Studies on Echo Counting for Estimation Fish Stocks-I. 
Overlap Counting and Reading of S-type Echo Counter

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A preliminary consideration and method is presented for calculating the fish abundance index in the S-type echo counter for estimating fish stocks of concentrated fish.

i) The mean density of fish per unit volume of water sampled is:

\[ n = 4\pi \sum \sum \frac{p_{E}^{1/2}}{p_{0}^{2}(r_{1} - r_{2})} \psi \rho_{f} \]

ii) The total quantity of fish distributed around the waters surveyed is:

\[ N_{S} = \sum (n_{i} + \frac{jn}{2})(r_{1} - r_{2})A_{i} \]

The next paper will deal with the practical application of the above method.

Recently, many studies on echo counting systems for estimating fish stocks have been developed in the United Kingdom, Norway, Canada, U.S.S.R., Japan and other countries. In Japan, the echo counters for individual fish and fish schools have just completed the experimental stages and are now proceeding to their field survey.

The statistical method of echo information on the length of individual fish was reported before in this bulletin. This report deals with some primary considerations on an echo counting system for fish school linked with a vertical echo sounder.

Fundamental considerations

As reported before, the echo counter for fish schools is an electro-acoustic system for the automatic estimation of fish stocks. The S-type echo counter is very simplified system. In this system, the ultrasonic impulse transmitted vertically from the sound source into sea water is scattered from all the targets within the sound beams and the echo directed towards the sound source is introduced into the counting circuit. In this circuit, the received signal above a certain reset level of echo from fish schools, which are scattered over an echo sampled volume of water, are reformed into square waves and the pulse lengths of echoes measured in meters are summed as an equivalent echo level at a constant value of echo amplitude, during a certain time interval. If a series of 100 echoes from a fish school of 10 m thick is received, the counter indicates "1,000". Accordingly, the final result of this system may be used as a sort of the fish density index.

1) Received sound pressure from fish school This section is based on Urick, Okada and Saito, and Hashimoto. The received sound pressure from an individual fish is:

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\[ p_R = \frac{p_0^2 \sigma}{4\pi} |R(\theta, \varphi)|^4/r^4 \]  

(1)

where, \( p_0 \) is the output sound pressure at the standard point of 100 cm from the source on the beam axis, \( p_R \) is the received sound pressure from a target in directional angle of \( \theta, \varphi \) to the beam axis, \( \sigma \) is the back-scattering cross section of the target fish, \( R(\theta, \varphi) \) is the directivity function of transducer in the direction of the target and \( r \) is the distance between sound source and the target fish.

When the back-scattering cross section of unit volume of water which contains a fish school is \( \sigma_V \), the received sound pressure from the water volume sampled by an impulse at an instant of time is:

\[ p_R = \frac{p_0^2 \sigma_V}{4\pi} |R(\theta, \varphi)|^4dV/r^4 \]  

(2)

where, \( dV \) is the elemental volume of sea water being an infinitesimal cylinder of finite length with ends normal to the direction of the incident wave. The area of the end face of this cylinder may be written as \( r^2d\Omega \), where, \( d\Omega \) is the elemental solid angle subtended by \( dV \) at the source. Hence the elemental volume \( dV \) is:

\[ dV = r^2(C\tau/2)d\Omega \]  

(3)

where, \( C \) is the sound velocity in sea water, and \( \tau \) is the pulse duration in time. Accordingly, from Eq. (2) and Eq. (3),

\[ |R(\theta, \varphi)|^4dV = \int |R(\theta, \varphi)|^4r^2(C\tau/2)d\Omega \]  

(4)

After Urick's work, let the two-way beam pattern of \( |R(\theta, \varphi)|^4 \) be related by an ideal beam pattern of unit relative response within a solid angle, \( \psi \), and zero relative response beyond \( \psi \), then the angle \( \psi \) of this equivalent ideal beam pattern is:

\[ \int_{-\pi/2}^{\pi/2} [R(\theta, \varphi)]^4d\Omega = \psi \]  

(5)

where, the solid angle of the equivalent ideal beam pattern in steradians is:

\[ 10 \log \psi = 10 \log_{10} \int_{-\pi/2}^{\pi/2} [R_s(\theta, \varphi)]^4 |R_\theta(\theta, \varphi)|^4 \cos \theta \theta d\theta d\varphi \]  

(6)

where, \( 10 \log_{10} \psi \) is in dB re steradian, and \( R_s(\theta, \varphi) \) and \( R_\theta(\theta, \varphi) \) are the directivity functions of the transmitting and receiving transducers, respectively.

