{\hat{q}\hat{p}})^T u_j - (g_{\hat{u}_i\hat{u}_i}^{\hat{q}\hat{p}})^T u_i + (g_{\hat{u}_j\hat{u}_i}^{\hat{q}})^T e_{s^q} - (g_{\hat{u}_j\hat{u}_i}^{\hat{p}})^T e_{s^p} \\ &\quad + e_{CA,u_j u_i}^{qp} \\ &\tilde{\Phi}_{L1,\hat{u}_j\hat{u}_i}^{\hat{q}\hat{p}} \\ &\equiv \Phi_{L1,u_j u_i}^{qp} + (g_{\hat{u}_j\hat{u}_i}^{\hat{q}})^T \tilde{s}^q - (g_{\hat{u}_j\hat{u}_i}^{\hat{p}})^T \tilde{s}^p \\ &\cong (g_{\hat{u}_j\hat{u}_i}^{\hat{q}\hat{p}})^T u_j - (g_{\hat{u}_i\hat{u}_i}^{\hat{q}\hat{p}})^T u_i + (g_{\hat{u}_j\hat{u}_i}^{\hat{q}})^T e_{s^q} - (g_{\hat{u}_j\hat{u}_i}^{\hat{p}})^T e_{s^p} \\ &\quad + \lambda_1 N_{L1,u_j u_i}^{qp} + \lambda_1 \varepsilon_{L1,u_j u_i}^{qp} \end{aligned} \quad (10)$$

where

$$\begin{aligned} g_{\hat{u}_j}^{\hat{q}\hat{p}} &\equiv g_{\hat{u}_j}^{\hat{q}} - g_{\hat{u}_j}^{\hat{p}}, & g_{\hat{u}_i}^{\hat{q}\hat{p}} &\equiv g_{\hat{u}_i}^{\hat{q}} - g_{\hat{u}_i}^{\hat{p}} \\ g_{\hat{u}_j\hat{u}_i}^{\hat{q}} &\equiv g_{\hat{u}_j}^{\hat{q}} - g_{\hat{u}_i}^{\hat{q}}, & g_{\hat{u}_j\hat{u}_i}^{\hat{p}} &\equiv g_{\hat{u}_j}^{\hat{p}} - g_{\hat{u}_i}^{\hat{p}} \\ \rho_{CA,u_j u_i}^{qp} &\equiv (\rho_{CA,u_j}^q - \rho_{CA,u_i}^q) - (\rho_{CA,u_j}^p - \rho_{CA,u_i}^p) \\ \Phi_{L1,u_j u_i}^{qp} &\equiv (\Phi_{L1,u_j}^q - \Phi_{L1,u_i}^q) - (\Phi_{L1,u_j}^p - \Phi_{L1,u_i}^p) \\ e_{CA,u_i}^{qp} &\equiv e_{CA,u_i}^q - e_{CA,u_i}^p \\ e_{CA,u_j}^{qp} &\equiv e_{CA,u_j}^q - e_{CA,u_j}^p \\ e_{CA,u_j u_i}^{qp} &\equiv e_{CA,u_j}^{qp} - e_{CA,u_i}^{qp}. \end{aligned}$$

For the case of $p = 1, q = 2, \dots, n_s$ and $u_i = u_1, u_j = u_2$ ($n_r = 2$), we have the following measurement equation for antennas of u_1 and u_2 and for n_s satellites [7, 17].

$$y_{u_2 u_1}^{n_s} \equiv C_{\hat{u}}^{\hat{s}1} \eta_{u_2 u_1}^{s1} + v_{u_2 u_1}^{s1} \quad (12)$$

where

$$\begin{aligned} y_{u_2 u_1}^{n_s} &\equiv \begin{bmatrix} y_{CA,\hat{u}_2\hat{u}_1}^{\hat{s}1} \\ y_{L1,\hat{u}_2\hat{u}_1}^{\hat{s}1} \end{bmatrix}, & \eta_{u_2 u_1}^{s1} &= \begin{bmatrix} u_1 \\ u_2 \\ N_{L1,u_2 u_1}^{s1} \end{bmatrix}, \\ C_{\hat{u}}^{\hat{s}1} &= \begin{bmatrix} -G_{\hat{u}_1}^{\hat{s}1} & G_{\hat{u}_2}^{\hat{s}1} & \mathbf{0} \\ -G_{\hat{u}_1}^{\hat{s}1} & G_{\hat{u}_2}^{\hat{s}1} & \lambda_1 \mathbf{I} \end{bmatrix}, \\ G_{u_i}^{\hat{s}1\hat{i}} &\equiv \begin{bmatrix} g_{\hat{u}_i}^{2\hat{i}} & \dots & g_{\hat{u}_i}^{\hat{n}_s, \hat{i}} \end{bmatrix}^T : (n_s - 1) \times 3. \end{aligned} \quad (13)$$

