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ON THE CHANGE OF ELASTIC CONSTANTS OF FERROMAGNETIC SUBSTANCES BY MAGNETIZATION.

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The elastic constants of a substance in a magnetic field is properly defined as the ratio of the strain applied and the stress thereby caused, the magnetic force constantly acting on it. The change of elasticity is the difference of the values when the magnetic field is on and when it is off. In the case of ferromagnetic substances for which the hysteresis is considerable, the change of elastic constants thus defined may or may not coincide with that when the stress is first applied and then the field. Little attention, however, seems to have hitherto been paid on this point. In many of the experiments hitherto made, the proper order of applying the stress and the magnetic field is inverted. In some experiments, the change of elasticity was deduced from the effect of tension on the magnetic elongation, with the assumption that the effect is due to the change of elasticity by magnetization. The acoustical method used by several experimenters is


also an indirect one. In H. Rensing's experiment, \(^1\) Kundt's tube was utilized; but the field in which the bar was placed was far from being uniform.

Our present experiments were undertaken with the main object of measuring the change of elastic constants of the same specimen by the direct as well as the indirect method and comparing the results thus obtained. Specimen to be tested was placed in nearly uniform field.

Specimens tested in our experiments were nickel, Swedish iron, tungsten steel and nickel steels of different percentages. They were generally tested in form of wires. For the change of rigidity, the rod was also used in the case of nickel and iron.

If \(\xi\) be the strain of a given type, \(X\) the external stress applied, and \(H\) the magnetic field, we have the relation

\[
\frac{\partial}{\partial H} \left( \frac{\partial \xi}{\partial X} \right) = \frac{\partial}{\partial X} \left( \frac{\partial \xi}{\partial H} \right)
\]  

(1) provided the change is independent of the order of applying the field and the stress. Thence we have

\[
\left( \frac{\partial \xi}{\partial X} \right)_n - \left( \frac{\partial \xi}{\partial X} \right)_0 = \frac{\partial (\xi_n - \xi_0)}{\partial H}
\]  

(2)

If \(K\) be the elastic constant for the same type of the stress and strain,

\[
\frac{\partial \xi}{\partial X} = \frac{c}{K}
\]

where \(c\) is a constant depending on the dimensions of the specimen.

Hence

\[
\frac{1}{K_n} - \frac{1}{K_0} = \frac{1}{c} \frac{\partial (\xi_n - \xi_0)}{\partial X}
\]

\[
= \frac{\frac{\partial (\xi_n - \xi_0)}{\partial X}}{K_0 \frac{\partial \xi_0}{\partial X}}
\]

\[
\therefore \frac{\delta K}{K_n} = \frac{\frac{\partial (\xi_n - \xi_0)}{\partial X}}{\frac{\partial \xi_0}{\partial K}}
\]  

(3)

If \(X\) be the tension and \(\xi\) the elongation, \(K\) corresponds to the modulus of elasticity. The change of elasticity may, therefore, be

\[^1\] Rensing, Ann. der Phys. 14, 363, 1904.
calculated from the effect of tension on the magnetic elongation, if the
relation (1) or (2) holds. Again, if $X$ be the twisting couple and $\xi$
the twist, $K$ corresponds to the modulus of rigidity and its change
may be calculated from the change of twist caused by magnetization,
provided (1) or (2) is true.

The validity of (3) depends on the relation (1) which is usually
employed in the theory of magnetostriction. It may, therefore, be a
matter of interests to test experimentally how far the relation actually
holds.

Our present investigation was carried out in the following order:—

I. Change of Elasticity by Magnetization.

Specimens were tested in the form of thin wires, well annealed.
Since the elongation due to the change of elasticity is a small fraction
of the total elongation, it was desirable to devise some differential
method. For this purpose, the elastic elongation, when the field is off,
was compensated by that of a nonmagnetic wire and the differential
elongation due to magnetization was observed.

