The Development of Spherical Gas Bearings for the 3DRW Operated by a Small Air Supplier

By Keita TANAKA1), Yuichi TSUDA2), Takanao SAIKI2), Yoji SHIRASAWA3), Ryo JIFUKU1)

1) Department of Aeronautics and Astronautics, The University of Tokyo, Tokyo, Japan
2) The Institute of Space and Astronautical Science / Japan Aerospace Exploration Agency, Sagamihara, Japan
3) JAXA Space Exploration Center / Japan Aerospace Exploration Agency, Sagamihara, Japan

(Received June 27th, 2011)

The three-dimensional reaction wheel (3DRW) is a three-axis free rotor reaction wheel, which can achieve a compact and high integrity attitude control systems. In the system we propose, a spherical rotor is levitated without mechanical contact by gas bearings and rotated by the torque generated by magnetic fields. To realize the precise control of the rotor, it is important to understand the characteristics of spherical gas bearings such as pressure distribution, load capacity, stiffness and flow rate. Some previous works clarified them but few researches are there relating the use of gas bearings in the space. What is required for the space gas bearing is having the minimum robust characteristic against various small disturbances. It is more appropriate to adopt small-light air compressor which can provide the minimum air pressure and flow than big-heavy powerful one. In this study, the authors have developed a spherical gas bearing with a circular slot restrictor and analyzed its performance by experiments and numerical calculations. The result shows that this type of bearing can generate necessary floating force with low supply pressure and low flow rate, and that leaves the possibility to realize 3DRW system operated by a compact pump such as piezoelectric type.

Key Words: Three Dimensional Reaction Wheel (3DRW), Spherical Gas Bearing

Nomenclature

- \( B \) : width
- \( g \) : gravity constant
- \( h \) : thickness
- \( l \) : length
- \( M \) : mass
- \( p \) : pressure
- \( q \) : flow rate
- \( R \) : radius
- \( W \) : load capacity

Greeks

- \( \alpha \) : correction term for slot width
- \( \beta \) : correction term for the air-film thickness \( h \)
- \( \mu \) : viscosity
- \( \theta \) : angle measured from load line

Subscripts

- \( a \) : atmosphere
- \( i \) : inlet
- \( s \) : supply
- \( sl \) : slot

1. Introduction

The three-dimensional wheel (3DRW) is a system for the control of the attitude of a satellite. It is characterized by a single spherical wheel, which is levitated and can rotate about any one of the axes. Compared with traditional reaction wheel systems, 3DRW has several advantages: (i) It requires only single flywheel and therefore can reduce the weight and the volume of the system. (ii) It can avoid cross-coupling effects resulting from the existence of the three separated wheel (Fig. 1).

The principal concept of 3DRW was originally proposed by Muller et al. They pointed out the possibility that the spherical-flywheel system would achieve a light-weight and powerful attitude control system of space vehicles, but the question of its feasibility remains unsettled.

Developing 3DRW requires two major challenges: (i) A technology which realize the stable control of the position of the rotor without any contact and (ii) a technology which make it possible to rotate the rotor about any one of axis to generate the necessary torque to stabilize the attitude of a satellite. In this paper, we concentrate on a position control system of the rotor, a bearing, and show the characteristic of it.

IWAKUKA et al. developed the first experimental model of 3DRW which used magnetic force to both levitate and rotate the rotor. They stated the system required great magnetic attraction to levitate the rotor counteracting the
gravity, but conversely, increasing levitation force disturbed the smooth rotation of the rotor.

SHIRASAWA and TSUDA proposed using a gas bearing for levitation system instead of attracting the rotor by electric magnet. The merit of introducing gas bearings is that there is no interference with electronic, magnetic and other force fields, and the bearing system can be completely separated from other components and that makes the design and analysis of it easy.

The biggest achievement of their work is that they specifically showed the feasibility of the 3DRW by developing gas-levitation-magnetic-rotation system. They also pointed out if the system was used in the space, the circulation of the air come to an issue.

