On the Motion of Fishing Gear Against Wave Action—I

Vertical Motion of a Float in Still Water

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A series of experiments was performed to study the motion of freely floating bodies in still water, and apply the data acquired here to the study on the motion of fishing gear. The displacement from the equilibrium position with the lapse of time was recorded on a 35 mm film with the aid of a stroboscope and was evaluated for the vertical oscillation. The configuration of the models used in the experiments were of three types; a circular cylinder, a circular cone and a sphere. The main problem was to describe the motion of a float with non-uniform cross section area. For the solution of this problem, an approximation was done by substituting the amount of change in submerged volume due to damped oscillation as a volume of a body with uniform cross section area. Accordingly, to use this approximation method, the constant coefficient in the equation of motion was calculated in every period, i. e. the equation was solved by means of the method of variable constant. The theoretical consideration was performed based on the experimental results of damped oscillation to examine the possibility of applying the theory for a circular cylinder to a circular cone and a sphere. Through the experimental results, it was ascertained that the theory for circular cylinder could be applied to a circular cone or a sphere for practical purposes.

A floating body exposed to waves may perform in complicated motion. In the case of fishing gear, particularly those placed near the water surface will take various shapes due to the height and period of surface waves that would act upon it. The fishing efficiency of a fishing gear is closely related to the shape it assumes in waters during operation. Therefore, it has been found necessary to have some knowledge of the response characteristics of a fishing gear which vary significantly when exposed to wave action.

In the case of fishing gear consisting of floats, webbing and sinkers, those deformation due to wave action may be closely connected with the motion of floats exposed to waves. Thus, the response characteristics of the floats exposed to wave action must be considered first of all. Basically, these floats can be classified into two categories; floats with uniform cross section area and floats with non-uniform cross section area. Experiments were conducted in this study to obtain elementary knowledge on the vertical motion of these floats in still water. Information acquired here will later be applied to the study on the motion of surface fishing gear.

The motion of floating body can not be described using the linear equation in a strict interpretation; however, to facilitate mathematical solution it

seems more efficient to start with linear equation. Thus, the approach to the problem is through linear equation. In addition, to assess its accuracy and practicality, linearized equations are used to calculate the vertical motion of the semi-submerged floating bodies (a circular cylinder, a circular cone and a sphere) in the still water. The relation between the amount of displacement and the period of oscillation will be discussed here.

Theoretical Consideration

In the case of a floating body oscillating vertically in water, various forces will act upon it. However, in order to simplify the solution of problem, the buoyant force and the resistance force proportionate to the frequency are considered as the forces that act upon the body. On the other hand, one of the difficulties in formulating the problem of motion of floating body is in geometrical configuration of the body, the equation of motion of a cylindrical float oscillating vertically in the still water is formulated here.

A schematic diagram of a floating body in a position of equilibrium is shown in Fig. 1. The x-coordinate is measured from the center of gravity of the body in the at-rest-position and denotes the amplitude of vertical oscillation.

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Fig. 1. Schematic diagram of cylindrical float in equilibrium position; G center of gravity; X_0 length of submerged portion; x direction of motion.

The equation of motion for the vertical oscillation of a body in still water is;

$$M \frac{d^2 x}{dt^2} + k \frac{dx}{dt} + \rho g V(x) = 0 \quad (1)$$

In the case of a circular cylinder with uniform cross section area to the oscillating direction, the change in buoyant force is given as an expression of a first degree in relation to the amount of displacement. Accordingly, the equation becomes;

$$M \frac{d^2 x}{dt^2} + k \frac{dx}{dt} + \rho g S x = 0 \quad (2)$$

This is a second order differential equation with constant coefficient. Here, (M) is effective mass of the body of vertical oscillation, (x) is displacement of the center of gravity from the equilibrium position, (k) is a constant for resistance force, (ρ) is density of water, (V) is submerged volume of body, (S) is cross section area at water line and (g) is acceleration of gravity.

However, in the case of the float with non-uniform cross section area such as a circular cone and a sphere, (S) is given as a functional value of displacement; therefore, equation (1) is solved by means of the method of variable constant which is described in detail elsewhere.

