Proposal of strength equation for gusset plates
subjected to compressive force in steel truss bridge

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Gusset plates of a joint in a steel truss bridge are the key parts of the structure system, because they are connecting several main truss members. However, the detailed failure mode of gusset plates subjected to compression has not been well examined yet. Therefore, this paper discusses the failure mechanism of the gusset plates of a joint subjected to compressive force through the diagonal member. The failure mode is examined by both numerical and experimental method. And It is clearly shown that the failure mode demonstrated by these two methods are quite consistent with each other. Then, a strength equation for the gusset plates subjected to compression is finally created on the basis of the failure mode.

**Key Words**: steel truss bridge, gusset plate, joint, failure mode

1. INTRODUCTION

Generally, a steel truss bridge is said to be less redundant than other types of bridges, because it is basically regarded as a statically determinate structure. Especially, a joint of a truss bridge is considered to be a structurally important part of a truss structure system, because it combines several main truss members. As represented by the collapse of I-35W steel truss bridge in United States in 2007, the failure of a joint in a steel truss bridge is likely to lead to the collapse of the entire bridge.

In this study, the authors study the behavior of gusset plates of a joint subjected to compressive force through a diagonal member. The mechanical behavior of gusset plates subjected to compression is much more complex than that of gusset plates subjected to tension. Referring to the existing research and report, the authors focused on the failure mode (deformation and displacement) of U10 joint in I-35W bridge. The purpose of this study is to clarify the failure mode of gusset plates subjected to compressive force through a diagonal member by conducting a loading test of a joint cut out from an old steel truss bridge in Japan with a view to the results of non-linear FEM analyses. In addition, a strength equation for the gusset plates subjected to compression is proposed.

2. FAILURE MECHANISM OF U10 NODE IN I-35W BRIDGE

2.1 Failure mode of U10 joint in I-35W bridge in U.S.

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The gusset plates of U10 joint which failure led to the entire collapse of I-35W bridge is shown in Fig. 1. The fracture originally occurred in the part of the gusset plate subjected to compressive force from the diagonal. In more detail, the shear failure is observed along the side of riveted area (Line A in Fig.1) and buckling-like mode can be seen in the compression area of the gusset plate (Line B in Fig.1.).

### 2.2 FEM analysis
In order to investigate the failure mechanism of U10 joint in I-35W bridge, a material and geometrical nonlinear FEM analysis was conducted. The FEM model for this analysis is shown in Fig.2. 3D FEM model of U10 joint was built with 4 node shell elements and it was attached into the global I-35W bridge model consisting of beam elements. The loading conditions and the boundary conditions at the time of collapse were applied to the global I-35W bridge model according to NTSB’s (National Transportation Safety Board) investigation reports12. The Young’s modulus of steel is 2.0×10^5 N/mm^2 and the stress-plastic strain relationship in nonlinear stage is shown in Fig.3. Then, von-Mises yield condition, associated flow rule and isotropic hardening rule were employed for the non-liner analysis.

Loading from the both sides of approach deck (280tf and 500tf) and 260t of construction materials and vehicles loaded upward of the U10 panel point is included in the loading conditions6. And boundary conditions are shown in Fig.2.

### 2.3 Failure mode of U10 node
The authors focused on the failure mode of U10 joint in I-35W bridge at the time of the collapse demonstrated by the FEM analysis. This mode is illustrated in Fig. 4. It is expected that the failure of U10 joint occurred by the following four behaviors in conjunction:

(a) The sway buckling occurs in the compressive parts of the gusset plates,
(b) The diagonal sways in the direction of out-of-plane of the gusset plate and moves into the axial direction of the member.
(c) The free edges of the gusset plates are curved.
(d) The shear yielding occurs along both sides of the riveted area.