The use of gas bearing as a levitation system is originally put for ground experiments which need to take account of the existence of the gravity. However, the system found it possible to be used if it is brushed up by eliminating waste. It would be required to understand the characteristics of gas bearings to make good one as well as to introduce a small light pump such as piezoelectric type.

The main features of gas bearing are as pressure distribution, load capacity, stiffness and flow rate. Some previous works clarified them but few researches are there relating the use of gas bearings in the space. What is required for the space gas bearing is having the minimum robust characteristic against various small disturbances. In addition, it is more appropriate to adopt small-light air compressor which can provide the minimum air pressure and flow than big-heavy powerful one.

In this study, the authors have developed a spherical gas bearing with a circular slot restrictor and analyzed its performance by experiments and numerical calculations. The result shows that this type of bearing can generate necessary force can be neglected.

The Reynolds equation of the time-invariant flow in a spherical coordinate can be obtained by replacing \( R \) with \( \sin \theta \) in Eq. (4),

\[
\begin{align*}
\frac{\partial}{\partial \theta} & \left( \frac{1}{\mu} \frac{\partial p}{\partial \theta} \right) + \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left( \frac{1}{2} \sin \theta \frac{\partial \omega}{\partial \theta} \right) = \frac{q_\theta}{\sin \theta} + \frac{q_x}{\cos \theta} \\
\text{where} \quad q_\theta & \text{is the flow in the } \theta \text{ direction,} \quad h \text{ is the thickness of the gap and } B \text{ is the width of the gap.}
\end{align*}
\]

2. The Static Property of A Spherical Gas Bearing

2.1. Air-film thickness

A spherical gas bearing is different from a disc bearing that the air-film thickness is not uniform but varies depending on the position on the bearing surface. As Fig. 2 shows, it is presumed that the bearing has the same radius of curvature as the rotor and it is displaced in the center when the rotor is levitated. The air-film thickness \( h \) is the function of the angle measured from the load line \( \theta \). Applying the law of cosines, the following equation can be obtained.

\[
h'(\theta) = h \cos \theta + R - \sqrt{R^2 - h^2 \sin^2 \theta} \tag{1}
\]

where \( h \) is the thickness of the air-film at the bottom of the bearing, that is, rotor-levitation width. When \( h \) is sufficiently small compared to the radius of the rotor \( R \), Eq. (1) becomes

\[
h'(\theta) = h \cos \theta \tag{2}
\]

This approximation works well in the bearing performance calculation.

2.2. Pressure distribution, load capacity, flow rate

The load capacity of the bearing \( W \), which means the ability to sustain a mass, is obtained by integrating the pressure over the bearing surface.

\[
W = \int_p (p(\theta) - p_i) \cos \theta \cdot 2\pi R \sin \theta \cdot R d\theta
\]

where \( \theta_1 \) is the edge angles and shown in Fig. 3.

It is satisfactory to simplify the fluid problem on aerostatic bearings by considering laminar, viscous and incompressible flow. Because the gap is very narrow, the velocity of the flow normal to the direction of the flow can be neglected and thus the pressure is assumed to be constant over that direction. The flow forms the complete laminar flow and there is no disturbance. The force acting on the air is mainly caused by the pressure gradient and the viscous shearing, and the inertia force can be neglected.

The Reynolds equation of the time-invariant flow in a narrow parallel gap is given by

\[
q_x = -B h \frac{\partial p}{12 \mu} \tag{4}
\]

where \( q_x \) is the flow in the \( x \) direction, \( h \) is the thickness of the gap and \( B \) is the width of the gap.

