A Study on the Planting Effect of Salmon—I

A Mathematical Model for the Derivation of their Rate of Return and its Applications

Kazumi SAKURAMOTO* and Sakutarō YAMADA*

(Received January 10, 1980)

It is important to estimate the true rate of return when we judge the effect of release. As a first step of the estimation, in this paper, we proposed a mathematical model by which we would like to clarify the variation mechanism of the rate of return.

We showed that the rate of return $r(t, y)$ could be written in the following formula under the assumption that it would be determined by the natural mortality coefficient M , the fishing mortality coefficient at the offshore fisheries $F(t)$ in year t and the rate of maturity $m(y)$ of age y :

$$r(t, 3) = m(3) \exp [-(2 + (3/4))M - (1/4)F(t)],$$

$$r(t, 4) = (1 - m(3))m(4) \exp [-(3 + (3/4))M - (1/4)F(t-1) - (1/4)F(t)],$$

$$r(t, 5) = (1 - m(3))(1 - m(4))m(5) \exp [-(4 + (3/4))M - (1/4)F(t-2) - (1/4)F(t-1) - (1/4)F(t)].$$

Using this model it is possible to judge the effectiveness of offshore fisheries. We showed an example of a method of judgement.

If $F(t)$ is constant in the above formula $r(t, y)$ is independent of t , so it is possible to estimate that values using the data of the released number and of the returned number by the least square method. As a result we obtained the estimated values using the data of all Iwate prefecture; $r(3) = 0.00107$, $r(4) = 0.00380$ and $r(5) = 0.000549$.

The simplest method for judging the effect of release is to calculate the rate of return. So far, the rate of return has been estimated as the number of salmon that return to the coast divided by the number released four years before. The age composition of chum salmon, *Oncorhynchus keta*, at the coast extends between 2~6 years. However, since the four year old fish constitute the main group the above method of calculating the rate of return is used. If the released number of salmon in each year is the same the above method is a good estimation procedure. For the age composition of each year class can be considered almost the same to that at the coast in same year. In case the number released varies greatly from year to year, even if the age composition of each year class is constant the age composition at the coast will vary. Therefore, to obtain the rate of return by the above method will yield serious error. It is thus necessary to get the rate of return for each age group. To make things more clear it is necessary to know the mechanism which governs the rate of return. Knowledge of this mechanism will lead us to the true rate of return.

We proposed a mathematical model for obtain-

ing a representation of the rate of return, which shows a mechanism dominating the variation of the rate of return. Under the assumption the rate of return is determined by the three factors of, natural mortality coefficient, fishing mortality coefficient and rate of maturity. The merit of introducing such a mathematical model is to be able to investigate theoretically, the variation of the rate of return when the value of each of the three factors varies. For example it is possible to estimate the rate of return when offshore fisheries is not undertaken. Using the rate of return in this case, we considered a method for judging the effect of offshore fisheries from the view point of effective use of the resources. Moreover, we paid attention to the special case of the model that the fishing mortality coefficient is constant throughout all years. In this case the rate of return can be discussed as a relation to the number released and the number that return. Estimation of the rate of return is done by the least square method.

Data and Methods

The whole catch statistics¹⁻³⁾ of chum salmon

* Laboratory of Population Dynamics, Tokyo University of Fisheries, Konan 4, Minato-ku, Tokyo 108
(桜本和美・山田作太郎：東京水産大学)。

Table 1. Number of fry and number of catch

Year	Number of fry ($\times 1000$) ²⁾	Number of catch ³⁾ *		Total
		River	Coastal water	
1958	23,725	—	—	—
1959	24,478	—	—	—
1960	25,196	—	—	—
1961	36,974	—	—	—
1962	70,608	—	—	—
1963	45,882	65,110	234,000	299,110
1964	46,531	61,215	182,571	243,786
1965	33,194	36,561	149,142	185,703
1966	57,435	54,623	270,571	325,194
1967	—	46,226	200,571	246,797
1968	—	47,601	142,000	189,601
1969	—	43,135	141,142	184,277

* The number of catch was calculated from the catch in weight by putting the weight per one chum salmon is 3.5 kg.

from Iwate prefecture is used. The statistics of the number of fish released extend from 1958 to 1966. Planting and feeding were not undertaken during this period. The statistics on the catch number extend from 1963 to 1969 (Table 1). The chum salmon present in the above catch statistics can be interpreted as the group that were released during 1958~1966 and returned. For estimating the mean weight of each age group, data of the 856 chum salmon caught in the Katagishigawa and Tsugaruigishigawa rivers in December 1976 are use (Table 2).