$y_{CA,u_2 u_1}^{\hat{s}1}$ and $y_{L1,u_2 u_1}^{\hat{s}1}$ are $((n_s - 1) \times 1)$ vectors which are DD observables from raw C/A code pseudoranges and L1 carrier phases, respectively. $s^1 (p = 1)$ is the reference satellite. $N_{L1,u_2 u_1}^{s1}$ and $v_{u_2 u_1}^{s1}$ are DD-based integer ambiguity and observation noise vector, respectively. $G_{\hat{u}_i}^{\hat{s}1}$ is the known $((n_s - 1) \times 3)$ matrix which consists of gradient vectors $g_{\hat{u}_i}^{\hat{s}i}$ from the estimated antennas position $\hat{u}_i$ to the estimated satellite position $\hat{s}_i$ by the receiver u_i .

2.1 State Equations for the Static Case

In the static case, we utilize the state vector $\eta_{u_2 u_1}^{s1}$ in Eq. (12) for antennas of u_1 and u_2 ($n_r = 2$) and for n_s satellites. In order to simplify the expression, superscripts $s, 1$ and subscripts u_1, u_2 are omitted hereafter. Then in the case of η , the state equation is described by

$$\eta_{t+1} = \eta_t, \quad \eta_t : n' \times 1, \quad n' = 5 + n_s, \quad (14)$$

and the measurement equation is

$$y_t = C_t \eta_t + v_t, \quad (15)$$

Therefore the positioning algorithms based on the Kalman filter for Eqs. (14) and (15) are given as follows

$$\hat{\eta}_{t+1|t} = \hat{\eta}_{t|t} \quad (16)$$

$$\hat{\eta}_{t|t} = \hat{\eta}_{t|t-1} + K_t \nu_t \quad (17)$$

$$\nu_t \equiv y_t - C_t \hat{\eta}_{t|t-1} \quad (18)$$

(: Innovation Process)

$$K_t = \Sigma_{\eta, t|t-1} C_t^T [C_t \Sigma_{\eta, t|t-1} C_t^T + R_t]^{-1} \quad (19)$$

(: Kalman Gain)

$$\Sigma_{\eta, t+1|t} = \Sigma_{\eta, t|t} \quad (20)$$

$$\Sigma_{\eta, t|t} = \Sigma_{\eta, t|t-1} - K_t C_t \Sigma_{\eta, t|t-1} \quad (21)$$

$$\text{Init. Cond.:} \quad \begin{cases} \hat{\eta}_{L,0|-1} = \bar{\eta}_{L,0} \\ \Sigma_{\eta,0|-1} = \Sigma_{\eta,0} \end{cases}$$

2.2 Updating by the Constraints

The constraint conditions are applied to update DD-based state vector estimation as follows. Namely, when we have obtained the filtering estimates $\hat{\eta}_{t|t}$ and the error covariance matrix $\Sigma_{\eta, t|t}$, we apply the true position of the reference receiver as

$$\tilde{u}_1 = u_1 + v_{u_1}, \quad (22)$$

where u_1 is the position of the reference antenna such that we almost know the position $\tilde{u}_1$ for u_1 . We assume that v_{u_1} is independent of v_t and a Gaussian white noise caused by the phase center variations of antennas with

$$v_{u_1} \sim N(0, R_{u_1}). \quad (23)$$

Therefore we can apply the useful formula in [18]. Thus we can consider the following relations of the conditional probability density function (CPDF):

$$\begin{aligned} p(\eta_t | Y^t, \tilde{u}_1) &= \frac{p(\eta_t, Y^t, \tilde{u}_1)}{p(Y^t, \tilde{u}_1)} = \frac{p(\eta_t, \tilde{u}_1 | Y^t) p(Y^t)}{p(Y^t, \tilde{u}_1)} \\ &= \frac{p(\tilde{u}_1 | \eta_t, Y^t) p(\eta_t | Y^t) p(Y^t)}{p(Y^t, \tilde{u}_1)} \\ &= K_0(Y^t, \tilde{u}_1) p(\eta_t | Y^t) p(\tilde{u}_1 | \eta_t) \end{aligned} \quad (24)$$

where

$$K_0(Y^t, \tilde{u}_1) \equiv \frac{p(Y^t)}{p(Y^t, \tilde{u}_1)}. \quad (25)$$

Then we have relations:

