Twist Geometry on Strand and 3-ply Cord—I
On Cross-sectional Form of Ply in the Cord and Cording Limit

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(Received May 26, 1978)

The theory of geometry of the 3-ply cord developed in this paper is based on the assumption that the cylindrical strand in the cord has the form of a simple helix around the cord axis. The geometrical analysis yields a ratio of the helix radius to the ply radius, $a/b$, with varying helix angle of ply axis, $\theta$, for defining the cross-sectional form of the ply in the cord.

As a result, the sectional form of the ply in the cord cross-section can not be regarded as an ellipse in the range of the helix angle of the ply axis of over $30^\circ$ up to $60^\circ$, $(\theta_m)$ maximum cord twist.

The twist geometries of the twisted yarns and the cords have been considered theoretically by TRELOAR¹, GRACIE², and FREESTON, and SCHOPPEE³, and studied experimentally by TATTERSALL⁴, and RIDING⁵. The term twisted yarn used in those papers would mean a strand based on Japanese fishing terminology. Generally, a strand is made by twisting an assembly of continuous filaments or yarns of staple fibers, and a cord is made by twisting together two or three strands (or plies as they are usually called in cording).

Now, in Gracie’s paper, he put the following equation for a 3-ply cord,

$$a=b\sqrt[3]{1-\sec^2\theta+1},$$

Eq. (1) can be recognized for a low-level-twisted cord where the plies are shown as elliptical sections in the cord cross-section, but Eq. (1) has a doubt to be recognized for a high-level-twisted cord. The purpose of this paper is to give an answer to above doubt, and thereby the cross-sectional form of the ply in the cord must be given more exactly.

Formulation of Cross-sectional Form of Ply

For the approach to the above purpose of this paper, the following assumptions will be introduced.

1. The cord is constructed of three plies symmetrically disposed around the cord axis.
2. The ply has a fairly circular cross-section which is perpendicular to its own axis during the twist.
3. The ply has the form of a simple helix around the cord axis.

The explanatory diagram of the cross-sections of such a ply is presented in Fig. 1, in which $OO'$ represents the cord axis, $x$-$y$ plane and $\alpha$-$\beta$ plane, the cross-sections which are perpendicular to the cord axis and the ply axis respectively, and $y$-$z$ plane, whose longitudinal section is parallel to the cord axis; $P''P'P'^{'''}$, an optional helix as it lies on the ply surface. The axes $\alpha$, $\beta$, $x$, $y$, and $z$ have the same point of origin. And $P^{'''}$ is the intersection of a line drawn perpendicularly from $P'$ to the $x$-$y$ plane. From the above assumption (2), the cross-section of the ply is a circule as $\alpha^2+\beta^2=b^2$, therefore the relation between coordinates $P(x, y)$ and $P'(\alpha, \beta)$ is expressed as the following equation,

$$x^2+(\alpha+y)^2=(\alpha\cos\theta)^2+(\alpha+\beta)^2$$

And in the nature of a simple helix, the helix angle $\gamma$ of the $PP'$ and its rotating angle $\phi$ are expressed respectively as

$$\tan\gamma=\frac{\sqrt{(\alpha\cos\theta)^2+(\alpha+\beta)^2}}{a}\tan\theta.$$  

And the angle $\xi$ and $\phi$ in Fig. 1 are clear to be written as

$$\xi=\cos^{-1}\frac{a+\beta}{\sqrt{(\alpha\cos\theta)^2+(\alpha+\beta)^2}}.$$  

$$\phi=\phi+\xi.$$  

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And the co-ordinates $P(x, y)$ is given as

$$x = \sqrt{(\alpha \cdot \cos \theta)^2 + (a + \beta)^2 \sin \phi}, \quad (7)$$

$$y = \sqrt{(\alpha \cdot \cos \theta)^2 + (a + \beta)^2 \cos \phi - a}. \quad (8)$$

Similarly the co-ordinates $P'(y, z)$ is given as

$$y = \sqrt{(\alpha \cdot \cos \theta)^2 + (a + \beta)^2 - a}, \quad (9)$$

$$z = \sqrt{(\alpha \cdot \cos \theta)^2 + (a + \beta)^2 \phi \cdot \cot \eta}. \quad (10)$$

Because all of the above equations include an unknown parameter of the cord helix radius $a$, those equations can not be solved easily. However, from the assumption (1), we know that the maximum angle of $\phi$ must be $60^\circ$. This condition for established 3-ply cord leads us to the numerical solution of the above equations.

