Visualization experiments employing rectangular cross-section curved channels were performed in order to examine the fundamental characteristics of a curved detonation wave propagating stably through an annular channel. A stoichiometric ethylene-oxygen mixture gas and five types of curved channels with different inner radii of curvature were used. The detonation waves propagating in the curved channels were curved due to the expansion waves from the inner walls of the curved channels. The ratio of the inner radius of curved channel ($r_i$) to the normal detonation cell width ($\lambda$) was an important factor determining the stability of the curved detonation waves. The detonation propagation mode in the curved channels transitioned from unstable to stable in the range $14 \leq r_i/\lambda \leq 26$. The normal detonation velocity ($D_n$) of the curved detonation wave propagating stably in a curved channel was approximately formulated. The approximated $D_n$ given by the formula agreed well with the experimental results. The front shock shape of the curved detonation wave could be reconstructed accurately using the formula. The value of $D_n$ nondimensionalized by the Chapman-Jouguet detonation velocity became a function of the local curvature of the curved detonation wave ($\kappa$) nondimensionalized by $\lambda$ regardless of the shape of curved channel. The front shock shapes of the detonation waves in the stable mode became similar to each other under constant $r_i/\lambda$ conditions.

Key Words: Curved Detonation, Curvature, Detonation Cell, Annular Channel, Self-similarity

Nomenclature

- $D$ : detonation velocity
- $m$ : exponent in Eq. (5)
- $p$ : pressure of mixture gas
- $r$ : distance from the polar coordinate origin to an arbitrary point on the detonation wave
- $t$ : time
- $\phi$ : angular difference between rotational and tangential directions at an arbitrary point on the detonation wave
- $\kappa$ : local curvature of detonation wave
- $\lambda$ : normal detonation cell width
- $\theta$ : angle from the initial line to an arbitrary point on the detonation wave
- $\omega$ : angular velocity of an arbitrary point on the detonation wave

Subscripts

- $0$ : initial condition
- $asy$ : asymptotic value at a distance sufficiently far from the inner wall of a curved channel
- $CJ$ : Chapman-Jouguet state
- $ex$ : experimental value
- $i$ : inner wall of the curved section of a curved channel

$k$ : index of a division point on the detonation wave
$n$ : normal direction
$rec$ : reconstructed value
$str$ : straight section of the curved channel

1. Introduction

A rotating detonation engine (RDE) is a propulsion system which generates thrust by propagating a detonation wave in an annular combustor\textsuperscript{1-4}. Since the RDE cycle operates with near-constant volume combustion, RDE theoretically has a higher thermal efficiency than other conventional propulsion systems. The RDE needs just one initiation process (ignition and deflagration-to-detonation transition) in principle because the RDE utilizes the detonation wave that propagates continuously within the combustible gas layer in an annular combustor. Since the annular combustor has a curved shape and the opened aft end, the propagation of the detonation wave is affected by expansion waves, and consequently the front of the detonation wave is curved\textsuperscript{5}. For the purpose of RDE design, it is important that the fundamental propagation characteristics of the curved detonation wave in the annular combustor should be well understood.

However, the experimental studies of a curved detonation wave are scarce. Kudo et al. visualized the detonation waves of a stoichiometric ethylene-oxygen mixture gas propagating...
through rectangular cross-section curved channels with constant inner and outer radii of curvature\(^6\)). They experimentally demonstrated that a detonation wave curved by expansion waves could propagate stably through a curved channel. Nakayama et al. simultaneously visualized the front shock shapes and trajectories of triple points of curved detonation waves propagating through curved channels by employing the same mixture gas and multi-frame short-time open-shutter photography (MSOP)\(^9\). They showed that the cell structure of a curved detonation wave propagating stably through a curved channel was maintained although the detonation cell width in the vicinity of the inner wall of the curved channel was enlarged due to the expansion waves from the inner wall. They also examined the relation between the front shock shape and normal detonation velocity of the curved detonation wave by using MSOP images\(^5\). However, since the detonation cell width was so large under the filling pressure conditions of the mixture gas in their study that it was difficult to define the front shock shapes precisely from the triple point trajectories recorded in MSOP images. Therefore, the condition where their quantitative examination on the relation was performed was limited to a certain high filling pressure condition\(^9\).

The propagation characteristics of a curved detonation wave have not yet been sufficiently understood due to complexity. Therefore, we examined its fundamental propagation characteristics by using curved channels which are a basic element of an annular combustor of RDE. A rectangular cross-section was employed in order to make the propagation of the curved detonation waves in the curved channels two-dimensional. The inner radii of the curved channels and/or the filling pressure of the mixture gas were parametrically altered in order to elucidate the condition where the curved detonation waves are stabilized in the curved channels. We focus our attention upon the stabilized curved detonation waves, propose a formula which gives rigorous geometrical front shock shapes for them, and examine the factors determining their normal propagation velocities and front shock shapes under a wide range of filling pressure conditions of the mixture gas.

