On Design of Homogeneous Feedback Controllers for Finite-Time Stabilization of Stochastic Systems

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Abstract

In this paper, we investigate the design of controllers for the finite-time stabilization of stochastic systems. In the stochastic finite-time stabilization using controllers designed by previous design methods, the input signals often exhibit the chattering-like behavior. This paper presents another method for designing controllers in stochastic finite-time stabilization problems to avoid the chattering-like behavior. Moreover, we discuss the cause of the chattering-like behavior in the finite-time stabilization through numerical results.

1 Introduction

This paper deals with designing controllers in the finite-time stabilization problem of stochastic systems. In particular, the proposed approach aims to avoid the chattering-like behavior in input signals.

The finite-time stability and stabilization have been dealt with by many authors in the nonlinear control community. The finite-time stability means that a system is stable and exhibits the finite-time convergence. The analysis of the finite-time stability of deterministic systems has been shown in [1] (see also [2, 3, 4]). The analysis provides a foundation of the finite-time stabilization. Those results enable the synthesis of the finite-time stable control systems, which are shown in [5, 6, 4, 7, 8].

Stochastic systems also can exhibit the finite-time stability. For stochastic systems, the study [9] investigates the analysis of the finite-time stability. Following the analysis, the studies [10, 11, 12, 13] deal with the finite-time stabilization problem. Those studies show the stabilization of systems such as triangular systems and are based on homogeneity or backstepping-like techniques. Those studies develop the design methods as extensions of those for deterministic systems. However, when the

design methods for deterministic systems are merely extended to stochastic systems, due to the stochastic noise, undesired behavior appears.

As to stochastic systems, the input signals often show the chattering-like behavior in the finite-time stabilization. This is caused by merely applying design methods developed for deterministic systems to stochastic systems. We note that the non-Lipschitz continuity is a necessary condition of the finite-time stability. Therefore, finite-time stabilizing controllers are given by non-Lipschitz continuous functions, and the non-Lipschitz continuity brings the non-smoothness structures into closed-loop systems. Then, the non-Lipschitz continuity and the stochastic noise cause the chattering-like behavior. Therefore, we need to develop another design to avoid the chattering-like behavior under the existence of the stochastic noises.

This paper shows a design method of finite-time stabilizing controllers such that the chattering-like behavior of input signals can be avoided. Previous studies heavily depend on the backstepping-like techniques in the design of the finite-time stabilizing feedback controllers [10, 13]. The backstepping-like techniques are difficult to avoid the structure of the closed-loop systems exhibiting the chattering-like behavior. Instead of the backstepping-like techniques, this paper employs a homogeneity-based method. The design in this paper is based on the results in [14].

The rest of this paper is organized as follows. The following section introduces a motivating example that shows the chattering-like behavior. Then, we review the stochastic finite-time stability and the homogeneity. The following section gives the problem statements of this study, and we show a design of finite-time stabilizing controllers. We subsequently show a numerical example of avoiding chattering-like behavior.

Notations. We use the following notations. $\mathbb{R}$ is the set of real numbers and $\mathbb{R}^n$ is the n -dimensional Euclidean space. With the signum function $\text{sgn} : \mathbb{R} \rightarrow \mathbb{R}$ which takes the values $\text{sgn}(x) = 1, -1, 0$ for $x > 0$, $x < 0$, and $x = 0$, respectively, we define the function

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$\text{sig}(x)^\alpha$ as $\text{sig}(x)^\alpha = \text{sgn}(x)|x|^\alpha$ with $\alpha \in \mathbb{R}$. Moreover, $\mathbb{R}_+ = (0, \infty)$. We denote the sample space, the σ -field, and the probability measure as Ω , $\mathcal{F}$, and $\mathbb{P}$, respectively. With the filtration of $\mathcal{F}$, denoted as $\{\mathcal{F}_t\}_{t \geq 0}$, we assume that the filtered probability space $\{\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geq 0}, \mathbb{P}\}$ satisfies the following conditions; the probability space $\{\Omega, \mathcal{F}, \mathbb{P}\}$ is complete, $\{\mathcal{F}_t\}_{t \geq 0}$ is right continuous with respect to t , and $\mathcal{F}_0$ includes all the $\mathbb{P}$ -null sets of $\mathcal{F}$.