$$\begin{aligned} p(\eta_t | Y^t) &= \frac{1}{(2\pi)^{n'/2} |\Sigma_{\eta, t|t}|^{1/2}} \\ &\times \exp \left\{ -\frac{1}{2} [\eta_t - \hat{\eta}_{t|t}]^T \Sigma_{\eta, t|t}^{-1} [\eta_t - \hat{\eta}_{t|t}] \right\} \end{aligned} \quad (26)$$

$$\begin{aligned} p(\tilde{u}_1 | \eta_t) &= \frac{1}{(2\pi)^{3/2} |R_{u_1}|^{1/2}} \\ &\times \exp \left\{ -\frac{1}{2} (\tilde{u}_1 - u_1)^T R_{u_1}^{-1} (\tilde{u}_1 - u_1) \right\} \end{aligned} \quad (27)$$

where $n' \equiv n_s + 5$. Therefore, $p(\eta_t | Y^t, \tilde{u}_1)$ is expressed as follows.

$$\begin{aligned} p(\eta_t | Y^t, \tilde{u}_1) &= K_0(Y^t, \tilde{u}_1) \frac{1}{(2\pi)^{n'/2} |\Sigma_{\eta, t|t}|^{1/2}} \\ &\times \exp \left\{ -\frac{1}{2} [\eta_t - \hat{\eta}_{t|t}]^T \Sigma_{\eta, t|t}^{-1} [\eta_t - \hat{\eta}_{t|t}] \right\} \\ &\times \frac{1}{(2\pi)^{3/2} |R_{u_1}|^{1/2}} \\ &\times \exp \left\{ -\frac{1}{2} (\tilde{u}_1 - u_1)^T R_{u_1}^{-1} (\tilde{u}_1 - u_1) \right\} \end{aligned} \quad (28)$$

Then we have the following quadratic form for the power term of the Ep.(28):

$$\begin{aligned} &\frac{1}{2} (\eta - \hat{\eta})^T \Sigma_{\eta}^{-1} (\eta - \hat{\eta}) + \frac{1}{2} \left\{ (\tilde{u}_1 - u_1)^T R_{u_1}^{-1} (\tilde{u}_1 - u_1) \right\} \\ &= \frac{1}{2} \left\{ \eta^T \Sigma_{\eta}^{-1} \eta - \eta^T \Sigma_{\eta}^{-1} \hat{\eta} - \hat{\eta}^T \Sigma_{\eta}^{-1} \eta + \hat{\eta}^T \Sigma_{\eta}^{-1} \hat{\eta} \right. \\ &\quad \left. + \tilde{u}_1^T R_{u_1}^{-1} \tilde{u}_1 - c_{u_1}^T \eta - \eta^T c_{u_1} + \eta^T M_{u_1} \eta \right\} \\ &= \frac{1}{2} \left\{ \eta^T (\Sigma_{\eta}^{-1} + M_{u_1}) \eta - \eta^T (\Sigma_{\eta}^{-1} \hat{\eta} + c_{u_1}) \right. \\ &\quad \left. - (c_{u_1} + \Sigma_{\eta}^{-1} \hat{\eta})^T \eta + \hat{\eta}^T \Sigma_{\eta}^{-1} \hat{\eta} + \tilde{u}_1^T R_{u_1}^{-1} \tilde{u}_1 \right\} \\ &= \frac{1}{2} \left\{ [\eta - (\Sigma_{\eta}^{-1} + M_{u_1})^{-1} (\Sigma_{\eta}^{-1} \hat{\eta} + c_{u_1})]^T (\Sigma_{\eta}^{-1} + M_{u_1}) \right. \\ &\quad \times [\eta - (\Sigma_{\eta}^{-1} + M_{u_1})^{-1} (\Sigma_{\eta}^{-1} \hat{\eta} + c_{u_1})] \\ &\quad \left. - (\Sigma_{\eta}^{-1} \hat{\eta} + c_{u_1})^T (\Sigma_{\eta}^{-1} + M_{u_1})^{-1} (\Sigma_{\eta}^{-1} \hat{\eta} + c_{u_1}) \right. \\ &\quad \left. + \hat{\eta}^T \Sigma_{\eta}^{-1} \hat{\eta} + \tilde{u}_1^T R_{u_1}^{-1} \tilde{u}_1 \right\} \end{aligned} \quad (29)$$

where M_{u_1} and c_{u_1} are given as follows,

$$c_{u_1} \equiv \begin{bmatrix} R_{u_1}^{-1} \tilde{u}_1 \\ 0_{n_s+2} \end{bmatrix}, \quad (30)$$

$$M_{u_1} \equiv \begin{bmatrix} R_{u_1}^{-1} & O_{3 \times (n_s+2)} \\ O_{(n_s+2) \times 3} & O_{(n_s+2) \times (n_s+2)} \end{bmatrix} \quad (31)$$