The chum salmon fry, which are released between February and April of a certain year, stay

Table 2. Mean weight of each age group of chum salmon in the Katagishi and Tsugaruishi rivers obtained by age-determination based on the measurement of scale

Katagishigawa				
Age	Sex	No. of specimens	Mean weight (kg)	SD
5	Female	14	4.75	1.24
	Male	10	5.02	0.60
4	Female	145	4.60	0.81
	Male	56	4.70	1.15
3	Female	96	3.18	0.61
	Male	133	2.76	0.68
Tsugaruigishigawa				
5	Female	10	5.70	0.68
	Male	3	7.17*	0.24
4	Female	167	4.96	0.84
	Male	64	4.65	1.22
3	Female	64	3.14	0.52
	Male	94	2.87	0.69

* In this paper we do not use the value of mean weight of the male chum salmon of age 5 in the Tsugaruigishigawa river in the calculation of mean weight for each age group because the sample number is very small.

within the bay for a while and then leave for the north sea. They grow with a succession of feeding migration. During May and July, this shoal is subjected to offshore fisheries.

It is assumed that only the mature fish that escape death during offshore fisheries return to the coast, and the immature fish that escape being

Fig. 1. Assumed schematic migration route of chum salmon originating from Hokkaido, Honshu, Sakhalin (southern Sakhalin) and the Kuril Is.⁴⁾ The sea regions A and B show the fishing ground of chum salmon during 1961~1965.

Year	t	t+1	t+2	t+3	t+4	t+5
	Taking eggs	Planting				
Month	11-1	3		5-7 12	5-7 12	5-7 12
Age			2	3	4	5

Fig. 2. Diagram shows the life history of chum salmon. $\square$; the period during which offshore fisheries was operated, $\downarrow$; the period which mature chum salmon returned to the coast.

caught during offshore fisheries continue the process of feeding-migration (see Fig. 1).

The age composition of the catch during offshore fisheries, consists of three, four, five and six year old fish. Four and Five year olds are the main constituents⁵⁾. The age composition of chum salmon returned to the coast consists of two, three, four, five and six year old fish. Three and four year olds are the main constituents⁶⁾. For simplicity, we assume in this paper that both the age groups consist of only three, four and five year old fish. In Iwate prefecture, the chum salmon return to the coast during the beginning of October and January of the following year. However, for simplicity, we assume that all of them return on December 1st. It is also assumed that all the fry are released on March 1st (see Fig. 2).

The notations that will be used from here on are explained below.

$H(t)$: the number of plantings in year t .*

$N(t, y)$: the individual number of y years' fish in May of year t .

$C(t, y)$: the catch number of y years' fish in year t .

$m(y)$: the rate of maturity, that is, the proportion of the number of fish which mature in y years for the first time to the total number of fish of y years.

$R(t, y)$: the individual number of y years' fish which return to the coast in year t .

M : the natural mortality coefficient during a year. We assume that this value is constant throughout all age groups and all years.

$F(t)$: the fishing mortality coefficient during a year, which is considered to fluctuate with year t .

$Z(t)$: the total mortality coefficient during a year, $Z(t) = F(t) + M$.

The individual number $N(t+3, 3)$ of chum salmon is given by

$$N(t+3, 3) = H(t) \exp[-(2 + (1/6))M]. \quad (1)$$

Three year old fish are the object of offshore fisheries. The individual number of three year olds that are caught during the three month duration of offshore fisheries is given by

$$\begin{aligned} C(t+3, 3) &= \frac{F(t+3)}{Z(t+3)} N(t+3, 3) \\ &\times \left[1 - \exp\left(-\frac{1}{4}Z(t+3)\right) \right] \\ &= \frac{F(t+3)}{Z(t+3)} H(t) \left[1 - \exp\left(-\frac{1}{4}Z(t+3)\right) \right] \\ &\times \exp\left[-\left(2 + \frac{1}{6}\right)M\right]. \end{aligned} \quad (2)$$