2. Geometric Front Shock Shape of Detonation Waves Propagating Stably Through a Curved Channel

As shown in Fig. 1, the center of the inner and outer radii of curvature of a curved channel is defined as the origin, and the boundary between the curved section and the straight section of the curved channel is defined as the initial line in a two-dimensional polar coordinate system. It is assumed in Fig. 1 that the front shock shape of a detonation wave changes gradually as the detonation wave propagates through the curved channel and eventually becomes a specific curved shape where the curvature increases toward the inner wall of the curved channel. If the detonation wave propagates stably while maintaining the specific curved shape, we can assume that the value of \(\omega\) is time-unvarying and constant everywhere on the detonation wave\(^5\)). Then, the value of \(\omega\) is given by \(D_n/r\). If \(dr, d\theta\), and \(d\theta\) are sufficiently small, one can consider the gray-colored areas in Fig. 1 to be right triangles. The following geometric relations can then be established at an arbitrary point \(P(r, \theta)\) on the detonation wave\(^5\)):

\[
\sin \phi = \frac{D_n}{r \omega}, \quad (1)
\]

\[
\tan \phi = \frac{\sin \phi}{\sqrt{1 - \sin^2 \phi}} = \frac{1}{r} \frac{dr}{d\theta}. \quad (2)
\]

The following differential equation is derived from Eqs. (1) and (2):

\[
\frac{d\theta}{dr} = \frac{\sqrt{(r \omega)^2 - D_n^2}}{D_n r}. \quad (3)
\]

If \(D_n\) is determined as a function of \(r\), the following relation between \(r\) and \(\theta\), which gives the front shock shape of the detonation wave propagating stably through a curved channel, is derived by integrating Eq. (3):

\[
\theta - \theta_i = \int \sqrt{(r \omega)^2 - D_n^2} dr. \quad (4)
\]

Fig. 1. Geometric relationship in a detonation wave propagating stably through a rectangular cross-section curved channel with constant inner and outer curvature radii.

The detonation velocity is slowed down in the vicinity of the inner wall of a curved channel due to the strong expansion effect\(^5\)). Hence, we suppose that \(D_n\) becomes the slowest on the inner wall where the influence of the expansion waves from the inner wall becomes the strongest. We also suppose that \(D_n\) increases as the distance from the inner wall increases and approaches asymptotically to a certain detonation velocity at a distance sufficiently far from the inner wall. As a formula which gives approximately such characteristics of \(D_n\), we propose the following formula:

\[
D_n = D_{n_{\infty}} - \left(D_{n_{\infty}} - D_{n_{i}}\right) \left(\frac{r}{r_i}\right)^m. \quad (5)
\]

\(D_n\) increases from \(D_{n_{i}}\) to \(D_{n_{\infty}}\) asymptotically with increasing
The values of \( d \) in Eq. (5).

By recording the front shock shape of the detonation wave at an arbitrary time interval and dividing it at a regular interval as shown in Fig. 1, one can pick up the coordinate values of the points of division and quantify the front shock shape of the detonation wave. In the present study, the front shock of the detonation wave recorded is divided into fifteen parts at a regular interval in the direction of \( r \) as shown in Fig. 1. The values of \( D_{n,asy} \) and \( m \) are determined by the trial-and-error technique in such a way that the residual sum of squares (RSS) between the front shock shape of the detonation wave reconstructed by using Eqs. (4) and (5) and the coordinate values of the front shock position of the detonation wave acquired in an experiment becomes the smallest. The RSS is calculated according to the following equation:

\[
\text{RSS} = \sum_{k=0}^{15} (r_{n,k} - r_{0,k})^2. \tag{6}
\]

In the present study, in order to find the appropriate pair of \( D_{n,asy} \) and \( m \) which gives the smallest RSS, the values of \( D_{n,asy} \) and \( m \) are altered randomly at the intervals of 0.005 times \( D_{CJ} \) and 0.05, respectively.

The \( \omega \) values of each division point can be determined observing the alternation of their positions from moment to moment. If a detonation wave propagates stably through a curved channel while maintaining a specific curved shape as shown in Fig. 1 at a constant curvature (5 mm, 10 mm, 20 mm, 40 mm, and 60 mm) were used. The inner and outer radii of curvature within the curved sections of the curved channels are maintained constant. The cross-section of these channels is rectangular (20 mm in width by 16 mm in depth). A deflagration wave transitions to a detonation wave within the Shchelkin spiral section mounted in the circular cross-section tube. A low-vacuum dump tank of 0.037 m\(^3\) is connected to the outlet port of the observation chamber, and a plastic diaphragm separates the dump tank and observation chamber. The detonation wave passing through the curved channel ruptures the diaphragm, and the high pressure and temperature gas generated by the detonation wave is caught in the dump tank.