2 Motivating Example

We here provide an example motivating to avoid the chattering-like behavior in the stochastic finite-time stabilization.

The following example is mathematical, however, the example gives a motivation of this study, which is to avoid the chattering-like behavior of input signals. The example is given by

$$\begin{aligned} dx_1 &= x_2 dt + 0.1 \text{sig}(x_1)^{7/8} dw, \\ dx_2 &= u dt, \end{aligned} \quad (1)$$

where $x = (x_1, x_2) \in \mathbb{R}^2$ is the state, $u \in \mathbb{R}$ is the input, and w is a one-dimensional standard Wiener process. For the system (1), we apply the feedback controller given as

$$u = k_1(x) := -3 \text{sig} \left(\text{sig}(x_1)^{9/4} + \text{sig}(x_2)^3 \right)^{2/9}, \quad (2)$$

which is designed by using the results in [6] with minor modifications. The feedback controller (2) guarantees the finite-time stability in probability (1) (see Definition 2 below).

Fig. 1 shows a sample path of $x(t)$ of (1) with (2). Fig. 1 indicates that $x(t)$ converges to $x = 0$ in finite time. Moreover, Fig. 2 shows a sample path of the input $u(t)$. It also shows the finite-time convergence to zero. However, the input signal exhibits the chattering-like behavior. In actual control problems, this will cause difficulties in the implementation of the finite-time stabilizing controllers. Therefore, we deal with a design of finite-time stabilizing controllers for avoiding the chattering-like behavior of the input signals.

3 Mathematical Preliminaries

3.1 Finite-time Stability of Stochastic Systems

This subsection reviews the finite-time stability of stochastic systems.

Given the state variable $x \in \mathbb{R}^n$, the coefficient functions $f : \mathbb{R}^n \rightarrow \mathbb{R}^n$ and $\sigma : \mathbb{R}^n \rightarrow \mathbb{R}^{n \times d}$, and a d -dimensional standard Wiener process w , we consider the system

$$dx = f(x)dt + \sigma(x)dw, \quad x(0) = x_0 \in \mathbb{R}^n, \quad (3)$$

Fig. 1: Sample path of state $x(t)$ of system (1) with (2)

Fig. 2: Sample path of input $u(t)$ of system (1) with (2)

where x_0 is the initial value of the state. Throughout this subsection, we consider the finite-time stability of (3). Moreover, throughout this paper, we introduce the following assumptions; $f(x)$ and $\sigma(x)$ are continuous in x , and $f(0) = 0$ and $\sigma(0) = 0$. Because we do not necessarily require the Lipschitz continuity of the coefficients $f(x)$ and $\sigma(x)$ of the system (3), the solutions of (3) are defined as the weak solutions (for a detailed discussion of the definitions of solutions, see [15, Ch. 5]). In the following, the solution of (3) is denoted as $x(t)$.

In the following, we discuss the stability of stochastic systems as in [16, 17].

Definition 1 (Stability in probability, [16, 17]). The origin $x = 0$ of (3) is stable in probability if the following holds; for any $\epsilon > 0$,

$$\lim_{x_0 \rightarrow 0} \mathbb{P} \left\{ \sup_{t \geq 0} \|x(t)\| > \epsilon \right\} = 0.$$

Then, we introduce the finite-time stability of stochastic systems as follows [9].

Definition 2 (Finite-time stability in probability, [9]). The origin $x = 0$ of (3) is finite-time stable in probability if it is stable in probability and for a stopping time τ_0 defined as

$$\tau_0 = \inf \{t \in (0, \infty) \mid \|x(t)\| = 0\},$$

it holds that for any $x_0 \neq 0$,

$$\mathbb{P} \{\tau_0 < \infty\} = 1.$$

The Lyapunov approach for finite-time stability has been established in [9]. For a twice continuously differentiable function $V : \mathbb{R}^n \rightarrow \mathbb{R}$, we introduce an operator given as

$$\mathcal{L}V(x) = \frac{\partial V}{\partial x}(x)f(x) + \frac{1}{2} \text{Tr} \left(\sigma(x)^T \frac{\partial}{\partial x} \left(\frac{\partial V}{\partial x}(x) \right)^T \sigma(x) \right).$$

The operator $\mathcal{L}$ is called the infinitesimal generator of the system (3). Then, the following theorem allows us to investigate the finite-time stability of stochastic systems with Lyapunov functions, and this becomes a basis in the stochastic finite-time stabilization.