In August, after the end of the offshore fisheries season, the individual number of mature three year old fish that escape death during the fishing season is given by $N(t+3, 3)m(3)\exp[-1/4Z(t+3)]$. Hence, the individual number of the three year old fish that return to the coast on December first is given by

$$\begin{aligned} R(t+3, 3) &= N(t+3, 3)m(3)\exp\left[-\frac{1}{4}Z(t+3)\right] \\ &\times \exp\left(-\frac{1}{3}M\right) \\ &= H(t)m(3)\exp\left[-\left(2 + \frac{3}{4}\right)M - \frac{1}{4}F(t+3)\right]. \end{aligned} \quad (3)$$

In August, the individual number of immature three year old fish that escape death during the fishing season is given by $N(t+3, 3)(1-m(3))\exp[-1/4Z(t+3)]$. The individual number, therefore, of four year old fish in May of the following year, $t+4$, is given by

* During the end of November of a certain year t and January of year $t+1$, fishermen collect new eggs for artificial hatching. The fry are then released between February and April of year $t+1$. We shall call these fry, in accordance with the custom, the planted fry in year t .

$$\begin{aligned}
 N(t+4,4) &= N(t+3,3)(1-m(3)) \\
 &\times \exp\left[-\frac{1}{4}Z(t+3)\right] \exp\left(-\frac{3}{4}M\right) \\
 &= H(t)(1-m(3)) \exp\left[-\left(3+\frac{1}{6}\right)M\right. \\
 &\quad \left.-\frac{1}{4}F(t+3)\right] \quad (4)
 \end{aligned}$$

The following relations are obtained in the same manner as above.

$$\begin{aligned}
 C(t+4,4) &= \frac{F(t+4)}{Z(t+4)} N(t+4,4) \\
 &\times \left[1 - \exp\left(-\frac{1}{4}Z(t+4)\right)\right] \\
 &= \frac{F(t+4)}{Z(t+4)} H(t)(1-m(3)) \\
 &\times \left[1 - \exp\left(-\frac{1}{4}Z(t+4)\right)\right] \\
 &\times \exp\left[-\left(3+\frac{1}{6}\right)M - \frac{1}{4}F(t+3)\right]. \quad (5)
 \end{aligned}$$

$$\begin{aligned}
 R(t+4,4) &= N(t+4,4)m(4) \\
 &\times \exp\left[-\frac{1}{4}Z(t+4)\right] \exp\left(-\frac{1}{3}M\right) \\
 &= H(t)(1-m(3))m(4) \exp\left[-\left(3+\frac{3}{4}\right)M\right. \\
 &\quad \left.-\frac{1}{4}F(t+3) - \frac{1}{4}F(t+4)\right]. \quad (6)
 \end{aligned}$$

$$\begin{aligned}
 N(t+5,5) &= N(t+4,4)(1-m(4)) \\
 &\times \exp\left[-\frac{1}{4}Z(t+4)\right] \exp\left(-\frac{3}{4}M\right) \\
 &= H(t)(1-m(3))(1-m(4)) \exp\left[-\left(4+\frac{1}{6}\right)M\right. \\
 &\quad \left.-\frac{1}{4}F(t+3) - \frac{1}{4}F(t+4)\right]. \quad (7)
 \end{aligned}$$

$$\begin{aligned}
 C(t+5,5) &= \frac{F(t+5)}{Z(t+5)} N(t+5,5) \\
 &\times \left[1 - \exp\left(-\frac{1}{4}Z(t+5)\right)\right] \\
 &= \frac{F(t+5)}{Z(t+5)} H(t)(1-m(3))(1-m(4)) \\
 &\times \left[1 - \exp\left(-\frac{1}{4}Z(t+5)\right)\right] \\
 &\times \exp\left[-\left(4+\frac{1}{6}\right)M - \frac{1}{4}F(t+3)\right. \\
 &\quad \left.-\frac{1}{4}F(t+4)\right]. \quad (8)
 \end{aligned}$$

$$\begin{aligned}
 R(t+5,5) &= N(t+5,5)m(5) \\
 &\times \exp\left[-\frac{1}{4}Z(t+5)\right] \exp\left(-\frac{1}{3}M\right) \\
 &= H(t)(1-m(3))(1-m(4))m(5) \\
 &\times \exp\left[-\left(4+\frac{3}{4}\right)M - \frac{1}{4}F(t+3)\right. \\
 &\quad \left.-\frac{1}{4}F(t+4) - \frac{1}{4}F(t+5)\right]. \quad (9)
 \end{aligned}$$

