A Relation between Experiments on Model and on Full Scale of Fishing Net.

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§ In the present paper, the author wishes to give a relation between experiments on model and on full scale of fishing net. The relation will be deduced from the following assumptions.

(i) The elongation of netting cords is negligible in the working conditions.

(ii) The pieces of net are perfectly flexible.

(iii) The change in the form of net occurs so slowly as the external forces acting on each elemental portion of net are taken to be in the quasi-equilibrium.

§ Now, let the axes of reference be Ox , Oy and Oz (Fig. 1), where O is the center of a knot, Ox is the bisector of the angle between two adjacent arms running from O , Oy at right angles to Ox in the plane of the elemental portion of net at O , Oz the outward normal to that plane. If the direction cosines of the movement of water relative to the portion are represented by (l_1, m_1, n_1) , the coefficients of x -, y - and z -components of the resisting force by k_x, k_y and k_z respectively, we get

Fig. 1.

$$\left. \begin{aligned} k_x &= a \left(\frac{D}{L} \right) l_1 \tan \varphi + \left(\frac{D}{L} \right)^2 f_x \\ k_y &= a \left(\frac{D}{L} \right) m_1 \cot \varphi + \left(\frac{D}{L} \right)^2 f_y \\ k_z &= a \left(\frac{D}{L} \right) \frac{n_1}{\sin \varphi \cos \varphi} + \left(\frac{D}{L} \right)^2 f_z \end{aligned} \right\} \quad (2.1)$$

where L is the length of an arm (the distance between centers of two consecutive knots), D its diameter, 2φ the angle between two adjacent arms, a some constant, and f_x, f_y and f_z depend on the direction cosines (l_1, m_1, n_1) and the angle φ .

The principal curvatures of the elemental portion lie on xz - and yz - planes. Let ρ_x and ρ_y denote the radii of curvature on xz - and yz -sections respectively, and let (l_2, m_2, n_2) denote the direction cosines of the gravitational force, w the apparent weight in water per unit area of the piece of net, and let T'_x and

¹⁾ TAUTI, M.: Bull. Japan. Soc. Sci. Fish., 3 (1), 1934, 1-4.

T_v denote the tension of net at O along x - and y -axes respectively. Then we have

$$\left. \begin{aligned} \frac{dT_x}{dx} + k_x v^2 + l_z w &= 0 \\ \frac{dT_y}{dy} + k_y v^2 + m_z w &= 0 \\ \frac{T_x}{\rho_x} + \frac{T_y}{\rho_y} &= k_z v^2 + n_z w \end{aligned} \right\} \quad (2.2)$$

where v is the velocity of water relative to net.

§ Consider that a model and full scale of net which have similar pieces of net in similar position be situated similarly in a model and full scale of stream respectively and that the above equations (2.2) be applied to a set of corresponding elemental portions, the symbols ' and ' ' being referred to model and full scale, then we have

$$\begin{aligned} \frac{dT'_x}{dx'} + k'_x v'^2 + l'_z w' &= 0 \\ \frac{dT'_y}{dy'} + k'_y v'^2 + m'_z w' &= 0 \\ \frac{T'_x}{\rho'_x} + \frac{T'_y}{\rho'_y} &= k'_z v'^2 + n'_z w' \end{aligned}$$

and

$$\begin{aligned} \frac{dT''_x}{dx''} + k''_x v''^2 + l''_z w'' &= 0 \\ \frac{dT''_y}{dy''} + k''_y v''^2 + m''_z w'' &= 0 \\ \frac{T''_x}{\rho''_x} + \frac{T''_y}{\rho''_y} &= k''_z v''^2 + n''_z w'' \end{aligned}$$

Let λ' and λ'' denote the linear dimensions of model and full scale and put $\lambda' : \lambda'' = A$, $v' : v'' = V$, $w' : w'' = W$, $k'_x : k''_x = K_x$, $k'_y : k''_y = K_y$, $k'_z : k''_z = K_z$, $T'_x : T''_x = T_x$, $T'_y : T''_y = T_y$. Then the necessary and sufficient condition under which the model and full scale maintain the similar and similarly situated form is that under which the two series of equations can be identified, A , V , W , K_x , K_y , K_z , T_x and T_y being assumed to be constants for all sets of corresponding portions, and being put $\rho'_x : \rho''_x = \rho'_y : \rho''_y = A$.