A general solution for the above differential equation where $\omega > k$ is as follow;

$$x = A e^{-kt} \cos(\sqrt{\omega^2 - k^2} t + \alpha) \quad (3)$$

Therefore, the displacement (x) of the center of gravity of the body from its equilibrium position shall be given as the function of time (t). Here, (A) is an amount of displacement from the equilibrium position of the float at initial condition, (ω) is a circular frequency of the body without re-

sistance force and (α) is the initial phase. The damping coefficient (k) is defined as the logarithmic value of the ratio of the amplitude,

$$k = \log \frac{x_i}{x_{i+2}} \quad (4)$$

In the above expression, x_i denotes the i -th maximum value with the lapse of time in damped oscillation, i. e. it shows the i -th amplitude. Thus, if the amount of displacement from the equilibrium point to the crest is taken as the value of x_i , the height of the next crest becomes x_{i+2} . On the other hand, in the case of the height of trough, it is taken as the value for x_i instead of the crest, x_{i+2} denotes the height of following trough. Actually, in the determination of practical (k) values of the ratio of amplitude, the following expression is used;

$$k = \log \left(\frac{\sum_{i=1}^n \frac{x_i}{x_{i+2}}}{n} \right)$$

where, (n) shows the number of trials employed.

Experiments

In the experiments, a circular cylinder, a circular cone and a sphere were used as experimental models. The schematic diagrams and dimensions for the models are shown in Fig. 2. The experiments were carried out in a channel 1 m wide by 1.2 m deep by 22 m long, with one side of the channel constructed of glass plates (1 m $\times$ 4 m) mounted in steel frames. The water depth and temperature during all experiments were 80 cm and 19°C, respectively. The vertical movement of the floating body was measured by recording the reflected light from the reflector attached to the side of the body on a film running at a regular speed using a 35 mm camera with a continuous moter drive. A stroboscope adjustable to produce 9, 11, 13 and 17 flashes per second was used as light source and the time interval of flashes was recorded on the oscillograph through a photo-cell. A representative experimental result in the case of circular cone is shown in Plate 1. The white dots on the photograph denote the position of float at time interval of 1/11 second. From the data obtained in this manner the amount of displacement was read.

Results

Cylinder Model

To change the natural period of the experimental

Fig. 2. Configuration of floats and dimensions; cylinder (vinyl chloride); cone (tin); sphere (polyethylene).

Plate 1. A representative experimental result; white dots show the position of float at time interval of 1/11 second.

body by increasing the body length of the cylinder model was not possible because of the limited depth in the channel. Therefore, as shown in Fig. 2 the volume of the increased length required was

transformed to a mass of the same volume with bigger diameter and attached to the lower end of the cylinder. Here, the experiments were done using four cylindrical floats of the same length

Fig. 3. Motion of a cylindrical float No. 2 in still water; (O) experimental values; (—) theoretical curve.

with different diameters. The attached portion at the lower end of the cylinder was constant in all four floats.

At first, from the obtained data, the damping coefficients were determined for each experimental model. Using these coefficients, the theoretical curves on the relation between the amplitude and period were drawn for all models which were then compared with their respective experimental values. Here, as an example, the result in the case of model No. 2 is shown in Fig. 3.

Circular Cone

In case of the circular cone, the variation of the buoyant force which changes according to its vertical motion can not be described by a simple equation. Here, to obtain the theoretical values, an approximation was done by substituting the amount of change in submerged volume due to the vertical motion as a volume of a body with uniform cross section area. Now, both the volumes were determined at x_i and x_{i+2} experimentally. Here, as the physical constant (C) showing the ratio of the variation on the volume according to the displacement in the damped oscillation is defined as follows;

$$C = \frac{V_{i+1}}{V_i}, \text{ thus } CV_i = V_{i+1} \quad (6)$$

And

$$C = \frac{V_{i+2}}{V_{i+1}}, \text{ thus } CV_{i+1} = V_{i+2} \quad (7)$$

From (6) and (7), the next expression is obtained as

$$C^2 V_i = V_{i+2} \quad (8)$$

Now we have

$$C = \sqrt{\frac{V_{i+2}}{V_i}} \quad (9)$$

where (V_i) denotes the amount of displaced volume

when the float moved vertically to (x_i) from the equilibrium position. Therefore, the amount of submerged volume with the lapse of time is given continuously using the following three values; logarithmic decrement (k), initial displacement (A) and physical constant (C). Now, an (S) value representing the cross section area can be obtained as a function of submerged volume. In this case, an (S) value indicated the mean value of cross section area for submerged volume. Therefore, we can rewrite the third term in equation (1) using an (S) value obtained as mean value, i. e. the third term in equation (1) becomes of the same order with that of equation (2). Consequently, the equation (1) can be solved analytically using the constant coefficient (S) which varies depending

Fig. 4. Comparison of experimental results with theoretical values for circular cone No. 1; (O) experimental values; (1) theoretical value without consideration of change in cross section area; (2) theoretical value computed from equation (9).

Fig. 5. Comparison of experimental results with theoretical values for circular cone No. 2.

on the amount of displacement, i. e. the equation (1) is solved by means of the method of variable constant.