Let us define the rate of return of chum salmon of age y in year t as

$$r(t,y) = \frac{R(t,y)}{H(t-y)}. \quad (10)$$

From (3), (6) and (9) we have

$$\begin{aligned}
 r(t,3) &= \frac{R(t,3)}{H(t-3)} = m(3) \exp\left[-\left(2+\frac{3}{4}\right)M\right. \\
 &\quad \left.-\frac{1}{4}F(t)\right]. \quad (11)
 \end{aligned}$$

$$\begin{aligned}
 r(t,4) &= \frac{R(t,4)}{H(t-4)} = (1-m(3))m(4) \\
 &\times \exp\left[-\left(3+\frac{3}{4}\right)M - \frac{1}{4}F(t-1)\right. \\
 &\quad \left.-\frac{1}{4}F(t)\right]. \quad (12)
 \end{aligned}$$

$$\begin{aligned}
 r(t,5) &= \frac{R(t,5)}{H(t-5)} = (1-m(3))(1-m(4))m(5) \\
 &\times \exp\left[-\left(4+\frac{3}{4}\right)M - \frac{1}{4}F(t-2)\right. \\
 &\quad \left.-\frac{1}{4}F(t-1) - \frac{1}{4}F(t)\right]. \quad (13)
 \end{aligned}$$

If there exists a value of F which is optimum from the view point of effective use of the resources, we should adopt the management procedure of fishing which has that value of F as the fishing mortality coefficient. A procedure, for judging the effect of offshore fisheries from the view point of effective use of the resources, is as follows: To compare the sum of the catch in weight for a certain year class by offshore fisheries, having a certain constant F as the fishing mortality coefficient, and the catch in weight at the coast with the catch in weight at the coast when offshore fisheries is not operated.

For this judging procedure we can use relations (11), (12) and (13) in the following way. The rate of return for each age when offshore fisheries is not operated, can be obtained from (11), (12) and (13) by putting $F=0$. $r(t,y)$ is independent of t

and is denoted by $r'(y)$. Thus the total weight of fish, which were released in year t and returned to the coast when offshore fisheries is not operated is given by

$$\sum_{y=3}^5 r'(y)H(t)W'(y),$$

where $W'(y)$ is the mean weight of one fish of age y at the time it returned to the coast. On the other hand, the total catch in weight of fish which were released in year t , when offshore fisheries, having a certain constant F as the fishing mortality coefficient, is operated is given by

$$\sum_{y=3}^5 [C(t+y,y)W(y) + r(y)H(t)W'(y)]$$

from (2), (5), (8), (11), (12) and (13). Here $W(y)$ is the mean weight of one fish of age y in the north

sea at the time of offshore fisheries and $r(y)$ is the rate of return of age y when $F(t)$ is the constant F . Let us denote

$$\alpha = \frac{\sum_{y=3}^5 [C(t+y,y)W(y) + r(y)H(t)W'(y)]}{\sum_{y=3}^5 r'(y)H(t)W'(y)} \quad (14)$$

According to whether the value α is larger or smaller than one, it is possible to some extent to judge if offshore fisheries is reasonable or not from the view point of effective utilization of the resources.⁷⁾

In Table 2 we showed the mean weight $W'(y)$ of age group y , using data obtained at Katagishigawa and Tsugaruigishigawa rivers by the method of age-determination by scale. From Table 2 the values of $W'(y)$'s at $W'(3)=2.95$ kg, $W'(4)=$

Fig. 3. The supposed rate of maturity for each age and the growth curve. The graph of $m(y)$ is a straight line (A,B). The graph of $m(y)$ is an exponential curve (C,D). The graph of $m(y)$ is a sigmoid curve (E,F). Mean weight of chum salmon (G); Mean weight of chum salmon at the coast (●) and in the north sea (■). The growth rate per month is 5% (□) and 15% (○).