2.2.1. A spherical slot-inlet bearing

Fig. 3 shows the geometry of the aerostatic bearing with a slot-type restrictor. \( \theta \) indicates the angle of the slot inlet, and \( \theta_1 \) is the edge angles. The Reynolds equation in the spherical coordinate can be obtained by replacing \( B \) with \( 2\pi \sin \theta \) and \( x \) with \( R \theta \) in Eq. (4),

\[
q_\theta = -2\pi R \sin \theta \left( \frac{h \cos \theta}{12 \mu} \right) \frac{1}{R} \frac{\partial p}{\partial \theta} \tag{5}
\]

This means the pressure within the slot-inlet is assumed to be constant, being equal to inlet pressure \( p_i \). Through integrating Eq. (5) under the boundary conditions, Eq. (6), the flow rate out of the bearing \( q_{out} \) is derived.

\[
q_{out} = \frac{\pi h^3}{3\mu} \frac{p_i - p_a}{\tan^2 \theta_1 - \tan^2 \theta_1 + 2\log \left( \frac{\tan \theta_1}{\tan \theta} \right)} \tag{7}
\]
where Eq. (3) can sustain the rotor weight, condition that the load capacity of the bearing calculated in unknown. It is calculated by considering the necessary bearing substituting written as follows.

The flow rate into the bearing equivalent to the flow in a narrow circular cylindrical gap. After redeploying Eq. (9), the inlet pressure

\[ p_{\text{i}} - p_s = \frac{\tan^2 \theta_1 - \tan^2 \theta_2 + 2 \log \left( \frac{\tan \theta_1}{\tan \theta_2} \right)}{\tan^2 \theta_1 - \tan^2 \theta_2 + 2 \log \left( \frac{\tan \theta_1}{\tan \theta_2} \right)} (p_i - p_s) \]  

(8)

Here, the pressure at the inlet of the bearing \( p_i \) is still unknown. It is calculated by considering the necessary condition that the load capacity of the bearing calculated in Eq. (3) can sustain the rotor weight, \( Mg \).

\[ Mg = \int_0^\phi (p_i - p_s) \cos \theta \cdot 2 \pi R \sin \theta \cdot Rd\theta \]  

\[ + \int_0^\phi (p_i - p_s) \cos \theta \cdot 2 \pi R \sin \theta \cdot Rd\theta \]  

\[ = \pi R^2 (p_i - p_s) \times \left[ -\frac{1}{2} \cos^2 \theta - 2 \sin^2 \theta \log \left( \frac{\tan \theta_1}{\tan \theta_2} \right) \sin \theta_2 + \frac{\tan^2 \theta_2 - \tan^2 \theta_1 + 2 \log \left( \frac{\tan \theta_1}{\tan \theta_2} \right)}{\tan^2 \theta_1 - \tan^2 \theta_2 + 2 \log \left( \frac{\tan \theta_1}{\tan \theta_2} \right)} \right] \]  

(9)

After redeploying Eq. (9), the inlet pressure \( p_i \) is derived as follows.

\[ p_i - p_s = \frac{\pi R^2 (p_i - p_s)}{1 + \cos^2 \theta - 2 \sin^2 \theta \log \left( \frac{\tan \theta_1}{\tan \theta_2} \right) \sin \theta_2 + \frac{\tan^2 \theta_2 - \tan^2 \theta_1 + 2 \log \left( \frac{\tan \theta_1}{\tan \theta_2} \right)}{\tan^2 \theta_1 - \tan^2 \theta_2 + 2 \log \left( \frac{\tan \theta_1}{\tan \theta_2} \right)} \} Mg \]  

(10)

This equation expresses that the inlet pressure \( p_i \) depends on the configuration of the bearing and the rotor weight. By substituting \( p_i \) of Eq. (10) into Eq. (7), the flow rate out of the bearing \( q_{\text{out}} \) is rewritten as

\[ q_{\text{out}} = \frac{h^3}{3 \mu} \frac{Mg}{R^2} \left( \tan^2 \theta_1 - \tan^2 \theta_2 \right) \]  

(11)

Next, the flow within the bearing, that is, from the chamber to the slot-inlet on the bearing surface is considered. It is equivalent to the flow in a narrow circular cylindrical gap. The flow rate into the bearing \( q_{\text{in}} \) can be obtained by replacing \( B \) with \( 2 \pi R \sin \theta_1 \) with \( h_a \) and integrating it.

\[ q_{\text{in}} = -2 \pi R \sin \theta_1 \frac{h^3}{12 \mu} \frac{p_i - p_s}{l_s} \]  

(12)

where \( h_a \) is the slot thickness and \( l_s \) is the slot length.