If the size of the rectangular transducer is \( L \times B = b \times a \),

\[ 10 \log_{10} \psi = 10 \log_{10} \left( \frac{\lambda^2}{4\pi ab} \right) + 7.4 \, (dB) \]  

(6')

Then, Eq. (2) may be written approximately, if \( V = (C\tau/2)r^2 \),

\[ p_R = \frac{p_0^2 \sigma_V}{4\pi} \frac{C\tau}{2} r^2 \psi |r^4 = \frac{p_0^2 \sigma_V}{4\pi} \frac{C\tau}{2} \psi |r^2 \]  

(7)
2) Echo integration In the counting circuit, the first integration is made for the received sound pressure from the sampled range between \( r_1 \) and \( r_2 \) during one transmission, where, \( r_1 \) is the lower limit and \( r_2 \) is the upper limit of the range. Then from Eq. (7),

\[
\int_{r_1}^{r_2} p_R^2 dr = p_0^2 \left[ \frac{r_2}{r_1} \frac{a_v}{4\pi} \frac{C_\tau}{2} \frac{\phi}{r^3} \right]
\]

if, \( dr = (C\tau/2) \) and \( p_R' = r p_R \) (via TVG of 20 log \( 10 \) \( r \)),

\[
\sum p_R^2 dr = p_0^2 \left[ \frac{\sigma_v}{4\pi} \phi dr \right]
\]

Then,

\[
\sum p_R^2 dr = p_0^2 \sigma_v (r_1 - r_2) \phi / 4\pi
\]

When the integration of counting is made at a given time duration from \( T_1 \) to \( T_2 \):

\[
\sum \sum p_R^2 dT = p_0^2 (r_1 - r_2) (T_2 - T_1) \sigma_v \phi / 4\pi
\]

where, \( (T_2 - T_1) \) is equal to the number of pulses \( n_p \) transmitted during the counting, if \( dT \) is equal to the pulse repetition period \( T_p \), and \( (r_1 - r_2) \) is shown in meters, if \( dr = 1 \) m. Accordingly, the back-scattering cross section of unit volume of water contained fish school is:

\[
\sigma_v = 4\pi \sum \sum p_R^2 / p_0^2 n_p (r_1 - r_2)
\]

Then the mean number of fish per unit volume of water is:

\[
n = \sigma_v / \sigma_f = 4\pi \sum \sum p_R^2 / p_0^2 n_p (r_1 - r_2) \phi \sigma_f
\]

where, \( \sigma_f \) is the mean of back-scattering cross section of fish distributed in the area of echo survey, and it should be determined on the basis of catches of test hauls in the same area.

The total quantity of fish distributed in this area, \( N_s \), is:

\[
N_s = \sum (n_i + \Delta n/2) (r_1 - r_2) A_i
\]

where, \( n_i \) is a certain estimated density of fish per unit volume of water within a sampled layer between \( r_1 \) and \( r_2 \) at a constant interval of integration, \( \Delta n = n_{i+1} - n_i \) and \( A_i \) is the enclosed geographic area by respective lines of \( n_i \) and \( n_{i+1} \) on the isogram of fish density drawn by \( \Delta n \) throughout the survey.

3) Overlap counting In this section, consideration for overlap counting is made upon assuming that the distribution of fish throughout fish school is random or homogeneous.