Fig. 4-A

Fig. 4-B

Fig. 4-C

Fig. 4-D

Fig. 4-E

Fig. 4-F

Fig. 4-G

Fig. 4-H

Fig. 4-I

Fig. 4-J

Fig. 4. The Isopleth diagram for judging effectualness of offshore fisheries from the view point of the effective use of resources. A-F have same growth rate and different rate of maturity as below, A; $m(3)=0.10$, $m(4)=0.55$, B; $m(3)=0.50$, $m(4)=0.75$, C; $m(3)=0.10$, $m(4)=0.30$, D; $m(3)=0.30$, $m(4)=0.55$, E; $m(3)=0.10$, $m(4)=0.80$, F; $m(3)=0.30$, $m(4)=0.85$. We assume $m(5)$ is equal to 1 in every cases. G and H have the rate of maturity as same as C above, G's growth rate per month is 5% and H's is 15%. I and J have the rate of maturity as same as E above, I's growth rate per months is 5% and J's is 15%.

4.76 kg and $W'(5)=5.11$ kg are estimated by calculating the total mean for each age group irrespective of the sex. TAGUCHI⁸⁾ showed that, by an analysis of the weight composition of catch by gill net fisheries using different mesh sizes, the weight of a chum salmon at the time of offshore fisheries extended over 1.7 kg to 2.8 kg. Hence we estimated the values of $W(y)$'s, using $W(3)=1.70$ kg, $W(5)=2.80$ kg and $W(4)=W'(4) \times [W(5)/W'(5)]=2.62$ kg.

There are limited detailed studies on the maturity

of chum salmon. We were thus compelled to calculate α by assuming some values of $m(y)$ in each of the following three cases: 1) The curve of $m(y)$ is a straight line (Fig. 3 A, B). 2) The curve of $m(y)$ is an exponential curve (Fig. 3 C, D). 3) The curve of $m(y)$ is a sigmoid curve (Fig. 3 E, F). Moreover we calculated the values of α , taking the values of Fig. 3 C and E as the values of the rate of maturity, by obtaining the values of $W(y)$ from those of $W'(y)$. For each case, the growth rate per month during the period that chum

salmon return to the coast from north sea is 5% or 15%.

In case $F(t)$ is constant the rate of return for each age group is determined by the age y only. This is considered to be a special case. Let $S(t)$ be the number of chum salmon at the coast that returned in year t . Then, from (10), we have

$$S(t) = r(3)H(t-3) + r(4)H(t-4) + r(5)H(t-5). \quad (15)$$

The rates of return $r(y)$ are obtainable by the least square method using the data of the number released and the number returned.

Results and Discussion

In Fig. 4 A-J, we show projections onto the (F, M) -plane from the α -axis. The parameter M should, in fact, be estimated by a biological inspection. The value of M is however not known and hence M is treated as a variable. From Fig. 4A-J, it is deduced that there are two regions in the (F, M) -plane which have a common boundary corresponding to some value of M , say M_0 . One of the regions indicates that offshore fisheries is effectual from the view point of effective use of the resources. The other region points to making offshore fisheries ineffectual. The value M_0 and the slope of the graph of α are found to be functions of $m(y)$ (Fig. 4 A-F). As the values of $m(y)$ of younger fish become larger, the area of the region known as effectual becomes narrower and the slope of the graph of α becomes smoother (Fig. 4 B, D, F). Moreover the graph of α varies

with the growth rate (Fig. G-J). As the values of the growth rate become larger, the area of the region which regards offshore fisheries as effectual becomes narrower and the slope of the graph of α becomes smoother (Fig. 4 H, J). Hence it turned out that, for judging the effect of offshore fisheries, we should consider the three factors of growth rate, the rate of maturity and the natural mortality coefficient. In reality, there exists a value M of natural mortality coefficient. Thus, we can judge offshore fisheries to be effective or not according to whether the value of M is larger than the value M_0 or not. When offshore fisheries is judged to be effectual α is an increasing function, for each fixed value of M , of F belonging to the interval $(0, 2)$. There is no optimum value of F that makes the value of α maximum. When offshore fisheries is judged to be ineffectual, α is a decreasing function, for each fixed value of M , of F belonging to the interval $(0, 2)$. In this case as in the last, there does not exist an optimum value for F .