In the present experiments, three experimental models were used as shown in Fig. 2. Here, to examine the relation between the amplitude and the period of damped oscillation, the circular frequencies for various amplitude were calculated as described. As shown in Figs. 4, 5 and 6, the circular frequencies for various amplitude obtained experimentally were compared with their respective theoretical value. In these figures, the theoretical value (1) was computed using the cross section area at water line in the equilibrium position; on the other hand the theoretical value (2) was computed using (S) values obtained for several

Fig. 6. Comparison of experimental results with theoretical values for circular cone No. 3.

displacement. These figures indicate the decrease of circular frequencies due to increase in the amount of amplitude. In addition, using the data from Fig. 5, the relation between the amplitude and the period of oscillation with the lapse of time was investigated and is shown in Fig. 7. Here, experimental values shifted away from the theoretical curve (1) which were obtained using the circular frequency computed from cross section area in the equilibrium position. On the other hand, there is no significant difference between the experimental plots and the theoretical curve (2).

Sphere

As mentioned before, in case of the variation in the buoyant force which can not be expressed by a simple equation, the volume of a cylindrical body was substituted for the volume changed per

Fig. 7. Motion of a circular cone No. 2 in still water; (O) experimental results; (1) theoretical curve without consideration of change in cross section area; (2) theoretical curve computed from equation (9).

unit displacement of the floating body. Thus, in case of the theoretical analysis for the problem caused by the non-uniformity in cross section area against the oscillating direction, the problem was solved theoretically in the same way in the case of the circular cone. The comparison with the experimental results and the theoretical values on the relation between the amplitude and natural period was investigated and shown in Figs. 8, 9 and 10. As shown in these figures, the circular frequencies have the tendency to decrease with the increase in the amplitude. These results indicate that the

Fig. 8. Comparison of experimental results with theoretical values for sphere No. 1; (O) experimental results; (1) theoretical value without consideration of change in cross section area; (2) theoretical value computed from equation (9).

Fig. 9. Comparison of experimental results with theoretical values for sphere No. 2.

Fig. 10. Comparison of experimental results with theoretical values for sphere No. 3.

periods of damped oscillation of the experimental floats decrease with the lapse of time.

Discussion

To test the possibility of applying the equation formulated for a circular cylinder to the float with non-uniform cross section area, a circular cone and a sphere were used in this study as the experimental models. In consideration of the above problem, the differential equation was solved by means of the method of variable constant under the limited condition where the body does not plunge underwater completely and the maximum diameter is usually in water.

In the case of a circular cone, as shown in Figs. 4, 5 and 6, the results of these tests indicate the variation on the circular frequencies according to change in the amplitude. In these figures, the experimental plots are scattered, but the trend seems to be well defined. Accordingly, from these figures, it was observed that the period of oscillation had a tendency to become short with the lapse of time. Therefore, as presented in these figures, the variation on the circular frequencies caused by the difference in the amplitude can not be expressed by the theoretical values (1). On the other hand, the theoretical values (2) shows the decrease in the circular frequencies depending on the increase in the amplitude. For these theoretical values, although there is a slight difference between the experimental results, as a whole, this was true for all cases. Thus, it is considered that

the theoretical values computed as a function of the submerged volume will give a good result as shown in figures mentioned above.

In the case of sphere, the results of experiments are presented in Figs. 8, 9 and 10 which show the variation on the circular frequencies due to the difference in the amplitude. As shown in these figures, the experimental plots are mostly scattered around theoretical values (2), with some of the data around theoretical values (1). However, it is obvious from these figures that the data shows a tendency to increase in the circular frequency according to decrease in the amount of amplitude. Therefore, the variation on the circular frequency with the difference in the amplitude can not be expressed by the theoretical values (1). On the other hand, the theoretical values (2) indicate the increase in the circular frequencies with the lapse of time. In this case, as shown in these figures the experimental values are more dispersed as compared with that of circular cone. The reason for these dispersion seems to be that the diameter of float was too small as models; and the non-stability of float caused by the configuration might have considerable effect on the experimental results. However, as shown in these figures the theoretical values (2) have in general view given a good agreement with the experimental results in all cases. Consequently, it could be considered that the use

of the method mentioned before will give a good results in investigating the change in the period of oscillation of float with the lapse of time.

As mentioned before, in the case of float with non-uniform cross section area, such as a circular cone and a sphere, it became clear that the variation on the circular frequencies according to the lapse of time can be expressed approximately by using the circular frequency obtained as a function of the submerged volume. Consequently, the validity of the method of the approximation to obtain the theoretical values of floats with non-uniform cross section area was ascertained experimentally.

Based on this study, it was observed that the resistance force acted upon a body proportionately to the frequency in the case of a circular cone and a sphere under the condition defined in this study. Therefore, it can be concluded that the change in the period of oscillation of float with non-uniform cross section area as a function of the submerged volume.

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