Theorem 1 ([9]). Let a function $V : \mathbb{R}^n \rightarrow \mathbb{R}$ be a twice continuously differentiable on $\mathbb{R}^n$. Moreover, assume that $V(x)$ is radially unbounded and positive definite on $\mathbb{R}^n$. For the system (3), if the inequality

$$\mathcal{L}V(x) \leq -cV(x)^\gamma \quad (4)$$

holds for some $c > 0$ and $\gamma \in (0, 1)$, then the origin of (3) is finite-time stable in probability.

3.2 Homogeneity

This subsection recalls the homogeneity of functions and vector fields (see [18, Chapter 5] for the details of the homogeneity). The homogeneity is employed in design methods of the finite-time stabilizing controllers in many studies [5, 6, 7].

We begin with the definition of the dilation mapping.

Definition 3 (Dilation, [18]). Given parameters $r = (r_1, \dots, r_n) \in \mathbb{R}_+^n$ and $\lambda > 0$, the mapping $\Delta_\lambda^r : \mathbb{R}^n \rightarrow \mathbb{R}^n$ defined as $\Delta_\lambda^r(x) = (\lambda^{r_1}x_1, \dots, \lambda^{r_n}x_n)$ is called the dilation mapping.

Then, with the dilation, we define the homogeneous functions and homogeneous vector fields.

Definition 4 (Homogeneous functions, [18]). Given parameters $r = (r_1, \dots, r_n) \in \mathbb{R}_+^n$, $\lambda > 0$, and a function $V : \mathbb{R}^n \rightarrow \mathbb{R}$, we call $V(x)$ a homogeneous function of degree m with respect to the dilation Δ_λ^r if the function V satisfies that $V(\Delta_\lambda^r(x)) = \lambda^m V(x)$.

Definition 5 (Homogeneous vector fields, [18]). Given parameters $r = (r_1, \dots, r_n) \in \mathbb{R}_+^n$, $\lambda > 0$, and a vector field $f : \mathbb{R}^n \rightarrow \mathbb{R}^n$, we call $f(x)$ a homogeneous vector field of degree k with respect to a dilation Δ_λ^r if each i -th element $f_i(x)$ of the vector field $f(x)$ satisfies that $f_i(\Delta_\lambda^r(x)) = \lambda^{r_i+k} f_i(x)$.

For the later use, we define the homogeneous norm as follows.

Definition 6. Given $r = (r_1, \dots, r_n)$ and sufficiently large $p > 0$, we call $\|\cdot\|_{\{r,p\}} : \mathbb{R}^n \rightarrow \mathbb{R}$ the homogeneous norm if the mapping is given by $\|x\|_{\{r,p\}} = (\sum_{i=1}^n |x_i|^{p/r_i})^{1/p}$.

Then, we recall results related to the homogeneity.

Proposition 1 ([14]). Given two homogeneous functions $V_1, V_2 : \mathbb{R}^n \rightarrow \mathbb{R}$ with respect to a dilation Δ_λ^r of degree m_1 and m_2 , respectively, suppose that the function V_1 is positive definite. Then,

$$k_1 V_1(x)^{m_2/m_1} \leq V_2(x) \leq k_2 V_1(x)^{m_2/m_1},$$

where

$$k_1 = \min_{\{x \in \mathbb{R}^n | V_1(x)=1\}} V_2(x),$$

$$k_2 = \max_{\{x \in \mathbb{R}^n | V_1(x)=1\}} V_2(x).$$

Proposition 2 ([11]). Given a twice continuously differentiable function $V : \mathbb{R}^n \rightarrow \mathbb{R}$ and a vector field $f : \mathbb{R}^n \rightarrow \mathbb{R}^n$, suppose that the function $V(x)$ is homogeneous of degree m with respect to a dilation Δ_λ^r and that the vector field $f(x)$ is homogeneous of degree k with respect to the dilation Δ_λ^r . Then, the following two functions are homogeneous of degree m and $m+2k$, respectively, with respect to the dilation Δ_λ^r ;

$$\frac{\partial V}{\partial x}(x)f(x), \quad f(x)^T \frac{\partial}{\partial x} \left(\frac{\partial V}{\partial x}(x) \right)^T f(x).$$

4 Problem Statements: Finite-Time Stabilization of Stochastic Systems

This section gives the problem statements of this study. We deal with the finite-time stabilization of an n -th order integrator-like system.