On the echo survey with commercial echo sounder, the sound beams are transmitted vertically at a constant time interval due to the pulse repetition rate, \( T_p \), while the transducer moves horizontally with a given speed of research vessel, \( V_s \), on the sea surface. During a constant time interval of one transmission, the vessel runs the distance of
VsTp. Then the horizontal radius of an equivalent ideal beam at a given depth of \( r \) at an instant of time may be written as \( r(\psi/\pi)^{1/2} \), if \( \psi \) is small enough. Hence the overlap counting is produced under following considerations:

\[
r(\psi/\pi)^{1/2} > VsTp/2
\]

Fig. 1 shows typical examples of overlapping aspects of horizontal sections of successive ideal beams in various relative radii. The ratio of horizontal radius of ideal beam pattern and respective distance of successive ideal beams,

\[
k = 2r(\psi/\pi)^{1/2}/VsTp,
\]

is 1 to 6 from top to bottom respectively.

Let us consider the overlap rate in an area limited by solid circular line in this figure.

The horizontal area enclosed by an equivalent ideal beam pattern, \( S \), which is limited by a solid circle in this figure, is written generally:

\[
S = r^2\psi = \pi(r \tan \alpha)^2
\]

if the angle of the equivalent ideal beam pattern, \( \psi \), is small enough,

\[
\alpha = \tan^{-1}(\psi/\pi)^{1/2}
\]

(14)

where, \( \alpha \) may be written as the effective beam angle as half angle in degrees.

Let us calculate the overlap rate of the equivalent ideal beam patterns in various relative radii of horizontal section in terms of the distance moved by the vessel during one transmission. The horizontal area of the ideal beam patterns with radius \( k \) is:

\[
S_k = \pi k^2
\]

(15)

When the horizontal radius is \( k \), the number of overlapped beams is \( 2(k-1) \). The area of overlapped section of sound beams is twice the area of the arc which is enclosed by a perfect circle and a chord through two crossing points of perfect circle to the other circle. When the distance between successive sound axes, which moved by the vessel during one transmission, is \( 2l \), the area of an overlapped section is:

\[
S'_k = k^2(2\delta - \sin 2\delta) = 2 \left[ k^2 \left( \tan^{-1} \left( \frac{(2kl - l^3)^{1/2}}{k - l} \right) - (k - l)(2kl - l^3)^{1/2} \right) \right]
\]
The overlap rate of horizontal areas of successive ideal beams with radii \( k \) is \( \frac{\sum S_k}{S_k} \), and the line calculated from Eq. (15) and (16) is shown in Fig. 2. In this figure, the \( x \)-axis represents the radius \( k \) and the \( y \)-axis shows the overlap rate of the equivalent ideal beam patterns. The rate may be approximated to 0.82 \((k-1)^{1/2}\) when \( k > 1 \). The rate is 1, when \( k \leq 1 \). Accordingly, \( S_e \) or the actual effective area of an ideal beam in horizontal section without overlap is obtained as the ratio of \( S_k \) to the overlap rate as shown in Fig. 2.

**Fig. 2. Overlap rate of successive ideal beams with various radii.**
\[ k = 2r_0(\phi/\pi)^{1/2}/V_s T_p \]

**Some calculation for S-type echo counter**

The principal factors for echo survey with the S-type echo counter are:

i) Echo sounder
   - frequency: 50 kHz; pulse repetition frequency: 112.5 pulses per min at 100 m range; transducer: circular type multiplied with several Langevin-transducers; half angle: 6.5° at \( R(\theta_{1/2}) = 0.5 \)

ii) Ship's speed on survey: 4 m/sec

iii) Counting range in depth: 5 to 200 meters

The equivalent diameter of this transducer assumed to be single circular disk, may be estimated by following equation:

\[ \theta_{1/2} = \sin^{-1}(7.1\lambda/d) \]  

(17)

where, \( \theta_{1/2} \) is the half angle in degrees, \( \lambda \) is the wave length in water and \( d \) is the diameter of transducer. After tank test, it is known that \( \theta_{1/2} = 6.5^\circ \) and \( \lambda = 3 \) cm. Substituting the above values into Eq. (17), \( d = 19 \) cm.

According to Urick, the solid angle of an equivalent ideal beam pattern for a circular transducer is:

\[ 10 \log_{10} \phi = 20 \log_{10} (\lambda/\pi d) + 7.7 \]  

(18)

Substituting, \( d = 19 \) cm and \( \lambda = 3 \) cm,

\[ \phi = 0.0154 \]
From Eq. (14), the effective beam angle in half angle is:
\[ \alpha = \tan^{-1} \left( \frac{\phi}{\pi} \right) = 4^\circ \]

According to HASHIMOTO*, the effective beam angle in half angle is:
\[ \theta_0 = 0.6 \theta_{1/2} = 3.9^\circ \]

Thus \( \alpha \) from \( \phi \) is nearly equal to \( \theta_0 \) on this echo sounder.