At the present stage we cannot discuss in detail the judgment on the effectiveness of offshore fisheries. But we hope that judgment becomes possible if the values of $m(y)$ and M can be estimated.

From (14), by the least square method, we obtained the estimated values of $r(y)$, $r(3)=0.00107$, $r(4)=0.00380$ and $r(5)=0.000549$. The estimated rate of return for a year class $r=r(3)+r(4)+r(5)$ was calculated to be 0.00542.

The age compositions of chum salmon for a year class are $r(3)/r=0.197$ for age three, $r(4)/r=0.701$ for age four and $r(5)/r=0.101$ for age five. The

Table 3. The number of chum salmon returned to the coast calculated using $r(y)$'s, and the age composition of chum salmon at the coast for each year

Year	Estimated values			Total $\hat{S}(t)$	Catch $S(t)$
	$t+3$	$t+4$	$t+5$		
1963	26,960	93,016	13,025	133,001	299,110
	20.3%	69.9%	9.8%		
1964	39,562	95,745	13,438	148,745	243,786
	26.6%	64.4%	9.0%		
1965	75,551	140,501	13,833	229,885	185,703
	32.9%	61.1%	6.0%		
1966	49,094	268,310	20,299	337,703	325,195
	14.5%	79.5%	6.0%		
1967	49,788	174,352	38,764	262,904	246,797
	18.9%	66.3%	14.7%		
1968	35,518	176,818	25,189	237,525	189,601
	15.1%	75.1%	10.7%		
1969	61,455	126,137	25,546	213,138	184,277
	28.8%	59.2%	12.0%

Fig. 5. The individual number returned to the coast, i.e. the total number caught in the rivers and within the bays, and its predicted value. ●; the individual number, ○; the predicted value.

number of chum salmon y year old at the coast that returned in year t is given by $r(y)H(t-y)$, $y=3, 4, 5$. The age composition of chum salmon at the coast is shown in Table 3, which is obtained by dividing $r(y)H(t-y)$ by $r(3)H(t-3)+r(4) \times H(t-4)+r(5)H(t-5)$, $y=3, 4, 5$.

In Fig. 5, the predicted values $\hat{S}(t)$ of $S(t)$'s with the total catch at the coast (i.e. the sum of number caught at the rivers and by fixed nets) are shown. The values of $\hat{S}(t)$ seem to be similar to those of $S(t)$. However, because the amount of data is small, the confidence interval of $S(t)$ is rather large. Incidentally, the significance test of the coefficients was not tried because there was no guarantee that the error terms would satisfy usual regular conditions and also because the amount of data was limited.

Acknowledgements

The authors thank Professor N. HIRAYAMA and Professor M. INOUE of Tokyo University of Fish-

eries for their kind advice. They are also thankful to Mr. Y. TAMAKI, Mr. M. SHIMADA and Mr. D. IRANI in various aspects.

References

- 1) Data on Salmon in Iwate Prefecture (1), Iwate-ken Keison Zōshoku Kyokai, Iwate-ken, 1975, pp. 1-519.
- 2) Data on Salmon in Iwate Prefecture (2), Iwate-ken Keison Zōshoku Kyokai, Iwate-ken, 1975, pp. 1-225.
- 3) Fundamental Planning of Fisheries in Iwate Prefecture, Iwate-ken, 1972, p. 110.
- 4) T. YONEMORI: *Bull. Int. North. Pac. Fish. Comm.*, **32**, 130-150 (1975).
- 5) A. ISHIDA and K. MIYAGUCHI: *Bull. Hokkaido Reg. Fish. Res. Lab.*, **18**, 11-22 (1958).
- 6) Hokkaido Salmon Hatchry: *Sake to Masu*, **15**, 41-47 (1973).
- 7) T. DOI: *Bull. Tokai Res. Lab.*, **34**, 1-11 (1962).
- 8) K. TAGUCHI: *Bull. Japan. Soc. Sci. Fish.*, **27**, 645-649 (1961).