A New Coefficient of Association between Two Attributes.

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Introduction. In statistics, the association between two attributes, that is the correlation between characters not quantitatively measurable has hitherto been studied by G. U. YULE⁽¹⁾ and other investigators. They have introduced several coefficients for the measurement of the intensity of association. In the present paper, a new form is added to them.

When each individual of the observations, numbering N in total, can

Table 1.

Characters	B	Non- B	Total
A	(AB)	$(A\beta)$	(A)
Non- A	(αB)	$(\alpha\beta)$	(α)
Total	(B)	(β)	N

be classified as being either A or α , i. e. as either possessing or not the attribute A , and at the same time, as being either B or β , i. e. as either possessing or not the attribute B , the frequencies of actual co-

incidences between A and B , A and β , α and B and α and β are expressed as (AB) , $(A\beta)$, (αB) and $(\alpha\beta)$ respectively, and they are set out in a "fourfold table" as shown in Table. 1.

In case of complete independence of both attributes, the actual chance of coincidences, such as $(AB)/N$ is equal to the product of two probabilities, such as $(A)/N$ and $(B)/N$. Usually the difference $\delta_{AB} = (AB) - (A)(B)/N$ has been employed to calculate the coefficients of association⁽²⁾. For a fourfold table, we have $\delta_{AB} = \delta_{\alpha\beta} = -\delta_{A\beta} = -\delta_{\alpha B}$ and this quantity is usually denoted by δ .

The new coefficient of association. Now, the association between A and B is *complete* or the most intense when $|\delta_{AB}|$ assumes the greatest possible value. Expressing the greatest and the least possible value of (AB) by $(AB)_{max}$ and $(AB)_{min}$ respectively, the new coefficient of association K_{AB} between A and B is given by the following formulae:—

$$K_{AB} = \frac{\delta_{AB}}{(\delta_{AB})_{max}} = \frac{(AB) - (A)(B)/N}{(AB)_{max} - (A)(B)/N}, \quad \text{when } \delta_{AB} > 0 \quad (1)$$

(1) YULE, G. U.: On the association of attributes in statistics. Phil. Trans. Roy. Soc. A 194, 1900, p. 257.

—: On the methods of measuring association between two attributes. Journ. Roy. Stat. Soc., 75, 1912, p. 579~642.

(2) Recently late Prof. T. TERADA used the ratio instead of the difference in his papers; TERADA, T.: Earthquakes and fisheries. Proc. Imp. Acad., 8, 1932, 83~86.

—: Earthquake and fisheries II. Bull. Earthq. Res. Inst., 11, 1933, 714~716.

and
$$K_{AB} = \frac{\delta_{AB}}{(\delta_{AB})_{min}} = \frac{(AB) - (A)(B)/N}{(AB)_{min} - (A)(B)/N}, \text{ when } \delta_{AB} < 0. \quad (2)$$

Table 2.

Characters	B	Non-B	Total
A	(B)	(A)-(B)	(A)
Non-A	0	(α)	(α)
Total	(B)	(β)	N

Consider, for example, the cases where $\delta_{AB} > 0$, and $(A) > (B)$. In such cases, δ_{AB} and $\delta_{a\beta}$ are maximum, while $\delta_{A\beta}$ and δ_{aB} are minimum at the same time, when the association table takes the form as shown in Table 2. From Table 2 it follows that four coefficients, K_{AB} , $K_{A\beta}$, K_{aB} and $K_{a\beta}$, are

all the same, because we have:—

$$K_{AB} = \frac{(AB) - (A)(B)/N}{(B) - (A)(B)/N} = \frac{N\delta}{(B)\{N - (A)\}} = \frac{N\delta}{(\alpha)(B)}, \quad (3)$$

$$K_{A\beta} = \frac{(A\beta) - (A)(\beta)/N}{(A) - (B) - (A)(\beta)/N} = \frac{-N}{(A)\{N - (\beta)\} - (B)N} = \frac{N\delta}{(\alpha)(B)},$$

$$K_{aB} = \frac{(\alpha B) - (\alpha)(B)/N}{-(\alpha)(B)/N} = \frac{-N\delta}{-(\alpha)(B)} = \frac{N\delta}{(\alpha)(B)}$$

and
$$K_{a\beta} = \frac{(\alpha\beta) - (\alpha)(\beta)/N}{(\alpha) - (\alpha)(\beta)/N} = \frac{N\delta}{(\alpha)\{N - (\beta)\}} = \frac{N\delta}{(\alpha)(B)}$$

$K_{A\beta}$ or K_{aB} does not take a negative sign even if $\delta_{A\beta}$ or δ_{aB} is negative. When it is desired to distinguish positive and negative association, the denominators may be substituted by their absolute values.

Similar expressions for K_{AB} as (3) can be obtained in the cases other than $\delta_{AB} > 0$ and $(A) > (B)$. But, the use of the formulae (1) and (2) is rather recommendable, where we have at once, from Table 1, that $(AB)_{max}$ is either (A) or (B) (whichever is the less) and $(AB)_{min}$ is either $(A) + (B) - N$ or 0 (whichever is the greater)⁽¹⁾.

The new coefficient K_{AB} fulfils, like other coefficients, the following conditions:— (i) $K_{AB} = 0$, when $\delta_{AB} = 0$; (ii) $K_{AB} = 1$, when the association is

Table 3.

Characters	b_1	b_2	t_j			Total
a_1	(a_1b_1)	(a_1b_2)		(a_1b_j)		(a_1)
a_2	(a_2b_1)	(a_2b_2)		(a_2b_j)		(a_2)
$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$
a_i	(a_ib_1)	(a_ib_2)		(a_ib_j)		(a_i)
$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$
Total	(t_1)	(b_2)		(b_j)		N

(1) YULE, G. U : An introduction to the theory of statistics. 10 ed. London 1932, p. 29.

most intense; and (iii) K_{AB} increases and decreases continually, and in fact linearly, with δ_{AB} , though $dK_{AB}/d\delta_{AB}$ is in general discontinuous at $\delta_{AB}=0$.

Manifold classification. When the classification with regard to two attributes is manifold and the individual observation has either one of the characters a_1, a_2, a_3 , etc. and at the same time either one of the second characters b_1, b_2, b_3 , etc., the association table such as Table 3 can be transformed into a fourfold one with respect to a_i and Non- a_i and b_j and Non- b_j , and $K_{a_i b_j}$ can be calculated as usual. But the result is obtainable directly from Table 3, because the formulae (1) and (2) can be write down by using only (a_i) , (b_j) , $(a_i b_j)$ and N .

When one of the attributes is dichotomous as in Table 4, we have always $K_{a_1 b_i} = K_{a_2 b_i}$ for all values of i .

Table 4.

Characters	b_1	b_2		b_i		Total
a_1	$(a_1 b_1)$	$(a_1 b_2)$		$(a_1 b_i)$		(a_1)
a_2	$(a_2 b_1)$	$(a_2 b_2)$		$(a_2 b_i)$		(a_2)
Total	(b_1)	(b_2)		(b)		N

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