Throughout this paper, given the state $x = (x_1, x_2, \dots, x_n) \in \mathbb{R}^n$ and the input $u \in \mathbb{R}$, we consider the stabilization of the n -th order integrator-like system given as

$$\begin{pmatrix} dx_1 \\ dx_2 \\ \vdots \\ dx_n \end{pmatrix} = \begin{pmatrix} x_2 \\ x_3 \\ \vdots \\ u \end{pmatrix} dt + \Sigma(x)dw, \quad (5)$$

where w is a one-dimensional standard Wiener process, and $\Sigma : \mathbb{R}^n \rightarrow \mathbb{R}^n$. We assume that the coefficient $\Sigma(x)$ is continuous in x and $\Sigma(0) = 0$.

The problem in this paper is primarily the finite-time stabilization of (5). That is, we deal with designing feedback controllers $u = k(x)$ such that the origin of (5) with the feedback controller $u = k(x)$ becomes finite-time stable in probability. In addition, we develop a design of finite-time stabilizing controllers to avoid the chattering-like behavior in the input signals, which we have seen in Section 2. Because previous design methods are based on the backstepping-like techniques and those methods causes the chattering-like behaviors, we show another approach to designing feedback controllers for the finite-time stabilization.

5 Main Results: Design of Finite-Time Stabilizing Controllers

This section describes a design of finite-time stabilizing controllers. The design is developed to avoid the

chattering-like behavior in the input signals. The design is an extension of that for the finite-time stabilization of deterministic systems with the homogeneity [14], which differs from the backstepping-like techniques.

In the following theorem, we denote the set S_1 as

$$S_1 = \{x \in \mathbb{R}^n \mid \|x\|_{\{r,p\}} = 1\},$$

where the value of $r = (r_1, \dots, r_n)$ is determined by a given dilation Δ_λ^r and $p > 0$. The following result gives feedback controllers for the finite-time stabilization of stochastic systems, which is based on those for deterministic systems in [14].

Theorem 2. Consider the system (5) and a feedback controller given in the form of

$$u = \kappa(x) = -k_1 \text{sig}(x_1)^{\alpha_1} \cdots -k_n \text{sig}(x_n)^{\alpha_n}, \quad (6)$$

where the parameters α_i satisfy

$$\alpha_{i-1} = \frac{\alpha_i \alpha_{i+1}}{2\alpha_{i+1} - \alpha_i}$$

for $i = 2, \dots, n$, $\alpha_n = \alpha$, and $\alpha_{n+1} = 1$ for some α . Further, assume that the vector field $\Sigma(x)$ is a homogeneous vector field of degree $k < 0$ with respect to the dilation Δ_λ^r with

$$r = \left(\frac{1}{\alpha_1}, \dots, \frac{1}{\alpha_n}\right) \quad (7)$$

for sufficiently small $|k|$ and satisfies that

$$\|\Sigma(x)\| < \delta, \quad x \in S_1 \quad (8)$$

for sufficiently small $\delta > 0$. Then, there exist the parameters α and k_i ($i = 1, \dots, n$) such that the origin of the system (5) with the feedback controller (6) is finite-time stable in probability.

Proof. To show the finite-time stability, we first consider the deterministic system

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_n \end{pmatrix} = \bar{f}(x) := \begin{pmatrix} x_2 \\ x_3 \\ \vdots \\ \kappa(x) \end{pmatrix}, \quad (9)$$

where $\kappa(x)$ is given by (6). The system (9) mathematically corresponds to the system (5) with the controller (6) and $\Sigma(x) \equiv 0$. Then, according to Proposition 8.1 of [14], there exists $\epsilon > 0$ such that the origin of (9) is finite-time stable with $\alpha \in (1 - \epsilon, 1)$. Moreover, $\bar{f}(x)$ of (9) is a homogeneous vector field of degree

$$\bar{k} = \frac{\alpha - 1}{\alpha} \quad (10)$$

with respect to the dilation Δ_λ^r with the value of r given in (7). Then, because of the homogeneity and the asymptotic stability of the system (9), according to