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![Failure mode of U10 node in I-35W](image1.png)

**Fig.1** Failure mode of U10 node in I-35W

![Stress-Plastic strain relationship](image2.png)

**Fig.3** Stress-Plastic strain relationship

![Analysis model](image3.png)

**Fig.2** Analysis model
3. LOADING TEST

3.1 Specimen cut out from steel truss bridge

To examine the actual ultimate behavior of a joint subjected to compressive force, a loading test was conducted\(^\text{10}\). A joint was cut out from a demolished steel truss bridge shown in Fig. 5. The steel truss bridge called Choshi Bridge which was built in 1962 across Tone River. It was 5-span steel through truss bridge with total length of 407.4m. The average amount of a daily traffic is about 20,000 vehicles including 10% of heavy trucks. Although repainting, strengthening and partial replacement of severely corroded members were conducted several times through its service life, it was finally replaced in 2009 at 47 years old, because the corrosion was unlikely to stop and it was considered to be impossible to maintain the safety of the bridge.

Fig. 6 shows the test specimen which was cut out from the upper chord joint (P25d) near the intermediate support P14. The sections of the two diagonals are both square box type. The steel grade is SM40 (400MPa nominal tensile strength).

3.2 Corrosion depth measurement

Section loss on the outer and inner surface of the specimen was measured by using a laser surface-measurement equipment\(^\text{11}\). Fig. 7 shows the measurement equipment and contours of corrosion states of the gusset plates. Red color area is representing deep section loss, and yellow color, non-corrosion areas. Although severe sections loss was not seen except for the rivets areas, large loss of section was observed on the inner surface, where humidity seems high and airborne salt is likely to accumulate. The average corrosion depth on the inner surface of the gusset plate on one side is 3.0mm, and that on the other side is 2.6mm. Therefore the average remaining thickness of the two gusset plates are 9.0mm and 9.4mm respectively, whereas the original design thickness of these gusset plates were both 12mm.

3.3 Test setup

Fig. 8 shows the outline of the specimen and loading frame. A 30MN loading machine was used for the compression loading and a loading frame with jacks was set for the tension loading to demonstrate the bi-axial loading. The photo of the experimental setup is shown in Fig. 9. The compressive and tensile loads were applied to the diagonal members at the same load increment step, because the absolute values of the design axial forces of both diagonals are almost equal. However, due to the restriction of capacity of tension jack, tensile load was fixed when it achieved 2000kN.
Fig. 5 General view of Choshi bridge

Fig. 6 Test specimen (P25d)

(a) Outside gusset plate  
(b) Inside gusset plate

Fig. 7 Thickness reduction of corroded specimen

Fig. 8 Outline of specimen and loading frame

Fig. 9 Test setup of P25d joint
3.4 Result of loading test

The result of the loading test showed exactly the same behavior of the gusset plates as the simulation of FEM analysis in Chapter 2, although the boundary condition of the specimen is different from that in the actual truss bridge. The failure mode of the specimen after the loading test is observed in **Fig.10**. The compressive areas of the gusset plates, which are underneath the diagonal, are buckling and the free edges of the gusset plates are curved. Then, **Fig.11 (a)** shows the out-of-plane displacement of the gusset plates and **Fig.11 (b)** indicates that the diagonal slightly slides into the out-of-plane direction. Thus, these mechanical behaviors are quite identical with the failure mechanism simulated by the FEM analysis on U10 joint of I-35W bridge. Specifically they are mentioned in (a), (b), and (c) in the section 2 of this paper. However, the shear fracture along both sides of the riveted area, which is mentioned in (d), didn’t occur even after the load-displacement curve fell down to some extent.

4. FEM ANALYSIS

4.1 Modeling and conditions

In order to obtain the numerical data on the mechanical behavior demonstrated by the loading test, a material and geometrical nonlinear FEM analysis was carried out. As shown in **Fig.12**, the tested joint model was built by using 4 nodes shell elements. The rivets are modeled by spring elements. A rivet was assumed as a column with height \( h = 12 \text{mm} \) and diameter \( D = 11 \text{mm} \), and spring constant of axial direction, and shear direction and rotation were determined by \( EA/h \), \( GA/h \), and \( EI/h \) respectively. Here, \( E \) is the elastic modulus of steel, \( G \) is the elastic shear modulus, and \( A \) is the section area of a rivet. The two loading points are formed with rigid beam elements at the ends of both compression and tension diagonal. To adopt the conditions of the corroded gusset plates into the FEM model, the thickness of the shell elements are varied for the each section of the gusset plates on the basis of the contours of corrosion states in **Fig.7**. The stress-strain curve of the steel which was obtained by the material test (**Fig.13**) and von-Mises yield condition, associated flow rule and isotropic hardening rule were adapted for the non-linear FEM analysis. Then, compressive load and tensile load were applied on the loading point A and B respectively so that compressive load was kept around 2000kN.