Then the update estimated vector $\check{\eta}$ and error covariance matrix $\check{\Sigma}_{\eta}$ of $\check{\eta}$ based on the minimum mean square estimate are given by

$$\check{\eta} = (\Sigma_{\eta}^{-1} + M_{u_1})^{-1} (\Sigma_{\eta}^{-1} \hat{\eta} + c_{u_1}), \quad (32)$$

$$\check{\Sigma}_{\eta} = (\Sigma_{\eta}^{-1} + M_{u_1})^{-1}. \quad (33)$$

3 EXPERIMENTAL RESULTS

In the experiments, two relative positionings of the conventional method [19], and the proposed method of updating by true positions were performed for static relative positioning, and the positioning accuracy was compared. The GPS data (see Table 1) were collected and provided by the GEONET (GNSS Earth Observation Network System) of Geospatial Information Authority of Japan (GSI).

Table 1: Experimental Conditions

Date	September 15, 2017
GPS-Time	03:00:00 ~ 04:00:00 11:00:00 ~ 12:00:00
Reference station: Name Receiver Antenna	HAMAOKA 1 TOPCON NETG3 TPSCR.G5 GSI
Rover station: Name Receiver Antenna	HAMAOKA 2 TRIMBLE NETR9 TRM59800.80 GSI
Baseline length	2.8813[km]
Epoch interval	$\Delta N = 30$ [s]
Elevation angel mask	15 [deg.]
Measurement Data	C/A code, L1 carrier-phase
Ionospheric delay estimation	No
Tropospheric delay estimation	No
Ambiguity Resolution	Lambda method

In the experiment, we calculated HAMAOKA 2 as the rover position by applying true position (HAMAOKA 1) as the reference position during 03:00:00 ~ 04:00:00 and 11:00:00 ~ 12:00:00 on Sept.15, 2017. The results show the Eastward-Northward errors of estimated positions (see Fig. 1 and Fig. 3) and upper errors (see Fig. 2 and Fig. 4) of rover position . Table 2 and Table 3 show RMS errors of positioning in each direction.

Fig. 1: EN Errors (03:00:00~04:00:00)

Fig. 2: Upper Positioning Errors (03:00:00~04:00:00)

Table 2: RMS errors (03:00:00~04:00:00)

	East [cm]	North [cm]	Up [cm]
New method	0.387	0.841	3.421
Conventional method	0.676	0.593	2.785

Fig. 3: EN Errors (11:00:00~12:00:00)

Fig. 4: Upper Positioning Errors (11:00:00~12:00:00)

Table 3: RMS errors (11:00:00~12:00:00)

	East [cm]	North [cm]	Up [cm]
New method	0.339	0.455	2.331
Conventional method	0.658	0.233	2.041

From Figs. 1 and 3, by applying the new method, we have the positioning accuracy less than 1.0 cm E-N directional errors. It should be remarked that the unit of the vertical coordinate is centi-meter. The difference of results by both methods is very small on the horizontal E-N plane. In terms of upper position errors, differences between two method are less than 1 cm; (see Fig. 2 and Fig. 4).

4 CONCLUSIONS

We have presented the algorithm of relative positioning based on alternative use of the positions of reference receivers in short baseline. The positioning error terms of the observable mathematical models are cancelled by DD techniques, and the satellite positions (uncanceled terms) are obtained from navigation messages in consideration of the estimated errors.

The experiments were conducted for short baseline (baseline length 2.8813 [km]) with negligible effects of ionospheric/tropospheric delay. Proposed method of relative positioning gives the positioning accuracy in several cm level as with the conventional method. In the future, we will verify the performance in long baseline (baseline length over 10 [km]) and the kinematic environment.