Accordingly, the horizontal radius of equivalent ideal beam pattern, \( R \), at a depth of \( r \) is:
\[ R = r \tan 4^\circ \]

On the other hand, the distance between sound sources during the interval from one transmission to another is obtained from the above conditions of echo survey:
\[ V s T p = 4(60/112.5) = 2.133 \text{ m} \]

then,
\[ k = \frac{R}{(V s T p/2)} = 0.0655 r \]

The trial calculation on the S-type counter at the survey conditions as mentioned above, is shown in Fig. 3. In this figure, the vertical axis is the overlap rate of successive sound beams and the other shows the distance on the sound axis from the source to a given section. Let us take an approximation from this figure:

if, \( r \leq 15.2 \text{ meters} \),

indicating no overlap,

and if, \( r > 15.2 \text{ meters} \),

\[ \left( \sum S \right) / S = 1 \]

\[ \left( \sum S_r \right) / S_k = 0.065 r \]

Accordingly, the effective horizontal area with corrected overlap of equivalent ideal beam pattern, \( Se \), is:
\[ Se = \frac{\pi k^2}{(\sum S_k) / S_k} = 0.235 r \]

* HASHIMOTO defined the term of effective beam angle \( \theta_0 \) that is referred to as a unit relative response within \( \theta_0 \) and zero relative response beyond \( \theta_0 \) from fish school. He also reported that the received sound pressure from fish school in planar distribution of jack-mackerel (TL=10 cm) and common mackerel (TL=15 to 21 cm), is proportional to the square root of the number of fish within the effective beam angle \( \theta_0 \).
Assuming, \( Se = r^2 \phi' \),

\[
\phi' = 0.235/r
\]

In order to obtain real estimates without overlap counting, \( \phi' \) is substituted into Eq. (7), and then,

\[
p_r^t = p_0^t \frac{\sigma_r}{4\pi} \frac{C_T}{2} \left( 0.235/r^2 \right)
\]

In this equation, if \( C_T/2 = 1 \) and the variable \( r^2 \) could be corrected automatically through the time varied gain control circuit, \( TVG \), which has a response of \((-10 \log_{10} 0.235 + 30 \log_{10} r)\), the total integration of echo counter is:

\[
\sum_r \sum_{\tau} p_{r,\tau}^t d\tau dT = p_0^t \frac{\sigma_r}{4\pi} n_r (r_1 - r_2)
\]  \( \text{(19)} \)

then, the number of fish per 1 cubic meter is:

\[
n = \frac{\sigma_r}{\sigma_f} = 4\pi \sum_r \sum_{\tau} p_{r,\tau}^t / p_0^t (r_1 - r_2) n_r \sigma_f
\]  \( \text{(20)} \)

The total number of counted fish within the sampled layer from \( r_1 \) to \( r_2 \) along the survey line is:

\[
N = n \times n_p (r_1 - r_2) \overline{Se} = 0.47 \pi (r_1 + r_2) \sum_r \sum_{\tau} p_{r,\tau}^t / p_0^t
\]  \( \text{(21)} \)

where, \( \overline{Se} = 0.235 (r_1 + r_2)/2 \)

**Discussion**

1) **Effective beam width for fish school**  
Urick's theory on the equivalent ideal beam pattern may be applied under the following conditions:  
i) The distribution of scatterers throughout the volume of \((C_T/2)r^2\) at any one instance of time is random or homogeneous.  
ii) A density of scatterers is so large that a large number of scatterers occur in an elemental volume \( dV \).

Hashimoto carried out several series of measurements on the received sound pressure from an artificial fish school formed by small fish less than 20 cm long and defined the effective beam angle, \( \theta_0 = 0.6 \theta_{1/2} \).

