Studies on Physico-Chemical Properties of Marine Products and Related Substances—V. Heat Conductivity of Fresh Fish Meats

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Introduction

The study of heat conduction in fish meats is of practical importance in connection with the heat sterilization of canned fish. Although many investigations have been made of this problem both theoretically and experimentally, almost all existing theories neglect the change in physico-chemical properties of fish muscle caused upon heating¹⁾. In a previous paper²⁾, FUJITA presented an approximate theory for dealing with the conduction of heat in fish meat packed in a flat can under the assumption that heating of a fresh fish muscle gives rise to a change in its heat conductivity, and applied it to a hypothetical case in which the heat conductivity increases with temperature following a sigmoidal curve. This sigmoidal temperature dependence of the heat conduction was presumed on the basis of the fact that fish meats usually coagulate thermally in the range 45 to 70°C. and maintain distinctly different states below and above this temperature region.

The purpose of the present paper is to describe an approximate method for determining the temperature dependence of the heat conductivity of a given fish meat from data obtained with the sample packed in a flat can and to illustrate it in terms of some preliminary experiments on mackerel.

Method

Consider the conduction of heat in a fish meat packed in a flat can of thickness $2X$ which is immersed in a liquid kept at constant temperature θ_e . If the heat flow occurs only in the direction of the thickness of the can, the temperature θ at time t and position x in the can is governed by the unidimensional heat conduction equation:

$$C\rho \frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x} \left[k(\theta) \frac{\partial \theta}{\partial x} \right], \quad \dots\dots\dots (1)$$

where the space variable x is taken in the direction of heat flow, and $k(\theta)$, C and ρ are the heat conductivity, the specific heat and the density of the meat, respectively. In the present treatment, $k(\theta)$ is considered as a function of θ only, and both C and ρ are assumed to be constant. Taking the origin of the x -axis at the center of the can, the boundary and initial conditions for θ are

$$\theta = \theta_e \quad (x = \pm X, t > 0), \quad \dots\dots\dots (2)$$

$$\theta = \theta_0 \quad (-X < x < X, t = 0). \quad \dots\dots\dots (3)$$

Here it has been assumed that initially the meat is maintained at uniform temperature θ_0 .

No analytic solution of Eq. (1) subject to conditions (2) and (3) is yet known for the case in which k varies with θ . For this reason, we shall, in what follows, adopt an approximate procedure which may be useful for k dependent not very strongly on temperature. This consists of replacing $k(\theta)$ by an average heat conductivity, $\bar{k}$, defined by

$$\bar{k} = (\theta_c - \theta_0)^{-1} \int_{\theta_0}^{\theta_c} k(\theta) d\theta. \quad (4)$$

Equation (1) with constant heat conductivity $\bar{k}$ can be solved under conditions (2) and (3) (See, for example, reference 3), giving for the temperature at the center of the can, θ_c , the equation:

$$\frac{\theta_c - \theta_0}{\theta_c - \theta_0} = 1 - \frac{4}{\pi} \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)} \exp \left[-\frac{(2n+1)^2 \pi^2 \bar{k} t}{4C\rho X^2} \right]. \quad (5)$$

For large enough t this is satisfactorily approximated by

$$\frac{\theta_c - \theta_0}{\theta_c - \theta_0} = 1 - \frac{4}{\pi} \exp \left(-\frac{\pi^2 \bar{k} t}{4C\rho X^2} \right), \quad (6)$$

from which one obtains

$$-\ln(\theta_c - \theta_0) = -\ln \frac{4}{\pi} (\theta_c - \theta_0) + \frac{\pi^2 \bar{k} t}{4C\rho X^2}. \quad (7)$$

Thus it may be expected that a plot of $-\ln(\theta_c - \theta_0)$ against t becomes linear for large t and that the slope of this straight line permits the evaluation of $\bar{k}$ with known values of C , ρ and X . In general, the value of $\bar{k}$ thus obtained depends both on θ_c and θ_0 .

Since Eq. (4) can be written

$$(\theta_c - \theta_0) \bar{k} = - \int_{\theta_r}^{\theta_0} k(\theta) d\theta + \int_{\theta_r}^{\theta_c} k(\theta) d\theta \quad (8)$$

with θ_r being chosen arbitrarily, plots of $(\theta_c - \theta_0) \bar{k}$ against θ_c for different initial temperatures θ_0 should be different, in the direction of the ordinate axis, by constant amounts given by

$$- \int_{\theta_r}^{\theta_0} k(\theta) d\theta$$

from that obtained with θ_r as θ_0 . This means that the plots for different θ_0 values can be made superpose by shifting them vertically by proper amounts to form a single composite curve. It is obvious that the composite curve thus formed is no more than the $(\theta_c - \theta_0) \bar{k}$ vs. θ_c plot which would be obtained at $\theta_0 = \theta_r$. For convenience, this composite curve is referred to as the **master intergral heat conductivity curve at reference temperature θ_r** . Since the first integral in Eq. (8) vanishes for $\theta_0 = \theta_r$, the equation for the master integral heat conductivity curve, denoted by $P(\theta_c)$, is given by

$$P(\theta_c) = \int_{\theta_r}^{\theta_c} k(\theta) d\theta. \quad (9)$$

It follows then that

$$k(\theta_c) = \frac{d}{d\theta_c} P(\theta_c) \quad (10)$$

Thus the problem of determining the heat conductivity, k , as a function of temperature

is reduced to constructing the master integral heat conductivity curve at a suitably chosen reference temperature according to the procedure described above.