According to the law of mass conservation, the flow rate into the bearing and the flow rate out of the bearing should be equal. Using Eq. (11) and Eq. (12), the relationship between supplying pressure \( p_s \) and air film thickness \( h \) can be obtained.

\[ h^3 = \frac{\pi R^2 h^3}{2 Mg l_s} \sin \theta_1 (\tan^2 \theta_1 - \tan^2 \theta_2) (p_i - p_s) \]  

(13)

Thus the pressure distribution on the bearing surface is completely obtained.

### 2.2.2. A spherical slot-inlet ring-type bearing

A spherical ring-type bearing is the bearing which has a hole/outlet at the bottom of it. The typical geometry of this kind of bearings is shown in Fig. 4. The flow from the slot-inlet is divided into an upward and a downward flow and then released to outside. The pressure at the outlet at the bottom of the bearing is equal to the air pressure. The load capacity of this kind of bearings is generally worse than that of non-ring type bearings which have the same configuration, but the performance analysis of them is meaningful because in many case it is inevitable to create a hole in the bearing surface to install sensors to detect the motion of the rotor.

To seek the pressure distribution on the surface, the same Reynolds equation can be used as non-ring spherical bearings, that is, Eq. (5), except that calculating a downward flow requires to multiply the right side of the equation by the negative number to express the backward flow. As for boundary conditions, there is little difference. They become

\[ \theta = \theta_i : p = p_s \]  

\[ \theta = \theta_a : p = p_i \]  

\[ \theta = \theta_2 : p = p_s \]  

(14)

\( \theta_i \) is the angle of the slot inlet, and \( \theta_a \) and \( \theta_2 \) are the edge angles. Here, following new functions \( f \) and \( g \) are introduced to make equations simple.

\[ f(x, y) = \tan^2 y - \tan^2 x + 2 \log \left( \frac{\tan y}{\tan x} \right) \]  

\[ g(x, y) = -1 + \frac{\cos^2 y}{\cos^2 x} - 2 \sin^2 x \log \left( \frac{\tan y}{\tan x} \right) \]  

(15)

Integrating the Reynolds equation under the boundary conditions Eq. (14) gives the flow rate out of the bearing.

\[ q_{\text{out}} = \frac{\pi h^3}{3 \mu} \frac{p_i - p_s}{l_s} \left( \frac{1}{f(\theta, \theta_1)} + \frac{1}{f(\theta, \theta_2)} \right) \]  

(16)
The load capacity can be calculated by integrating the pressure over the whole surface of the bearing.

\[ Mg = \int_{\theta_0}^{\theta_1} (p_d(\theta) - p_a) \cos \theta \cdot 2 \pi R \sin \theta \cdot Rd\theta + \int_{\theta_0}^{\theta_2} (p_u(\theta) - p_a) \cos \theta \cdot 2 \pi R \sin \theta \cdot Rd\theta \]

\[ = \pi R^2 (p_i - p_a) \left(\frac{g(\theta_1, \theta_0)}{f(\theta_1, \theta_0)} + \frac{g(\theta_0, \theta_0)}{f(\theta_0, \theta_0)}\right) \]

(17)

where \( p_d \) and \( p_u \) respectively represent the pressure distribution of the downward flow and the upward flow.

After redeploying Eq. (17), the inlet pressure \( p_i \) is derived as follows.

\[ p_i - p_a = \frac{Mg}{\pi R^2} \left(\frac{g(\theta_1, \theta_0)}{f(\theta_1, \theta_0)} + \frac{g(\theta_0, \theta_0)}{f(\theta_0, \theta_0)}\right) \]

(18)

The outflow rate is rewritten as

\[ q_{out} = \frac{h^3}{3 \mu R^2} \frac{Mg}{f(\theta_1, \theta_0)g(\theta_1, \theta_0) - f(\theta_0, \theta_0)g(\theta_0, \theta_0)} \]

(19)

The inflow rate is given by Eq. (12). In summary, air-film thickness \( h \) can be calculated as a function of the inlet pressure \( p_i \).

\[ h^3 = \frac{\pi R^2 h_i^3 (p_i - p_a) \times}{2 Mg} \frac{\sin \theta_1 f(\theta_1, \theta_0)g(\theta_1, \theta_0) - f(\theta_0, \theta_0)g(\theta_0, \theta_0)}{f(\theta_0, \theta_0) - f(\theta_1, \theta_0)} \]

(20)

3. Experiment of Levitation

Some spherical gas bearing tests are performed for verification of the theory developed above.