Hence the necessary and sufficient condition is represented by

$$T_x = T_y = AW \quad (3.1)$$

and

$$K_x = K_y = K_z = \frac{W}{V^2} \quad (3.2)$$

But from (2.1)

$$k_x' = a \left(\frac{D'}{L'} \right) l_1 \tan \varphi' + \left(\frac{D'}{L'} \right)^2 f_x'$$

$$k_y' = a \left(\frac{D'}{L'} \right) m_1 \cot \varphi' + \left(\frac{D'}{L'} \right)^2 f_y'$$

$$k_z' = a \left(\frac{D'}{L'} \right) \frac{n_1}{\sin \varphi' \cos \varphi'} + \left(\frac{D'}{L'} \right)^2 f_z'$$

and

$$k_x'' = a \left(\frac{D''}{L''} \right) l_1 \tan \varphi'' + \left(\frac{D''}{L''} \right)^2 f_x''$$

$$k_y'' = a \left(\frac{D''}{L''} \right) m_1 \cot \varphi'' + \left(\frac{D''}{L''} \right)^2 f_y''$$

$$k_z'' = a \left(\frac{D''}{L''} \right) \frac{n_1}{\sin \varphi'' \cos \varphi''} + \left(\frac{D''}{L''} \right)^2 f_z''$$

Then the equation (3.2) can be satisfied when $\varphi' = \varphi''$, $\frac{D'}{L'} = \frac{D''}{L''}$ and $\frac{w'}{v'^2} = \frac{w''}{v''^2}$ are satisfied for all sets of corresponding portions.

When $\varphi' = \varphi''$ and $\frac{D'}{L'} = \frac{D''}{L''}$, $\frac{w'}{w''} = \frac{D'(\rho' - 1)}{D''(\rho'' - 1)}$, because $w = \frac{\pi D^2 L + c D^3}{L^2 \sin 2\varphi} \times (\rho - 1)$, where ρ is the specific gravity of netting cord.

Therefore, $\frac{w'}{v'^2} = \frac{w''}{v''^2}$ becomes $\frac{v'^2}{v''^2} = \frac{D'(\rho' - 1)}{D''(\rho'' - 1)}$ and the equation (3.1) becomes $\frac{T_{x(y)'}}{T_{x(y)''}} = \frac{\lambda'}{\lambda''} \cdot \frac{v'^2}{v''^2}$.

Hence the condition is represented by

$$\left. \begin{aligned} \varphi' &= \varphi'' \\ \frac{D'}{L'} &= \frac{D''}{L''} \\ \frac{v'^2}{v''^2} &= \frac{D'(\rho' - 1)}{D''(\rho'' - 1)} \\ \frac{T_{x(y)'}}{T_{x(y)''}} &= \frac{\lambda'}{\lambda''} \frac{v'^2}{v''^2} \end{aligned} \right\} \quad (3.3)$$

In practice, the value of $\frac{v'}{v''}$ is the same for all sets of corresponding portion. Accordingly, the ratio $\frac{D'(\rho' - 1)}{D''(\rho'' - 1)}$ must have the same value for all sets of corresponding pieces of model and full scale.

If the tension of two pair of netting cords circumscribing a mesh be denoted by τ_1 and τ_2 respectively, then

$$T_x = \frac{(\tau_1 + \tau_2) \cos \varphi}{2L \sin \varphi}$$

and

$$T_y = \frac{(\tau_1 + \tau_2) \sin \varphi}{2L \cos \varphi}$$

Accordingly,

$$\frac{\tau_{1(2)}'}{\tau_{1(2)}''} = \frac{T_{x(y)}'}{T_{x(y)}''} \frac{L'}{L''}$$

Or

$$\frac{\tau_{1(2)}'}{\tau_{1(2)}''} = \frac{\lambda'}{\lambda''} \frac{D'}{D''} \frac{v'^2}{v''^2} \quad (3.4)$$

Fig. 2

§ In the case where the shape of the boundary of net changes during the fishing operation, the change in the form of net occurs from time to time. And, in this case, the additional relation must be satisfied for maintaining similar and similarly situated form, besides the relation (3.3) above obtained.

In this case, the velocity vector of water relative to net, v , is the resultant of the velocity vector of stream, v_1 , and the time rate of movement vector of elementary portion of net, v_2 .

Now, granting that the model at the instant t' and the full scale at t'' have a similar and similarly situated form and that $\frac{t'}{t''} = \text{constant}$, the next relation must be held for all sets of corresponding portions.

$$\frac{v't'}{\lambda'} = \frac{v''t''}{\lambda''} \quad (4.1)$$

Accordingly, the directions of the movement of every set of corresponding portion at every set of corresponding times must be parallel to each other and the next relations must be satisfied for all sets of corresponding portions

$$\frac{v_1'}{v_1''} = \frac{v_2'}{v_2''}$$

and

$$\frac{v_2't'}{\lambda'} = \frac{v_2''t''}{\lambda''}$$

Hence the additional relation required is represented by

$$\left. \begin{aligned} \frac{v_2'}{v_2''} &= \frac{v_1'}{v_1''} \\ \frac{t'}{t''} &= \frac{\lambda'}{\lambda''} \frac{1}{\frac{v_1'}{v_1''}} \end{aligned} \right\} \quad (4.2)$$

§ The rope for managing net and that for holding the buoys and sinkers

have two kinds of influence upon the form of net. One is due to that they have the unchangeable length and the other due to that they have the apparent weight in water and that they are under the resistance of water.