![Fig.10 Failed specimen after the loading test](image1)

![Fig.11 Deflected mode of compressive area](image2)
4.2 Result of FEM analysis

The FEM analysis well simulated the ultimate behavior of the gusset plates which was actually demonstrated by the loading test. Fig.14 shows the deformation shape of the FEM model when the load is maximum. Local buckling is observed in the compressive parts of the both gusset plates, and bowing of the free edges can be seen clearly. And, as shown in Fig.15, the peak of load-displacement (at loading point in axial direction) curve was almost agreed with the result of the loading test. The difference of initial stiffness between experimental result and analysis result is possibly caused by the initial imperfection of the specimen which was cut out of the over 40 years old bridge. Then, in order to examine the shear stress level along the path in Fig.16, the numerical data of the shear stress was extracted from the shell elements which are located near the riveted points and in the middle of the two riveted points along the critical path. The maximum value out of the 4 gauss integral points in a shell element was representatively selected. Fig.17 shows the shear stress level along the critical path when the load-displacement curve reached the peak. Most of the points are between the yield shear stress level $\tau_y (\sigma_y \sqrt{3})$ and the ultimate shear stress level $\tau_u (\sigma_u \sqrt{3})$. 

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**Fig.12** P25d FEM model

**Fig.13** Stress-Strain curve of structural steel

**Fig.14** Deformation shape and strain contour of FEM Model

**Fig.15** Load-Displacement curve

**Fig.16** Position No. of critical path for shear

**Fig.17** Shear stress level along the critical path
5. STRENGTH EQUATION

5.1 Proposal of strength equation

The existing researches and reports\(^{36}\) mentions the shear fracture of gusset plates in compression. However, in the guidelines of FHWA\(^{78}\), which is one of the most influential standard for steel truss bridges, the strength of the gusset plates in compression is evaluated by the Whitmore buckling of compressive area. The evaluation of shear strength of gusset plates is separated from the evaluation of buckling strength of gusset plates in compression. From the fact the shear fracture of U10 gusset occurred in the both side of the riveted area of gusset plates in compression, the strength of the gusset plates in compression should be evaluated as the combination of shear strength and buckling strength. Therefore, according to the experimental and the analytical verifications, a strength equation for the gusset plates subjected to compressive force in diagonal is proposed as follows;

\[
R = R_c + \frac{1}{\sqrt{3}} f_y A_{gy}
\]  

(1)

Here, \(R_c\) is the buckling strength over the compressive area represented by blue line in Fig.19. And \(R_c\) is calculated by using the column buckling theory with the effective column length \(l\). Then, \(f_y\) is the yield strength of the steel, and \(A_{gy}\) is the gross section area of the critical path represented by red line in Fig.18. Therefore the strength \(R\) is summation of buckling strength over the compressive area and the shear yield strength of the critical path, which is based on the idea of a strength equation of the block shear failure\(^{90}\).