3.1. Experimental set-up

Fig. 5 shows the rotor and the gas bearings which we use in the experiments. The steel flywheel ball has a radius of 17.46mm, whose weight is 173.5g. Two types of bearings are used for the test: The bearing No.1 is a perfect spherical bearing without any hole. The bearing No.2 is a spherical bearing with a hole at the bottom of it, but it can be sealed by an attachment not to leak the air. Table 1 summarizes the specifications of each bearing. It must be noted that these two types of bearings have the difference in the inlet and edge angle.

The compressed air is supplied from a hydraulic air pump and the air-film thickness is measured by a non-contact electrostatic displacement gauge. The experimental set-up is shown in Fig. 6.

<table>
<thead>
<tr>
<th>Table 1. Specification of the bearings.</th>
</tr>
</thead>
<tbody>
<tr>
<td>No.1</td>
</tr>
<tr>
<td>Curvature Radius: ( R ) [mm]</td>
</tr>
<tr>
<td>Inlet angle: ( \theta_1 ) [deg]</td>
</tr>
<tr>
<td>Outer edge angle: ( \theta_2 ) [deg]</td>
</tr>
<tr>
<td>Slot width: ( h_s ) [( \mu )m]</td>
</tr>
<tr>
<td>Slot length: ( l_s ) [mm]</td>
</tr>
</tbody>
</table>

3.2. Experimental results

3.2.1. Bearing No.1

Fig. 7 and Fig. 8 indicate respectively the air-thickness and flow rate with regard to the supplying pressure for the bearing No.1. The theoretical value is calculated using Eq. (13). The plots are the theoretical and experimental values. As these graphs show the calculation can predict almost accurately the levitation height but contains large error concerning the flow rate.
3.2.2. Bearing No.2

The experimental and theoretical results for the bearing No.2 appear in Fig. 9 and Fig. 10. The bearing No.2 can change its performance by attaching/detaching a seal-like component to/from the hole at the bottom of it. In the following graphs, Fig. 9 and Fig. 10 show the results of the with-seal-version and Fig. 11 and Fig. 12 represent without-seal (ring-type) version.

We see from these graphs that there are much error between the theoretical values and the experimental values compared with the result of the experiment using the bearing No.1. The experiment results in achieving less levitation height with less air flow rate than expected.

In addition, these results indicate that making a hole at the bottom of the bearing can much affect its load capacity but not its flow rate.

3.3. Modified analytic calculation

As Figures above show, there are errors between the analytic calculation and the experimental results. These errors can be considered to be as results of manufacture errors and some approximations which are used to develop the equations. Especially the slot width, which significantly affects the flow rate, is designed to be only 5μm and this degree of accuracy is very difficult to achieve by the existing techniques. Thus, in this section, we introduce two correction terms, α for the slot width $h_{sl}$,

$$h_{sl, modified} = \alpha h_{sl} \quad (21)$$

and β for the air-film thickness $h$.

$$h_{modified} = \beta h \quad (22)$$

Changing α mainly affects the flow rate of the system and β expresses the efficiency of the levitation which considers the effectiveness of the approximation and other manufacture errors.

3.3.1. Bearing No.1

When α is 1.19 and β is 0.8, the analytic results are shown in Fig. 13 and Fig. 14. The calculation can predicts more precisely than that of the previous section. Thus, the bearing 1 is considered to have 1.19 times wider slot width than designed and its efficiency is 0.8.
3.3.2. Bearing No.2

The analytic results are shown in Fig. 15 to Fig. 18 when $\alpha$ is 0.64 and $\beta$ is 0.35. You can see from these graphs that calculation can make better predictions. Different from the bearing No.1, the slot width is considered to be narrow (estimated $h_{sl} = 3.18\ \mu m$) and the efficiency is down to 0.35.