Theorem 2 of [18], there exists a function $\bar{V} : \mathbb{R}^n \rightarrow \mathbb{R}$ that is radially unbounded, positive definite, continuously differentiable of order p on $\mathbb{R}^n$ for some p , and homogeneous with respect to the dilation Δ_λ^r and satisfies

$$\dot{\bar{V}}(x) := \frac{\partial \bar{V}}{\partial x}(x) \bar{f}(x) < 0 \text{ for } \forall x \neq 0. \quad (11)$$

Because the function $\bar{V}(x)$ and the vector field $\bar{f}(x)$ are a homogeneous function and a homogeneous vector field, respectively, with respect to the dilation Δ_λ^r , according to Proposition 2, the function $\dot{\bar{V}}$ is also a homogeneous function with respect to the dilation Δ_λ^r .

Then, the closed-loop system (5) with the controller (6) is expressed as

$$dx = \bar{f}(x)dt + \Sigma(x)dw, \quad (12)$$

where $\bar{f}(x)$ is given in (9). When the value $|k|$ of the homogeneous degree $k < 0$ of the vector field $\Sigma(x)$ with respect to the dilation Δ_λ^r is sufficiently small, we can choose the value of α in (10) so that $k = \bar{k}/2$ holds. Then, with $k = \bar{k}/2$, (12) is a stochastic homogeneous system of degree $\bar{k}$ with respect to the dilation Δ_λ^r . Then, for the function $\bar{V}(x)$, the infinitesimal generator of (12) results in

$$\mathcal{L}\bar{V}(x) = \frac{\partial \bar{V}}{\partial x}(x) \bar{f}(x) + \frac{1}{2} \Sigma(x)^T \frac{\partial}{\partial x} \left(\frac{\partial \bar{V}}{\partial x}(x) \right)^T \Sigma(x).$$

Then, with $\dot{\bar{V}}(x)$ in (11), $\mathcal{L}\bar{V}(x)$ is rewritten as

$$\mathcal{L}\bar{V}(x) = \dot{\bar{V}}(x) + \frac{1}{2} \Sigma(x)^T \frac{\partial}{\partial x} \left(\frac{\partial \bar{V}}{\partial x}(x) \right)^T \Sigma(x). \quad (13)$$

Then, with Proposition 2, the first and second terms on the right-hand side of (13) are both homogeneous functions of the same homogeneous degrees with respect to the dilation Δ_λ^r . Since the function $\dot{\bar{V}}(x)$ is negative definite, we consider the constant $\bar{\delta} > 0$ defined as

$$\bar{\delta} = \min_{x \in S_1} \left\{ -\dot{\bar{V}}(x) \right\}. \quad (14)$$

Then, with $\bar{\delta}$ of (14) and δ in the condition (8), for $x \in S_1$,

$$\mathcal{L}\bar{V}(x) \leq -\bar{\delta} + \frac{1}{2} \delta^2 \max_{x \in S_1} \left\{ \left\| \frac{\partial}{\partial x} \left(\frac{\partial \bar{V}}{\partial x}(x) \right)^T \right\| \right\} \quad (15)$$

holds. Then, the inequality (15) implies that for the sufficiently small value of δ ,

$$\mathcal{L}\bar{V}(x) < 0 \quad x \in S_1 \quad (16)$$

holds. Then, with the inequality (16) and the homogeneity of the function $\mathcal{L}\bar{V}(x)$, it is concluded that the function $\mathcal{L}\bar{V}(x)$ is negative definite by using Proposition 1. Moreover, Proposition 1 and the homogeneity

of $\mathcal{L}\bar{V}(x)$ guarantee the existence of constants $c > 0$ and $\gamma \in (0, 1)$ such that

$$\mathcal{L}\bar{V}(x) \leq -c\bar{V}(x)^\gamma \quad (17)$$

holds. Finally, the inequality (17) implies the finite-time stability in probability of the closed-loop system (12). $\square$

Remark 1. The proof of Theorem 2 is based on the converse homogeneous Lyapunov theorem of [18]. Therefore, it does not provide the estimates of the values of δ in (8) and of k , which is the homogeneous degree of $\Sigma(x)$. In the numerical example shown in the next section, we provide the explicit form of the Lyapunov function $\bar{V}(x)$ for an example system and use the Lyapunov function to guarantee the stability.