**Numerical Methods and Results**

Under the fixed ply radius $b$, cord helix angle $\theta$ and cord helix radius $a$, we calculate $\phi$ with varying $\beta$ from $b$ to $-b$ using the above equations in the following order,

$$\beta \rightarrow \alpha \rightarrow \sqrt{(\alpha \cdot \cos \theta)^2 + (a + \beta)^2 \tan \eta \rightarrow \eta$$

$$\rightarrow \xi \rightarrow \phi \rightarrow \psi.$$ And, with $b$ and $\theta$ unchanged with only $a$ varied a little, we made repeated calculations to bring $\phi$ close to $60^\circ$. Finally we find the value of $a$ to produce $\phi$ value approximately $60^\circ$. This $a$ is the cord helix radius under one cord helix angle $\theta$ and ply radius $b$. Numerical example is shown in Table 1 at $b=1$ mm, $\theta=45^\circ$ and then just fit $a=1.395$ mm.

By the above method, the numerical results of $a/b$ (=ratio of the cord helix radius to the ply radius) with varying cord helix angle $\theta$ are shown in Table 2, and the cross and longitudinal sections in Fig. 2.

And, for comparison, the curves of $a/b$ obtained in this paper and $a/b = \sqrt{(1/3) \sec^2 \theta + 1}$ are shown in Fig. 3. As seen clearly from this figure, there is little error to set $a/b = \sqrt{(1/3) \sec^2 \theta + 1}$ untill about $\theta=30^\circ$. 
$$\frac{a}{b} = \sqrt{\frac{1}{3}} \sec^2 \theta + 1\text{ in Gracie's paper.}$$

**Limit to Cord Twist**

The illustrations of the longitudinal section and plan view of a 3-ply cord are shown in Fig. 4. As described by Gracie, both $XY$ and $H$ decrease as $N$, cord twist in turns per unit length, increases and it is fairly clear that they reach the same value at some cord twist $N_m$, maximum cord twist. And as is clear from Fig. 4, $XY$ is always less than $H$.

Thus

$$H \geq XY. \quad (11)$$

And clearly

$$H = \frac{1}{3N}, \quad (12)$$

and substitution of $\tan \theta = 2\pi Na$ leads to

<table>
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<th>$\beta$</th>
<th>$\alpha$</th>
<th>$\sqrt{(\alpha \cos \beta + \alpha + \beta)}$</th>
<th>$\tan \eta$</th>
<th>$\xi$</th>
<th>$\psi$</th>
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</tbody>
</table>

* This value is nearly equal to the maximum value of $\sqrt{\alpha + \beta}$, therefore in this case $a/b$ is given 1.395 approximately.
And using the maximum cord helix angle $\theta_m$, the maximum cord twist $N_m$ is written as

$$N_m = \frac{\sin \theta_m}{6b} \quad (15)$$

or

$$bN_m = \frac{\sin \theta_m}{6} \quad (15)'$$

The curves $H/b$ and $XY/b$ which were obtained from Eqs. (13)' and (14)' using the values $a/b$ in Table 2 are shown in Fig. 5. From the point of agreement of two curves, we know $\theta_m = 60^\circ$, and therefore $bN_m = 0.1443$ by Eq. (15)'. These values differ very much from the values, $\theta_m = 40.42^\circ$ and $bN_m = 0.1080$, in Gracie's paper. This difference is caused by that assuned $a/b = \sqrt{(1/3)} \sec^2 \theta + 1$, the plies to be elliptical in 3-ply cord cross-section.

Conclusions

From the above geometrical considerations, we may conclude that the form of ply in the cord cross-section can not be regarded as an ellipse in range of the cord helix angle over $30^\circ$. Therefore, the calculation for the twist geometry must be done using $a/b$, ratio of the cord helix radius to the ply radius, as in Table 2. And we may conclude that $\theta_m$, cord helix angle for the maximum cord twist, is $60^\circ$.

Acknowledgements

The author is grateful to Dr. O. SATO and Dr. K. NASHIMOTO (Faculty of Fisheries, Hokkaido University) for their suggestions and for reviewing the manuscript.

References