4. Conclusion

In this study, the authors have developed a spherical gas bearing and tested its performance by both experiments and numerical calculations. The result leads us to the conclusion that the gas bearing which have been tested in this study can serve as a position-control system of 3DRW. The following is its reasoning.

The calculation can almost exactly predict the bearing performance, the levitation height and the flow rate when the supply pressure is given (Fig. 7 and Fig. 8). It also suggests that the small supplying pressure, only 0.04 MPa, makes the rotor levitated. This fact shows the possibility that it would be able to be used as a sustainment system for 3DRW.
The second experiment shown in Fig. 9 to Fig. 12 insists that the bearing No. 2 cannot get as good performance as No. 1. It requires 0.2 MPa to levitate the flywheel. It is likely that this performance inferior is caused by the manufacture error. Because the experiment system requires very high degree of accuracy as much as 1.0 \( \mu \)m, it is not easy to eliminate all the manufacture error.

In the section 3.3, we introduce two correction terms relating the slot width \( h_s \) and levitation height \( h \) to make up for the difference between the real case and ideal case. As a result, the error of the analytic calculation in the levitation height and the flow rate can be modified in both experiments of the bearing No.1 and No.2. The bearing No. 2 is considered to have a narrower slot than the designed and that makes the flow of air interrupted. (Fig. 10, 12, 16 and 18)

Another notable achievement from these experiments is to confirm that it is possible to float the ball with ring-type bearings. When developing 3DRW, it is very important to monitor the movement of the ball while controlling it. The problem of how to install sensors always accompanies the design procedure. The result we have got here indicates that making the bearing a ring-type does not necessarily disturb the development of 3DRW.

As just described, all the results can support the feasibility of 3DRW.

There are some studies relating 3DRW. Muller, et al.\textsuperscript{1)} firstly proposed the concept of it and then we\textsuperscript{2,3)} have made several test models and conducted experiments. However, the optimum solution about the ball-supporting system is not obtained yet. For now, as we have insisted, the aerostatic bearing is considered to be the most possible system because of its usability.

Several researches have been made on thrust gas bearings. KAWASAKI\textsuperscript{4)} and SASAKI et al.\textsuperscript{5)} developed the theoretical solutions of pressure distribution, flow rate, load capacity of a spherical bearing. MORI et al.\textsuperscript{6)} analyzed theoretically the thrust collar bearings with a slit-supply. All their studies are about the bearings which are planned to be used on the ground which sustain a big-heavy ball with powerful air supply. Thus it is necessary to confirm whether their theory can be applied to small, low-air-supply systems.

The achievement of this study would be useful in developing more practical 3DRW. Applying slot-inlet gas bearings requires less air-supply power than other types of inlets. In addition, the fact the ring-type bearings work well expands the range of design of 3DRW.

The problem which we have to consider next is to estimate the stability of bearings such as the stiffness and the damping. To theoretically calculate the values of them is not difficult, but to measure is another story. Verifying the data integrity of them between theories and experiments is demanding, because they are exactly what are required to check to create the practical 3DRW which can be operated in the space of gravity-less.

In this study, we have conducted the test on the ground where the gravity exists and we have developed the bearing system which can resist the gravity of the ball. In fact, however, it is not necessary for the bearing to generate such a huge power when it is used in space. What is required for the space gas bearing is having the minimum robust characteristic against various small disturbances. Thus how to simulate the space condition and how to execute experiments are problems to be solved.

Acknowledgments

We would like to express our thanks to Ono Denki Co. for cooperating in developing the gas bearings.

References

2) Yoji SHIRASAWA and Yuichi TSUDA: EXPERIMENTAL STUDY AND ANALYSIS ON THREE DIMENSIONAL REACTION WHEEL FOR MICRO-SATELLITES, AAS 08-132, 2008.