6 Numerical Example

We show an example of the finite-time stabilization of stochastic systems with the design discussed in Section 5.

In this section, we consider the continuation of the stabilization of the system (1) in Section 2. Based on the results in Section 5, we consider the feedback controller given by

$$u = k_2(x) := -k_1 \text{sgn}(x_1)|x_1|^{1/2} - k_2 \text{sgn}(x_2)|x_2|^{2/3}. \quad (18)$$

The feedback is obtained by setting $\alpha = 2/3$ for (6). Moreover, with the value of r given by

$$r = (1/\alpha_1, 1/\alpha_2) = (2, 3/2),$$

the vector field $\Sigma(x)$ of the system (3) is a homogeneous vector field of degree $-1/4$ with respect to the dilation Δ_λ^r . Then, we consider the function given by

$$\bar{V}(x) = \frac{7}{4}|x_1|^3 + \frac{1}{4}x_2^4 + x_2 \text{sig}(x_1)^{\frac{9}{4}}. \quad (19)$$

We note that $\bar{V}(x)$ of (19) is radially unbounded, positive definite, and twice continuously differentiable on $\mathbb{R}^n$. The function $\bar{V}(x)$ of (19) is a Lyapunov function indicating the finite-time stability of the closed-loop system. Indeed, a straightforward calculation shows that the infinitesimal generator of the closed-loop system (1) with the feedback controller (18) is negative definite on $\mathbb{R}^2$. In addition, by using the homogeneity, we can show the existence of $c > 0$ and $\gamma \in (0, 1)$ such that the condition (4) of the finite-time stability holds.

Figs. 3 and 4 show the sample paths of the state and the input, respectively, of the system (1) with the finite-time stabilizing controller (18) using the values $k_1 = 3$ and $k_2 = 3$, and the initial value of the state x is given as $x(0) = (1, 1)$. In Fig. 3, $x(t)$ reaches $x = 0$ in finite time. Moreover, the input signal $u(t)$ does not show the chattering-like behavior, which is different from the behavior of the controller (2) shown in Fig. 2. Therefore, the proposed design achieves the stabilization without the chattering-like behavior in the input signals.

Fig. 3: Sample path of state $x(t)$ of system (1) with (18)

Fig. 4: Sample path of input $u(t)$ of system (1) with (18)

7 Discussions

Based on the results in Sections 5 and 6, this section discusses the difference between the finite-time stabilizing controller in Section 5 and the controllers designed by the backstepping-like methods, for example, the controller (2) in Section 2, in the view of the behavior of the input signals.

We examine the origin of the chattering-like behavior, which we have seen in Section 2. The feedback controller (2) is designed so that the state variable x converges to the set given as

$$S = \left\{ x \in \mathbb{R}^2 \mid x_2 = -\text{sgn}(x_1)|x_1|^{3/4} \right\}. \quad (20)$$

This can be shown in the plot of the function $k_1(x)$ of (2) (Fig. 5). In Fig. 5, the function $k_1(x)$ shows abrupt changes in the neighborhood of the set S and this leads to the rapid convergence to the set S . In other words, the set S plays similar roles to the sliding surfaces in the sliding-mode control. However, the state variable $x(t)$ crosses the set S frequently because of the stochastic noise and this causes the chattering-like behavior. On the other hand, we consider the function $k_2(x)$ of (18) which also gives controller (1), which guarantees the finite-time stability. Fig. 6 shows the plot of the function $k_2(x)$. The function $k_2(x)$ does not show abrupt changes unlike the function $k_1(x)$ shown in Fig. 5. This leads to avoiding the chattering-like behavior of the input signal with the finite-time stabilizing controller (18).

8 Conclusions

In this paper, we showed an approach for the finite-time stabilization of stochastic systems. In particular, we discussed the chattering-like behaviors in the input signals of finite-time stabilizing controllers designed by

Fig. 5: Plot of function $k_1(x)$

Fig. 6: Plot of function $k_2(x)$

backstepping-like methods and we considered another approach for designing the finite-time stabilizing controller for stochastic systems. The controllers shown in this study has a different form from the controller obtained by the backstepping-like methods. Through the numerical examples, we discussed that the controller obtained in this study could avoid the chattering-like behavior in the input signals.

We will deal with a constructive design of the finite-time stabilizing feedback controllers, which are shown in Theorem 2, in future works.

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