Twist Geometry on Strand and 3-ply Cord—I On Cross-sectional Form of Ply in the Cord and Cording Limit

Katsutaro YAMAMOTO*

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The theory of geometry of the 3-ply cord developed in this paper is based on the assumption that the cylindrical strand in the cord has the form of a simple helix around the cord axis. The geometrical analysis yields a ratio of the helix radius to the ply radius, a/b , with varying helix angle of ply axis, θ , for defining the cross-sectional form of the ply in the cord.

As a result, the sectional form of the ply in the cord cross-section can not be regarded as an ellipse in the range of the helix angle of the ply axis of over 30° up to 60° , (θ_m) maximum cord twist.

The twist geometries of the twisted yarns and the cords have been considered theoretically by TRELOAR¹⁾, GRACIE²⁾ and FREESTON, and SCHOPPEE³⁾, and studied experimentally by TATTERSALL⁴⁾ and RIDING⁵⁾. The term twisted yarn used in those papers would mean a strand based on Japanese fishing terminology. Generally, a strand is made by twisting an assembly of continuous filaments or yarns of staple fibers, and a cord is made by twisting together two or three strands (or plies as they are usually called in cording).

Now, in Gracie's paper, he put the following equation for a 3-ply cord,

$$a = b \sqrt{\frac{1}{3} \sec^2 \theta + 1}, \quad (1)$$

where a is a cord helix radius (=ply axis helix radius), b is a ply radius and θ is a cord helix angle (=angle between the ply and cord axes), and he obtained 40.42° as the cord helix angle for the maximum cord twist, θ_m , using this equation.

Eq. (1) can be recognized for a low-level-twisted cord where the plies are shown as elliptical sections in the cord cross-section, but Eq. (1) has a doubt to be recognized for a high-level-twisted cord. The purpose of this paper is to give an answer to above doubt, and thereby the cross-sectional form of the ply in the cord must be given more exactly.

Formulation of Cross-sectional Form of Ply

For the approach to the above purpose of this paper, the following assumptions will be introduced.

1. The cord is constructed of three plies symmetrically disposed around the cord axis.

2. The ply has a fairly circular cross-section which is perpendicular to its own axis during the twist.

3. The ply has the form of a simple helix around the cord axis.

The explanatory diagram of the cross-sections of such a ply is presented in Fig. 1, in which OO' represents the cord axis, x - y plane and α - β plane, the cross-sections which are perpendicular to the cord axis and the ply axis respectively, and y - z plane, whose longitudinal section is parallel to the cord axis; $P''P'P''''$, an optional helix as it lies on the ply surface. The axes α , β , x , y and z have the same point of origin. And P''' is the intersection of a line drawn perpendicularly from P' to the x - y plane. From the above assumption (2), the cross-section of the ply is a circle as $\alpha^2 + \beta^2 = b^2$, therefore the relation between coordinates $P(x, y)$ and $P'(\alpha, \beta)$ is expressed as the following equation,

$$x^2 + (a+y)^2 = (\alpha \cdot \cos \theta)^2 + (a+\beta)^2 \quad (2)$$

And in the nature of a simple helix, the helix angle η of the PP' and its rotating angle ϕ are expressed respectively as

$$\tan \eta = \frac{\sqrt{(\alpha \cdot \cos \theta)^2 + (a+\beta)^2}}{a} \tan \theta. \quad (3)$$

$$\phi = \frac{\alpha \cdot \sin \theta \tan \eta}{\sqrt{(\alpha \cdot \cos \theta)^2 + (a+\beta)^2}} \quad (4)$$

And the angle ξ and ψ in Fig. 1 are clear to be written as

$$\xi = \cos^{-1} \frac{a+\beta}{\sqrt{(\alpha \cdot \cos \theta)^2 + (a+\beta)^2}}. \quad (5)$$

$$\psi = \phi + \xi \quad (6)$$

* Faculty of Fisheries, Hokkaido University, Minato-cho Hakodate, 041, Japan (山本勝太郎: 北海道大学産学部).

Fig. 3. The curves of a/b obtained in present paper and $a/b = \sqrt{(1/3) \sec^2 \theta + 1}$ in Gracie's paper.²⁾

Limit to Cord Twist

The illustrations of the longitudinal section and plan view of a 3-ply cord are shown in Fig. 4. As described by Gracie, both XY and H decrease as N , cord twist in turns per unit length, increases and it is fairly clear that they reach the same value at some cord twist N_m , maximum cord twist. And as is clear from Fig. 4, XY is always less than H .