5.2 Strength Calculation

In order to confirm the accuracy of the strength \(R\) obtained from the proposed equation (Eq.1), the calculation of the strength \(R\) for the gusset plates in Fig.18 is performed below;

Firstly, \(R_c\) is calculated by theoretical Euler’s buckling strength equation with an effective column length \(l\) (Here, \(\beta\) is the coefficient of effective buckling length and is set as 1.2 because the buckling mode is sway mode as shown in Fig.11 (b)). Therefore, the buckling strength of the compressive area \(R_c\) is

\[
R_c = \frac{\pi^2 E l}{l^2} \\
= \pi^2 \times 2.1 \times 10^5 \times 270 \times 9.0^3 / 12 \\
= \frac{\pi^2 \times 2.1 \times 10^5 \times 270 \times 9.4^3 / 12}{(1.2 \times 213)^2} \\
= 520663 + 592875 \\
= 1113(kN)
\]

However, if the slenderness ratio \(\lambda = \frac{\sigma_y}{\pi \sqrt{E/r}} \leq 1.0\)

, then \(R_e = \sigma_y A_s\), where \(r\) is the radius of gyration of area, \(\sigma_y\) is the yield strength of the steel, and \(A_s\) is the section area of the compressive area.

Here, the average thicknesses of the two gusset plates are 9.0mm and 9.4mm respectively. Then, the shear strength of the critical paths is represented by the second term of Eq.1;

\[
\frac{1}{\sqrt{3}} f_y A_{gy} = \frac{1}{\sqrt{3}} \times 255 \times (9.0 \times 814 + 9.4 \times 814) = 2205(kN)
\]

Hence, the strength \(R\) is

\[
R = R_c + \frac{1}{\sqrt{3}} f_y A_{gy} = 1113 + 2205 = 3318(kN)
\]

This calculated strength \(R\) is consistent to the experimental result(3598kN). The comparison of the FEM result, the experimental result, Whitmore buckling strength, and calculated strength \(R\) is listed in the table-1.

![Effective column length; \(l = \beta l_1 + l_2 + l_3\) (mm)]

Fig.18 Compressive area and critical path

| Table 1 Comparison of the FEM result, the experimental result, Whitmore buckling strength and calculated strength \(R\) (kN) |
|----------------|----------------|-----------------|---|
| FEM result   | Experimental result | Whitmore buckling (FHWA) | \(R\) |
| 3690        | 3598            | 1819            | 3318 |
5.3 Parametric analyses

In order to confirm the accuracy of the proposed strength equation, the parametric analyses was performed by changing the thickness of the gusset plates of the joints P25d and P72d. The analytical maximum load and the strength R estimated by the proposed strength equation were listed in Table-2 and Table-3 for comparison. And Fig. 19 indicates that the proposed strength equation provides reasonably conservative value R against the result of FEM analyses, because the strain hardening is not taken into account for the proposed strength equation.

Table 2 Comparison between Maximum Load in FEM Analyses and R Calculated by Proposed Strength Equation (P25d)

<table>
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<th>Thicknesses of Gusset Plates</th>
<th>Maximum Load (kN)</th>
<th>R (kN)</th>
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<td>4529</td>
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<tr>
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Table 3 Comparison between Maximum Load in FEM Analyses and R Calculated by Proposed Strength Equation (P72d)

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<th>Thicknesses of Gusset Plates</th>
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<th>R (kN)</th>
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<td>2144</td>
</tr>
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6. CONCLUDING REMARKS

(1) The failure mode of the gusset plates in I-35W bridge simulated by FEM analysis was also demonstrated by the loading test of the gusset plates subjected to compressive force through the diagonal except for the fracture of the riveted area. The failure mode of the gusset plates is given as follows:

(a) Sway buckling of the compressive area of the gusset plates
(b) Out-of-plane sway and axial direction movements of the diagonal member.
(c) Curved free edges of the gusset plates
(d) Shear yielding of the gusset plates along the both sides of the riveted area.

(2) The shear stress levels in the critical path of the gusset plates are exceeding the shear yield stress level when the load-displacement curve reached its peak, which is consistent with the theory of the proposed equation.

(3) A strength equation for gusset plates subjected to compressive force was proposed on the basis of failure mechanism demonstrated by FEM analysis and the loading test. And the proposed strength equation can estimate the strength of the gusset plates on slightly conservative side in comparison with the FEM analyses results. This is because the effect of strain hardening of the steel is not considered in the proposed strength equation, whereas the non-liner stress-strain relationship of the steel is applied for the FEM analyses.
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References


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