The magnetic wire to be tested and the compensating copper wire,
hang side by side in the vertical magnetizing coil. To each end of
the wires was brazed a thick brass rod. The upper rods hang sepa-
rately on two horizontal brass beams resting on pairs of knife edges
whose distances could be adjusted at will. To the lower rod for the
copper wire, a light carriage for a mirror system, was clamped on.
Two fine spiral springs of german silver attached to the sides of the
carrier, supported horizontally the axis of the light mirror with a
suitable counterpoise. The lower rod for the magnetic wire carried
a flexible band made of a bundle of very fine copper wires, which
pressed the axis of the mirror lightly upon the plane vertical side of
the carriage. To the lower end of the band and that of the other rod,
a flexible silk cord was attached and a pulley hanging on this cord was
pulled vertically downward by desired tension which was transmitted
through a system of frictionless pulleys, in order to avoid an injurious
shock accompanying the loading and unloading. When the resultant
depressions of the lower rods due to the elongation of wires and the
bending of the upper beams caused by a given tension, are exactly
equal, there is no rotation of the axis of the mirror. If, however, the
compensation be disturbed in any way, the rotation may be observed by means of a telescope and a vertical scale in the usual manner. The sensibility of the arrangement was considerably increased by using a long wire and a long scale distance. The scale was graduated on a grinded glass and illuminated from its back by a mantled gas flame. The magnetizing coil was waterjacketed to prevent any heating effect.

The dimensions of the compensating copper wire was so chosen as to produce nearly equal elongation with that of the specimen. The final compensation, however, was always made by properly adjusting the distance of two knife edges supporting the horizontal beams, from which the wire were suspended. A certain tension is applied and removed, and the knife edges were adjusted until the addition and removal produced no rotation of the mirror. Since the elastic elongation is not exactly the linear function of the tension, we were obliged to readjust the compensation for each different initial tension and additional weight.

The present arrangement also enabled us to measure the magnetic change of length of the specimens under constant tensions, by applying different magnetic fields and observing the deflections of the mirror due to the elongation or contraction. From the results thus obtained, the change of elasticity may be calculated on the assumption of the relation (3). The change thus calculated was compared with that obtained by our direct method.

Since the hysteresis effect of tension on the length change was found to be considerable, the tension was varied cyclically between zero and its maximum value, before beginning the experiment, to remove any initial effect.

Our usual procedure in the experiments was as follows. The change of length was first observed. The wire was first loaded with the smallest weight, which was generally 1 kg; the demagnetization was then carefully effected. A series of gradually increasing current was passed through the magnetizing coil and corresponding deflections of the mirror was observed. The wire was then completely demagnetized. Another kilogram of weight was added, demagnetized again, and similar processes were repeated.

Next the change of elasticity was measured. We applied an initial load, demagnetized the wire, added and removed a weight of 1 kg and the compensation was adjusted as good as possible. Small deflection due to the additional weight for zero field was recorded. Then,
the magnetic field was applied and the deflection caused by the additional weight was observed. After complete demagnetization, the deflection due to the same weight for zero field was again tested. The difference between the deflections for no field and for a certain field, gives the change of elasticity by magnetization, provided we know the total elongation of the magnetic wire due to the same additional weight, or in other words, if we know the modulus of elasticity.

To measure the modulus of elasticity of the specimens tested, a special arrangement was devised. The wire was stretched vertically by a suitable initial tension. Two pivoted axes for two light mirrors were brought in contact with the wire near its upper and lower ends respectively. A vertical scale was placed in front of the upper mirror, and its image formed by successive reflexion by the two mirrors was observed by a telescope in front of the lower mirror. The initial weight was cyclically varied from 1 to 7 kg, and the elongations due to the increment of 100 gr. were observed for each initial weight. We found the variation of the modulus of elasticity with respect to the initial load considerable for nickel but small for steel.

In the calculation of the change of elasticity by magnetization, the values of the modulus of elasticity for zero field was taken from the results of these experiments corresponding to different initial tensions.

In a previous paper by Messrs S. Shimizu, S. Kusakabe, and one of us, the change of elasticity of magnetic bars determined by the method of flexure are given. It is interesting to compare these results with those obtained by the present direct method. Iron and steel wires in our experiment were drawn from the same specimen as in the previous experiment; but unfortunately, nickel specimens in these two experiments were quite different from each other, the rod in the previous experiment being magnetically very hard.

