H. NAGAOKA:—THE RIGIDITY OF THE EARTH AND THE VELOCITY OF SEISMIC WAVES.
(Read April 29, 1905).

The opinions of physicists as to the mean rigidity of the earth are widely different from those of geologists. The smallness of the elastic tide due to the action of sun and moon has been put forward as a reason indicating the great rigidity of earth. According to Lord Kelvin,\(^1\) the earth is more rigid than steel in order to keep up the present shape. Since the discovery of the variation of latitude and the investigation of its period, the question can be approached from another standpoint. In addition to this, the recent development in seismic measurements renders it possible to peep into the inaccessible subterranean abyss, by analysing the elastic vibrations arising from shocks of earthquakes.

The ordinary theory of gyrostats teaches us that the period of small oscillation of a rigid ellipsoid of rotation with moments of inertia \(A, A, C\) is \(\frac{C}{C-A}\times\) period of rotation.\(^2\) For the earth considered as a rigid body, the said Eulerian period amounts to about 10 months, while the investigations by Chandler\(^3\) indicates the existence of 14 months period in the variation of latitude. By the researches of Newcomb,\(^4\) it is now beyond dispute that the prolongation of the period is to be attributed to the elastic behaviour of the earth.

The same question was further examined by Hough,\(^5\) who investigated the elastic deformation of an homogeneous incompressible gravitating spheroid. He concludes that the Eulerian period is prolonged by \(\frac{\varepsilon_2}{\varepsilon_1}\), where

\[
\varepsilon_1 = \frac{5}{4} \frac{\omega^2 R}{g}
\]

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1) Lord Kelvin, Mathematical and Physical Papers, 3.
\[ \varepsilon = \frac{15}{38} \frac{\omega^2 R^2}{E} \]

\( \rho \) denoting the density, \( \omega \) the angular velocity, \( R \) the mean radius, \( E \) the Young's modulus of the substance composing the spheroid. The prolongation amounts to

\[ \frac{\varepsilon}{\varepsilon_1} = \frac{6}{19} g R \frac{\rho}{E} \]

Consequently

\[ 1 + \frac{\varepsilon}{\varepsilon_1} = 1 + \frac{6}{19} g R \frac{\rho}{E} = \text{Chandler's Period} \times \frac{\text{Eulerian Period}}{} \]

Putting the Eulerian period = 304 days and that of Chandler = 428 days, \( g = 981 \text{ cm} \text{ sec}^{-2} \), \( R = \frac{2}{\pi} \times 10^6 \text{ cm} \), we obtain

\[ \frac{\varepsilon}{\varepsilon_1} = 0.408 \]

and the ratio of the mean elastic constant to density

\[ \frac{E}{\rho} = 48.3 \times 10^{10} \text{ cm}^2 \text{ sec}^{-2} \]

Assuming the density \( \rho = 5.5 \), we find \( E = 2.6 \times 10^{12} \text{ C.G.S. units} \), which exceeds that for steel.

Since the elastic behaviour of that portion of the earth, which lies far from the polar axis is effective in causing the prolongation of the Eulerian period, the ratio of elastic constant to density above deduced would correspond to that of the portion lying near the surface and more in the equatorial region than near the poles. If we suppose the earth composed of numerous stata of different densities and elastic constants, and when we take into consideration that these strata are often subject to faults, it is evident that the velocity of longitudinal waves travelling in such a stratum should be found from the formula

\[ V_t = \sqrt{\frac{E}{\rho}} \]

instead of \( \sqrt{\frac{k+\frac{4}{3}n}{\rho}} \) (using Lord Kelvin's notation). Following this hypothesis, the value of \( \frac{E}{\rho} \) found above gives at once as the mean velocity of propagation of longitudinal wave

\[ V_t = 7.0 \text{ km/sec} \]
without introducing any new assumption as to the value of mean density $\rho$.

From the examination of earthquake diagrams, seismologists have found the maximum velocity to be about $14 \frac{\text{km}}{\text{sec}}$, while the slowest velocity may lie under a km., so that $7 \frac{\text{km}}{\text{sec}}$ is about the mean of the observed velocities. Thus the values of mean $V_t$ deduced from Chandler’s period and from seismograms are nearly equal to one another. The remarkable coincidence of these two values deduced from two apparently different phenomena gives further evidence in support of the theory as regards the rigidity of the earth, propounded by Hopkins and Lord Kelvin. This close connection between the problems of astronomy and of seismology will open a new field of research.

As to the velocity of the transversal waves, we have, on introducing the condition of incompressibility, which would be nearly satisfied in the interior of the earth’s crust, and which underlies the hypothesis in Hough’s calculation, $V_t = \sqrt{\frac{\mu}{\rho}} = \sqrt{\frac{E}{3\rho}} = 4.0 \frac{\text{km}}{\text{sec}}$. Singularly enough, the velocity of $7 \frac{\text{km}}{\text{sec}}$ corresponds to that of second preliminary tremor and that of $4 \frac{\text{km}}{\text{sec}}$ are often present in the seismograms, but these are probably mere chance coincidences.

It must not however be too hastily concluded that Chandler’s period ought to have some connection with the seismic activity. The recurrence of the pole to the same meridian is a simple characteristic of the periodic motion; if the seismic activity be of such a magnitude as to be noticeable in the motion of the pole, it will be traced in the variation of the amplitude in the motion of the pole. The period, if there is any, is therefore that of the amplitude variation, and not of the prolonged Eulerian motion.

On different grounds, we have reason to believe that the principal vibrations, which appear prominent in seismograms of different earthquakes, are due to surface waves travelling with a velocity of about $3.3 \frac{\text{km}}{\text{sec}}$. The velocity of such waves on the surface of an incompres-
sible elastic solid is according to Lord Rayleigh\(^1\) \(0.96 \sqrt{\frac{\mu}{\rho}}\). The value above calculated for \(\sqrt{\frac{\mu}{\rho}}\) is little above that usually observed, but when we take into consideration that the portion of the earth lying inside the crust is accounted for in the calculation of \(\frac{E}{\rho}\) from latitude variation, while the mean value of \(\frac{\mu}{\rho}\) of the crust appears on seismograms, the deviation between these values will favour the view that the interior is more rigid than the outside portion.

The complex nature of the variation of latitude as revealed by recent observations undertaken by the international geodetic association will perhaps find explanation by the complex elastic nature of the earth combined with the variation of surface traction, such as the variation of barometric pressure, the ocean current, the tidal friction and the like. By extending Hough’s investigations to a spheroid composed of shells of different density and elasticity, and by careful comparison of seismograms of distant earthquakes extending over a considerable length of time, we may aspire after a more accurate solution of the problem as regards the internal condition of the earth, if there are means of discriminating and eliminating the subsidiary effects attending the motion of the pole.