Fig. 4. The longitudinal section and plan view of 3-ply cord. This is the same figure as drawn by Gracie²⁾.

Thus

$$H \geq XY. \quad (11)$$

And clearly

$$H = \frac{1}{3N}, \quad (12)$$

and substitution of $\tan \theta = 2\pi Na$ leads to

Table 1. Numerical example to determine the cross and longitudinal sections of 3-ply cord at $b=1$ mm, $\theta=45^\circ$ and $a=1.395$ mm

β	α	$\sqrt{(\alpha \cdot \cos \theta)^2 + (\alpha + \beta)^2}$	$\tan \eta$	η	ξ	ϕ	ϕ	ϕ	x	y	y	z
1 mm	0 mm	2.395 mm	1.717	59.78°	0°	0°	0°	0°	0 mm	1 mm	0 mm	
0.9	0.4359	2.316	1.660	58.94	7.72	12.76	20.48	28.42	0.810	0.921	0.499	
0.8	0.6	2.236	1.603	58.04	10.99	17.43	28.42	30.10	1.064	0.572	0.692	
0.6	0.8	2.074	1.487	56.07	15.86	23.24	30.10	46.44	1.308	0.215	0.952	
0.4	0.9165	1.908	1.368	53.83	19.82	26.62	46.44	51.95	1.383	-0.080	1.131	
0.2	0.9798	1.739	1.247	51.26	23.48	28.47	51.95	55.92	1.369	-0.323	1.265	
0	1	1.564	1.121	48.27	26.88	29.04	55.92	58.54	1.295	-0.519	1.361	
-0.2	0.9798	1.381	0.990	44.71	30.08	28.46	58.54	59.66*	1.178	-0.674	1.425	
-0.4	0.9165	1.187	0.851	40.39	33.04	26.62	59.66*	58.70	1.024	-0.795	1.458	
-0.6	0.8	0.976	0.700	34.98	35.46	23.25	58.70	52.95	0.834	-0.888	1.429	
-0.8	0.6	0.731	0.524	27.66	35.52	17.43	52.95	44.55	0.583	-0.955	1.289	
-0.9	0.4359	0.583	0.418	22.68	31.89	12.66	44.55	0	0.409	-0.980	1.085	
-1	0	0.395	0.283	15.80	0	0	0	0	0	-1	0	

* This value is nearly equal to the maximum value of ϕ ($=60^\circ$), therefore in this case a/b is given 1.395 approximately.

Table 2. Values of a/b with varying the helix angle θ

θ°	0	5	10	15	20	25	30	35	40	45	50	55	60
a/b	1.155	1.156	1.159	1.165	1.175	1.190	1.210	1.250	1.305	1.395	1.495	1.645	1.910

Fig. 5. Changes of H/b and XY/b with the cord helix angle θ .

$$H = \frac{2}{3} \pi a \cdot \cot \theta \quad (13)$$

or

$$\frac{H}{b} = \frac{2}{3} \pi \frac{a}{b} \cot \theta, \quad (13)'$$

and

$$XY = 2b \cdot \operatorname{cosec} \theta \quad (14)$$

or

$$\frac{XY}{b} = 2 \operatorname{cosec} \theta. \quad (14)'$$

And using the maximum cord helix angle θ_m , the maximum cord twist N_m is written as

$$N_m = \frac{\sin \theta_m}{6b} \quad (15)$$

or

$$bN_m = \frac{\sin \theta_m}{6} \quad (15)'$$

The curves H/b and XY/b which were obtained from Eqs. (13)' and (14)' using the values a/b in Table 2 are shown in Fig. 5. From the point of agreement of two curves, we know $\theta_m = 60^\circ$, and therefore $bN_m = 0.1443$ by Eq. (15)'. These values differ very much from the values, $\theta_m = 40.42^\circ$ and $bN_m = 0.1080$, in Gracie's paper. This difference is caused by that assumed $a/b = \sqrt{(1/3)\sec^2 \theta + 1}$, the plies to be elliptical in 3-ply cord cross-section.

Conclusions

From the above geometrical considerations, we may conclude that the form of ply in the cord cross-section can not be regarded as an ellipse in range of the cord helix angle over 30° . Therefore, the calculation for the twist geometry must be done using a/b , ratio of the cord helix radius to the ply radius, as in Table 2. And we may conclude that θ_m , cord helix angle for the maximum cord twist, is 60° .

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