REFERENCES

- [1] S. Sugimoto and Y. Kubo: GNSS Regressive Models and Precise Point Positioning, *Proc. of the 36th ISCIE Int. Symp. on Stochastic Systems Theory and Its Applications*, Saitama, pp. 47-48, Nov. (2004).
- [2] S. Sugimoto and Y. Kubo: Carrier-Phase-Based Precise Point Positioning - a Novel Approach Based on GNSS Regression Models, *Proc. of 2004 Int. Symp. on GPS/GNSS*, Sydney, Dec. (2004).
- [3] Y. Kubo, A. Kitao, S. Fujita, and S. Sugimoto: A New RTK Algorithm for Carrier-Phase-Based Precise Point Positioning Based on GNSS Regression Model, *Proc. ION-GNSS 2005*, pp. 1492-1499, Long Beach, Sept. (2005).
- [4] S. Sugimoto and Y. Kubo: Unified Methods of Point and Relative Positioning Based on GNSS Regression Equations, *Proc. 19th Int. Tech. Meeting of the Satellite Division of The Institute of Navigation (ION GNSS 2006)*, pp. 345-358, Fort Worth, Texas, Sep. (2006).
- [5] S. Sugimoto, Y. Kubo, S. Fujita and T. Kazuno: A New GNSS Regressive Positioning Algorithm Based on GR Models, *Proc. of the 38th ISCIE Int. Symp. on Stochastic Systems Theory and Its Applications*, Suwa, Nagano, Japan, (Nov. 2006).
- [6] S. Sugimoto: GNSS Positioning Algorithms based on GR Models, *Section 7.3.1 in GPS Handbook (in Japanese)* ed. by S. Sugimoto and R. Shibasaki, Asakura-Shoten, Tokyo, Japan (2010).
- [7] S. Sugimoto, Y. Suzuki, Y. Karatsu, M. Ozaki, A. Mouri, Y. Kubo: VPPP Algorithms with Multiple Antennas and their Applications, *Proc. of the 27th Int. Tech. Meeting of The Satellite Division of the Institute of Navigation (ION GNSS+ 2014)*, pp. 1073-1084, Tampa, Florida, Sept. (2014)
- [8] B. Hofmann-Wellenhof, H. Lichtenegger and J. Collins: *GPS; Theory and Practice*, 5th revised edition, Springer-Verlag (2001).
- [9] Y. Gao, and X. Shen: A New Method for Carrier-Phase-Based Precise Point Positioning, *Journal of the Institute of Navigation*, Vol 49, No. 2, pp. 109-116 (Summer 2002).
- [10] A. Mouri, Y. Karatsu, S. Sugimoto, Y. Kubo, M. Ohashi and G. Okuda: New PPP/VPPP Algorithms by using Multiple Antennas, *Trans. of The Institute of Systems, Control and Information Engineers*, in Vol. 29, No. 12 pp. 525 - 535 (Dec. 2016).
- [11] A. Mouri, G. Okuda, Y. Kubo and S. Sugimoto: Novel VPPP Algorithms with Multiple Antennas and Attitude Estimation, *Proc. of the 30th Int. Tech. Meeting of The Satellite Division of the Institute of Navigation (ION GNSS+ 2017)*, pp. 474 - 491, Monterey, California, February (2017).
- [12] G. Okuda, A. Mouri, H. Hasegawa, Y. Arakawa, Y. Kubo and S. Sugimoto: VPPP Algorithms with Multiple Antennas and Highly Accurate Attitude Estimation by Ambiguity Resolution Methods, *Proc. of the 30th Int. Tech. Meeting of The Satellite Division of the Institute of Navigation (ION GNSS+ 2017)*, (will appear), Portland, Oregon, Sept. (2017).
- [13] P. J. G. Teunissen and A. Kleusberg (Eds.): *GPS for Geodesy, 2nd Edition*, Springer-Verlag, New York (1998).
- [14] A. Leick: *GPS Satellite Surveying, 3rd Edition*, John Wiley & Sons, New York (2004).
- [15] P. Misra and P. Enge: *Global Positioning System - Signals, Measurements, and Performance*, Ganga-Jamuna Press, Massachusetts (2001).
- [16] Y. Gao and Z. Z. Lin: Precise Ionosphere Modeling Using Regional GPS Network Data, *J. of Global Positioning Systems*, Vol. 1, No. 1, pp. 18-24 (2002).
- [17] Peter J. G. Teunissen: A-PPP: Array-Aided Precise Point Positioning with Global Navigation Satellite Systems, *IEEE Trans. on Signal Processing*, Vol. 60, No. 6, pp. 1-12, June (2012).
- [18] S. Sugimoto: Useful Formulas of the Kalman Filter for Measurement noises with Uncorrelated Sub-elements, Archives in HP of SAPT(Society for the Advancement of Positioning Technology): <http://jsapt.net/ja/>. (Oct. 2017).
- [19] S. Sugimoto and R. Shibasaki (Eds.): *GPS Handbook (in Japanese)*, Asakura-Shoten, Tokyo (2010)