Results and Discussion

The flat can used was a copper-made cylinder, 6 cm. in diameter and 2.5 cm. in thickness. To keep the heat flow in the can unidimensional in the direction of its thickness, its cylindrical surface portion was thermally insulated by covering it with thick layers of asbesto. A thin thermometer, inserted through the cylindrical surface portion so as to keep the bulb just at the center of the can, was used to measure θ_c . After the sample meat had been packed in, the can was allowed to stand for one hour in air kept at the desired initial temperature θ_0 . The can was then quickly transferred into a water-bath kept at the desired external temperature θ_e , and the subsequent change of θ_c was measured as a function of time. The water-bath was vigorously stirred so that the the surface temperature of the can could be maintained equal to the external temperature during a particular run.

The specific heat of the contents was assumed to be equal to that of water at 15°C. for all the samples studied. The density of each sample was calculated from its known weight and volume.

The sample fish studied in this work was mackerel purchased at a market in Maizuru. On every series of measurements fresh fish were purchased and used immediately to make the experimental results free of any effect brought about by their natural denaturation. Only muscle portions were cut out of the sample fish and, after mincing them thoroughly, packed tightly in the can.

Fig. 1. Temperatures of the center of the can, θ_c , plotted semi-logarithmically against time t (min.) for various values of the external temperature, θ_e , at a fixed value of the initial temperature, θ_0 .

Fig. 2. The master integral heat conductivity curve reduced to reference temperature $\theta_r = 21.6^\circ\text{C}.$; $P(\theta_e)$ is actually k (mean heat conductivity between temperatures of θ_e and θ_0) multiplied by $\theta_e - \theta_0$

obtained, which give the heat conductivity as a function of temperature for mackerel containing no added oil (solid line) and for the one containing 5% added oil (dashed line). It is seen that both curves have a sigmoidal shape, in agreement with the prediction made in a previous paper²). In addition, they both pass through an inflection point in the range 50 to 60°C. This behavior appears to be in line with the fact that fresh fish muscles are generally coagulated thermally in this region of temperature. It is also observed that addition of soya-bean oil brings about a lowering of the heat conductivity over the temperature range herein investigated. This is expected because the heat conductivity of soya-

Fig. 4. Comparison of calculated curves with observed values; solid line represents the curve for a variable heat conductivity and dashed line curve for a constant heat conductivity

By differentiating graphically these curves in Figure 2, the curves shown in Figure 3 are

Fig. 3. Temperature dependence of the heat conductivity k for mackerel. Solid line represents the data for mackerel containing no added oil, and dashed line the data for the mackerel containing 5% added soya-bean oil

bean oil is about one third that of water at ordinary temperatures. It is of some interest to note that the heat conductivity of water at 30°C. is $1.47 \times 10^{-3} \text{ cal. cm.}^{-1} \text{ sec.}^{-1} \text{ } ^\circ\text{C.}^{-1}$ and that of mackerel with no added oil at the same temperature is, as Figure 3 shows, $1.05 \times 10^{-3} \text{ cal. cm.}^{-1} \text{ sec.}^{-1} \text{ } ^\circ\text{C.}^{-1}$. This difference may be accounted for mainly by the water content and the oil content of the sample fish.

Finally, the θ_e vs. t curve for $k(\theta)$ represented by the solid line in Figure 3 is calculated directly from the differential equation (1) and com-

pared with the observed data. The solution of Eq. (1) was carried out for the case in which $\theta_0=26^\circ\text{C.}$ and $\theta_\infty=90^\circ\text{C.}$, by making use of the graphical method of solution proposed by Osida⁴⁾. The resultant θ_c vs. t curve is plotted in Figure 4 by solid line and is seen to agree well with the observed values indicated by open circles, except in the region of small t . The dashed line in the same figure represents the curve calculated from Eq. (5) by taking for $\bar{k}$ the value of k at 90°C. Comparison of this with the solid line demonstrates clearly that even such a relatively weak dependence of k on temperature as observed here has a pronounced influence on the rate of increase of temperature at the center of a flat can (probably for cans of other types, too). This results may be of practical significance to those who are engaged in canning processing.

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