II. Change of Rigidity by Magnetization.

(a) One of our methods consisted in giving torsional oscillation to the wires magnetized in different fields and calculating the corresponding values of the modulus of rigidity from the periods of oscillation. The same wires used in the above experiments were tested. The wire was hung vertically in the uniform field of the
magnetizing coil. Its upper end was brazed to a rigid brass rod which was clamped to the frame above. To the lower end, a similar rod was brazed, to which the oscillating weights were fixed. In order to sufficiently diminish the disturbance due to the resistance of the air, the Foucault current, etc., the period of oscillation was made very long by using the weight with considerable moment of inertia. The weights were a rectangular brass bar and several lead weights, proper combination of which enabled us to adjust the period to convenient values for different tensions. A light mirror was fixed to the lower rod. The image of a horizontal scale placed in front of the arrangement was observed by a telescope in the usual manner. The amplitude of oscillation was adjusted so that it was nearly the same throughout the experiments.

The wire was first demagnetized by reversals, the zero of the scale set to the position of equilibrium, a field applied, and then started to oscillation. Ten and fifteen sets of the time required for fifty complete vibrations were taken, from which the mean value was extracted. We may consider the result thus obtained accurate to \( \frac{1}{1000} \) of a second. The process was repeated for different fields and tensions. The wire was demagnetized each time before beginning the new sets.

The magnetizing coil was waterjacketed. Draught was screened off. Current remained nearly constant for each sets of observations. Effect of the Foucault current was found to be negligibly small, by oscillating with a copper wire in sufficiently strong fields.

(b) The same specimens, on the other hand, were tested by the differential method used by Barus, in which the wire was first twisted and then magnetized. Compensating wire in our experiment was copper one, whose total twist due to a given couple was nearly equal to that of the magnetic wire in zero field. The results obtained by this method was compared with those obtained by the oscillation method.

(c) In a paper by Messrs S. Shimizu, S. Kusakabe and one of us, the change of rigidity of ferromagnetic bars are given, which was determined by first applying the torsion and then the magnetic field. To investigate whether the results are the same or not, if the order of applying the torsion and the field is reversed, the similar arrangement as that used in the experiment above cited were availed of. The essential parts of the arrangement remained unchanged, except
for the mirror system. The rotating cylinder, to which the mirror was fixed, was horizontally supported by means of two weak spiral springs attached to the sides of a carriage similar to that used in our experiment for the change of elasticity. To the carriage, was rigidly fixed a horizontal axis, the conical ends of which fitted to the agate cups on the arm of a Y-shaped bar. This bar could be adjusted to the desired position, such that the axis of the mirror was pressed by the plane side of the carriage perpendicularly on a point of the circumference of the torsion wheel rigidly fixed to the magnetic bar. To adjust the pressure properly, a sliding weight was put on a pin protruding above the carriage. Two vertical scales were erected in front of the mirrors in the same vertical line; the one about one meter above the other. An initial couple was applied and the mirror was so adjusted that the image of the lower scale was observed by the telescope. A suitable weight was then chosen for giving an additional couple, which twisted the rod to such an amount that by the consequent rotation of the mirror, the image of the upper scale just appeared in the field of the telescope. The readings of the two scales corresponding to the removal and addition of the additional weight, were taken. Next, a field was applied and the procedure was repeated. The difference of the deflections for zero field and a certain field measures the change of rigidity corresponding to that field. The specimens were the same as in the previous experiment above quoted, and their modulii of rigidity were known.

On the other hand, ordinary method of first applying the couple and then the field, was also made, and the result was compared with those obtained by the direct method.

III. Summary of the Result of Experiments.

The followings are the brief summary of the result obtained.

1. Nickel.

The change of elastic constants is remarkably large, amounting to about 15% in the case of elasticity and 7% in the case of rigidity. The elastic constants first decreases, passes a minimum and then increases with the field. The minimum for the elasticity is more distinct than that of the rigidity. The decrease and increase of the
modulus of elasticity obtained by indirect method is generally greater than those obtained by the direct one. For the rigidity, the changes by indirect method is generally greater algebraically than those by the direct one, for small tension; but for large tension, the relation of the two values resembles to that for the change of elasticity.

The change of elasticity obtained by the flexure method is very different from that by the direct method. But it is to be remarked that the course of the curve of the change of elasticity to field resembles very much to that for the change of rigidity in low tension obtained by Barus's method.