As mentioned above, \( \theta_{1/2} \) of S-type counter is nearly equal to \( \tan^{-1}(\phi/\pi)^{1/2} \), and accordingly, Eq. (7) may be applied extensively to a random or homogeneous distribution of fish less than 20 cm long, such as anchovies, sardines and small mackerels.

The distribution of fish forming fish school in open sea may be at random or homogeneous, and the respective distance between fish in a fish school are usually longer than the wave length of commercial echo sounder.

The fish school may be divided schematically into three types, i.e., random, homogeneous and clumped distribution, but the geographic distribution of fish school is usually clumped with various size.
Assuming that the radius of respective fish school along the survey line is \( R' \) and \( r' = \frac{r}{R} \), the number of echo pulses from the fish school is given as \( 2(k' - 1) \), where, \( k' = \frac{2R'}{VsTp} \) (\( k = \frac{2R}{VsTp} \)). When \( R' > R \), then \( k' > k \), Eq. (7) may be applied for the received sound pressure from the fish school. However, if \( R' < R \) or \( k' < k \), URICK's assumptions are not satisfied and Eq. (7) cannot be applied for the received sound pressure from such a small shoal.

Let us integrate the received sound pressure in field survey of a) individual fish, b) small fish shoals and c) large fish schools. The total integration of received sound pressure from the above is:

\[
\sum P_R = \sum P_{R1} + \sum P_{R2} + \sum P_{R3}
\]

where, \( P_{R1} \) is the received sound pressure from an individual fish, \( P_{R2} \) is that from a small fish school and \( P_{R3} \) is that from a large fish school.

In this equation, the total integration \( \sum P_R \) is the sum of received sound pressure belonging to three different sonar equations as follows:

\[
p^0_{R1} = p^0\sigma_1 |R(\theta, \varphi)|^4/4\pi r^4
\]

\[
p^0_{R2} = p^0\sigma_2 m |R(\theta, \varphi)|^4/4\pi r^4, \text{ where, } m \text{ is number of fish}
\]

\[
p^0_{R3} = p^0\sigma_3 \frac{C\tau}{2} \phi/4\pi r^2
\]

When the number of fish throughout small fish shoal which concentrates at a point of \((\theta, \varphi)\) in angular distribution from beam axis, \( m \), and if the horizontal area of fish school is \( \Lambda' \), \( \sigma_V = (m\sigma/\Lambda')(C\tau/2) \) and then,

\[
p^0_{R3} = p^0\sigma_3 \frac{C\tau}{2} |R(\theta, \varphi)|^4/4\pi r^4
\]

The variable \( R(\theta, \varphi) \) varies with the angular distribution of fish \((\theta, \varphi)\) against the beam axis and is different in effect that of \( \varphi \) which is constant. Therefore, the number of echo pulses from individual targets and from small shoal varies with their size and angular distribution.

2) **Lowestoft method** Recently, the main current on the echo counting method for estimating fish stocks is to split all the multiple targets as fish school into individual fish by a narrow beam, minimum pulse length and towing the transducer in deep water near the targets, as suggested by CUSHING. Using this method, the informations from fish are always given as the received sound pressures and swimming depths of individual fish.

The echo counting system of the Fisheries Laboratory, Lowestoft, by CUSHING can identify the received sound pressures from individual fish, to that from multiple targets and sea bed echo comparing their pulse lengths with that of transmitted pulse, and can integrate respective values excluding sea bed echo.

However, the above standard method may not be applied for dense fish school in
which small fish such as anchovies are concentrated as dense as beyond the minimum-resolving range of echo sounder for splitting multiple targets into single targets. Therefore, the S-type echo counter was planned for such a dense fish school.

3) Back-scattering cross section of unit volume of water Eq. (7) may be applied only for a fish school throughout which fish are scattered but not for densely concentrated fish school, because in the latter the interrelation between $\sigma_v$ and $\sigma_f$ is not so simple that $\sigma_v$ is affected by rescattering and absorption of sound energy within the fish school. Accordingly, as reported by TRUSKANOV and SHERBIN0, the above relation could be determined experimentally after comparing the value of $\sigma_v$ by acoustic measurement and the number of fish per unit volume of water simultaneously observed by an automatic underwater camera.

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References