Now, take the line tangent to a rope at any point as ξ -axis, the outward normal to the rope in the plane of curvature at that point as ζ -axis. and the normal to that plane as η -axis.

Denote the tension of net on one side of the rope by T_{x1} and T_{y1} , that on the other side by T_{x2} and T_{y2} , the direction cosines of T_{x1} , T_{y1} , T_{x2} and T_{y2} by (l_{x1}, m_{x1}, n_{x1}) , (l_{y1}, m_{y1}, n_{y1}) , (l_{x2}, m_{x2}, n_{x2}) and (l_{y2}, m_{y2}, n_{y2}) , respectively, the tension of the rope by F , the radius of curvature of the rope by ρ_ξ , the apparent weight of the rope with buoys (or sinkers) in water by w_1 per unit length, the ξ -, η - and ζ -components of the resistance of water on the rope by r_ξ , r_η and r_ζ respectively, and the direction cosines of the gravity by (l_g, m_g, n_g) . Then the following relations hold :

Fig. 3

$$\frac{dF}{d\xi} + l_{x1}T_{x1} + l_{y1}T_{y1} + l_{x2}T_{x2} + l_{y2}T_{y2} + r_\xi + l_g w_1 = 0$$

$$m_{x1}T_{x1} + m_{y1}T_{y1} + m_{x2}T_{x2} + m_{y2}T_{y2} + r_\eta + m_g w_1 = 0$$

$$\text{and} \quad n_{x1}T_{x1} + n_{y1}T_{y1} + n_{x2}T_{x2} + n_{y2}T_{y2} + r_\zeta + n_g w_1 = \frac{F}{\rho_\xi}$$

Accordingly, the necessary and sufficient condition under which the model and full scale maintain the similar and similarly situated form is that the next relations hold for all sets of corresponding point.

$$\frac{F'}{F''} = \frac{\lambda'}{\lambda''} \frac{T'_{x(y)}}{T''_{x(y)}} \quad (5.1)$$

$$\text{and} \quad \frac{T'_{x(y)}}{T''_{x(y)}} = \frac{r'_{\xi(\eta,\zeta)}}{r''_{\xi(\eta,\zeta)}} = \frac{w'_1}{w''_1} \quad (5.2)$$

Substituting (3.3), the relation (5.1) is expressed by

$$\frac{F'}{F''} = \frac{\lambda'^2}{\lambda''^2} \frac{v'^2}{v''^2} \quad (5.1')$$

Let the diameter and the specific gravity of the rope be represented by D_1 and ρ_1 respectively, the linear dimension and the specific gravity of the buoys (or sinkers) by D_2 and ρ_2 respectively, and the number of them per unit length of the rope by n .

$$\begin{aligned} \text{Then} \quad r_{\xi(\eta,\zeta)} &\propto (r_{\xi(\eta,\zeta)} D_1 + a_{2\xi(\eta,\zeta)} D_2^2 n) v^2 \\ w_1 &\propto b_1 D_1^2 (\rho - 1) + b_2 D_2^3 (\rho_2 - 1) n \end{aligned}$$

where $a_{1\xi}$, $a_{1\eta}$, $a_{1\xi}$, $a_{2\xi}$, $a_{2\eta}$, $a_{2\xi}$, b_1 and b_2 are constant.

Hence, the relation (5.2) is expressed by

$$\frac{\lambda' v'^2}{\lambda'' v''^2} = \frac{(D_1' + a_{\xi(\eta, \zeta)} D_2'^2 n') v'^2}{(D_1'' + a_{\xi(\eta, \zeta)} D_2''^2 n'') v''^2} = \frac{D_1'^2 (\rho_1' - 1) + b D_2'^3 (\rho_2' - 1) n'}{D_1''^2 (\rho_1'' - 1) + b D_2''^3 (\rho_2'' - 1) n''} \quad (5.2')$$