2. Swedish Iron and Tungsten Steel.

Elastic constants generally increase by magnetization. The change is extremely small, scarcely amounting to 0.5% at most. The values obtained by the indirect method are several times greater than those obtained by the direct one. It is interesting to notice that the change of elasticity and that of rigidity almost coincide with each other not only in their general aspects, but also in numerical values.

In its general aspects, the change of elasticity obtained by the flexure method resembles to that obtained by the direct method, but is much greater in amount.

3. 28.74% Nickel Steel.

Elastic constants increase by magnetization in a small amount. Results for the direct and indirect method nearly coincide with each other for a moderate tension.

4. 50.72% and 70.32% Nickel Steels.

Elastic constants generally increase by magnetization except for a very low field, where they show a slight decrease for some tensions. They increase rapidly with the field and soon approach to asymptotic values which are remarkably large for both specimens, almost approaching to the amount of change for nickel. The indirect results are generally greater than the direct one; but the difference becomes less as the tension is increased. For a tension of 3 or 4 kg, it nearly vanishes. The change of rigidity seems to be generally greater than that of the elasticity.

The effect of annealing is to increase remarkably the amount of changes.

From the results above enumerated, it is evident that except for nickel steels under high tension, the relation (3) given in the earlier
part of the paper does not hold even approximately, so that the equation (1) can not be freely used in any quantitative discussion.

There are many analogous cases in the problem of magnetostriction. It is well known that the change of magnetization by pulling a magnetized wire does not agree with the change deduced from the results obtained by magnetizing the wire stretched by different constant tensions. Similar phenomena are also observed in the change of magnetism caused by the twist. Next, we may cite the case of magnetization of a wire traversed by an electric current. The change of magnetization by the longitudinal current is considerably greater, in the case of magnetizing the wire traversed by the current, than in the case of passing the current through the magnetized wire). Again take the case of Wiedemann effect. It is well known that in iron and nickel, the twist produced by magnetizing the wire traversed by an electric current is greater or less than the twist caused by passing the current through magnetized wire according to the strength of the field. The difference is remarkable; in some cases the former is several times greater than the latter. In nickel-steels), the difference is, however, very small. All these phenomena might perhaps arise from the hysteresis effect of magnetization; i.e. the final states attained by a magnetic substance are different for different order of applying the fields, or the field and the stress.

In conclusion, it may be remarked that, since the change of the elastic constants by magnetization is not so small as is generally believed, in any theory of magnetostriction aiming at the quantitative agreement between the theory and experiments, these changes must necessarily be taken into account. This makes the development of the theory very difficult. In addition to this, an equality such as

\[ \frac{\partial^2 Q}{\partial x \partial y} = \frac{\partial^2 Q}{\partial y \partial x} \]

in which Q is a quantity which depends upon two apparently independent variables x and y, can not be used in the theory of magnetism without experimental verification.

Finally, we may add following remarks in passing. If we consider X in the equation (1) as the temperature t, and \( \xi \) the thermal expansion, we have

where $a_H$ and $a_0$ are the coefficient of thermal expansion in the field $H$ and zero respectively, and $(l_H - l_0)/l_0$ the magnetic elongation at the field $H$. Thus, the change of the coefficient of thermal expansion by magnetization is equal to the temperature coefficient of the magnetic elongation, if the relation (1) holds in this case. The values of $\partial(l_H - l_0)/l_0 \partial t$ may be deduced from the experiments by Mr. S. Shimizu and one of us. They are the changes of $a$ by magnetization, when the temperature is first raised and then the field applied. However, any experiment for the determination of the change of $a$ by magnetization, in which the thermal expansion is directly observed in magnetic fields, must be welcomed as affording a counterpart for the analogous comparison.

Again, if we put the temperature $t$ for $H$ in (1), the tension $T$ for $X$, and the length of the specimen for $\xi$, we obtain an equation between the effect of tension upon the coefficient of thermal expansion and the effect of temperature upon the modulus of elasticity, as already given by Dahlander. In this case, the equation was found to be nearly satisfied. In problems which does not relate to magnetism, the hysteresis effect is generally small, so that the agreement might well have been expected.