§ From (5.2'), the relation to hold between model and full scale of rope is expressed by

$$\left. \begin{aligned} \frac{D_1'}{D_1''} &= \frac{\lambda'}{\lambda''} \\ \frac{\rho_1' - 1}{\rho_1'' - 1} &= \frac{1}{\lambda'} \frac{v'^2}{v''^2} \end{aligned} \right\} \quad (6.1)$$

and

If the resistance of water on the rope may be neglected, as in the case of the wire rope used in fixed nets, the relation is expressed by

$$\frac{\lambda' v'^2}{\lambda'' v''^2} = \frac{D_1'^2 (\rho_1' - 1)}{D_1''^2 (\rho_1'' - 1)} \quad (6.2)$$

If the apparent weight in water may be neglected, as in the case of the head rope of trawl nets, the relation is expressed by

$$\frac{D_1'}{D_1''} = \frac{\lambda'}{\lambda''} \quad (6.3)$$

When the resistance and the apparent weight may be neglected, as in the case of the hemp rope used in fixed nets, it is needed only to use a model rope of a size as small as possible.

§ From (5.2'), the relation to hold between model and full scale of buoys and sinkers is expressed by

$$\frac{\lambda' v'^2}{\lambda'' v''^2} = \frac{D_2'^2 n' v'^2}{D_2''^2 n'' v''^2} = \frac{D_2'^3 (\rho_2' - 1) n'}{D_2''^3 (\rho_2'' - 1) n''} \quad (7.1)$$

Or

$$\frac{n' D_2'^2}{n'' D_2''^2} = \frac{\lambda'}{\lambda''} \quad (7.2)$$

and

$$\frac{D_2' (\rho_2' - 1)}{D_2'' (\rho_2'' - 1)} = \frac{v'^2}{v''^2}$$

In the case where cork buoys (or lead sinkers) are used, the resistance of water may be neglected. Then the relation is expressed by

$$\frac{D_2'^3 (\rho_2' - 1) n'}{D_2''^3 (\rho_2'' - 1) n''} = \frac{\lambda' v'^2}{\lambda'' v''^2} \quad (7.3)$$

If the total number of cork buoys is the same in every set of corresponding boundaries of model and full scale, then the relation is expressed by

$$\frac{D_2'^3 (\rho_2' - 1)}{D_2''^3 (\rho_2'' - 1)} = \frac{\lambda' v'^2}{\lambda'' v''^2} \quad (7.3')$$

putting $n'\lambda' = n''\lambda''$ in the equation (7.3).

If the total number of cork buoys is proportional to the total number of mesh on the boundary, then the relation is expressed by

$$\frac{D_2'^3(\rho_2' - 1)}{D_2''^3(\rho_2'' - 1)} = \frac{D'v'^2}{D''v''^2} \quad (7.3'')$$

putting $\frac{n'}{n''} = \frac{\lambda'/L'}{\lambda''/L''}$ in the equation (7.3).

In the case where bamboo buoys are used in fixed nets, the resistance of water can not be neglected and the total number of buoys must be the same in every set of corresponding boundaries of model and full scale, then the relation is expressed by

$$\text{and } \left. \begin{aligned} \frac{D_2'}{D_2''} &= \frac{\lambda'}{\lambda''} \\ \frac{\rho_2' - 1}{\rho_2'' - 1} &= \frac{1}{\frac{\lambda'}{\lambda''}} \frac{v'^2}{v''^2} \end{aligned} \right\} \quad (7.4)$$

putting $n'\lambda' = n''\lambda''$ in (7.2).

The relation expressed by the second equation can be realized with a mechanical apparatus as shown in Fig. 4.

§ It is easily seen that the next relation must be satisfied between model and full scale of an apparatus spreading trawl nets, e. g., Otter board.

$$\frac{F'}{F''} = \frac{\lambda_1'^2 v'^2}{\lambda_1''^2 v''^2} = \frac{\lambda_1'^3 (\rho_3' - 1)}{\lambda_1''^3 (\rho_3'' - 1)} \quad (8.1)$$

$$\text{Or } \left. \begin{aligned} \frac{\lambda_1'}{\lambda_1''} &= \frac{\lambda'}{\lambda''} \\ \frac{\rho_3' - 1}{\rho_3'' - 1} &= \frac{1}{\frac{\lambda'}{\lambda''}} \frac{v'^2}{v''^2} \end{aligned} \right\} \quad (8.2)$$

Fig. 4.

B: Model buoy,
W: Weight,
S: Spring,
P: Pulley.

where λ_1 and ρ_3 represent the linear dimension and the mean specific gravity of the spreading apparatus.

In the case where the apparatus is made of canvas, the apparent weight in water may be neglected.

Then the relation is expressed by

$$\frac{\lambda_1'}{\lambda_1''} = \frac{\lambda'}{\lambda''} \quad (8.3)$$