Five types of curved channels with different inner radii of curvature (5 mm, 10 mm, 20 mm, 40 mm, and 60 mm) were used. The inner and outer radii of curvature within the curved sections of the curved channels are maintained constant. The cross-section of these channels is rectangular (20 mm in width by 16 mm in depth).

A stoichiometric ethylene-oxygen mixture gas was used in the present study since it was easy to stabilize the propagation of detonation waves of this mixture gas in the curved channels. The mixture gas is filled at a given pressure into the observation chamber in which the air is evacuated. The range of the filling pressure is from 20±1 kPa to 100±1 kPa and the mixture gas is filled at room temperature.

In the previous study\(^{(7-9)}\), we employed MSOP in order to investigate the relation between the behavior of the front shock shape and the detonation cell structure of a detonation wave propagating through a curved channel. In the present study, our attention is focused on the front shock shape of a curved detonation wave propagating stably through a curved channel. Therefore, we employed the shadowgraph method which is suitable for visualization of the front shock shape of a detonation wave under a wide range of filling pressure conditions of the mixture gas. The front shock shapes of detonation waves propagating through the curved channels were recorded using a high-speed video camera (Shimadzu HPV-2). All the images were taken at 2-\( \mu \)s time intervals in each experiment. The spatial resolution of the images is approximately 0.3 mm.
4. Experimental Results

4.1. Planarity and velocity of a detonation wave in the straight section of a curved channel

The front shock shape of a detonation wave propagating through the straight section of a curved channel can be defined from a shadowgraph image. It was verified that the detonation waves propagating through the straight sections of the curved channels were almost planar. Figure 3 shows the measurement results of $D_n\alpha$. The symbols represent the measured values and the solid line is the value of $D_{CJ}$ calculated using CEA\textsuperscript{13}. The error bar represents the typical possible systematic error of measurement resulting from the spatial resolution of the high-speed video camera. (The error bars used in figures that follow Fig. 3 represent the same definition.) Although the values of $D_n\alpha$ measured under the condition of $p_0 = 20$ kPa are a little bit overdriven and larger than $D_{CJ}$, since the deflagration-to-detonation transition may occur immediately before the straight section of a curved channel under this condition, almost all the values of measured $D_n\alpha$ are in good agreement with $D_{CJ}$. Therefore, we employ $D_{CJ}$ as a reference velocity in the present study when we nondimensionalize $D_n\alpha$.

![Fig. 3. Velocities of the detonation waves propagating through the straight sections of the curved channels.](image)

4.2. Definition of detonation propagation modes in a curved channel

The typical variations in $D_n\alpha$ are shown in Fig. 4. The values of $D_n\alpha$ are nondimensionalized by $D_{CJ}$. In the present study, the propagation of detonation waves through the curved channels are categorized into three modes based on how high the values of $D_n\alpha/D_{CJ}$ are as in the case of the study of Kudo et al.\textsuperscript{60} The propagation mode that consistently satisfies the relation $D_n\alpha/D_{CJ} \geq 0.8$ is defined as the stable mode, the mode that can not satisfy the relation $D_n\alpha/D_{CJ} \geq 0.8$ but can consistently satisfy the relation $D_n\alpha/D_{CJ} \geq 0.6$ is defined as the critical mode, and the mode in which $D_n\alpha/D_{CJ} < 0.6$ even just once is defined as the unstable mode. In Fig. 4, circles correspond to the stable mode, triangles to the critical mode, and crosses to the unstable mode. As shown in Fig. 4, the variation range of $D_n\alpha/D_{CJ}$ is narrow in the case of the stable mode, however that is wide in the case of the critical and unstable mode. In the present study, attention is focused on the detonation waves in the stable mode.

![Fig. 4. Typical variations in the normal detonation velocities on the inner wall of a curved channel ($r_i = 20$ mm).](image)

As an example of an image which shows a detonation wave propagating stably in a curved channel, the shadowgraph image of the detonation wave corresponding to the stable mode shown in Fig. 4 is depicted in Fig. 5. The MSOP image taken in the same condition employing the curved channel of
1-mm in depth\(^{13}\) is also shown in this figure for reference. One can see in Fig. 5(a) that although the detonation wave is curved due to the expansion waves from the inner wall of the curved channel, its propagation is stabilized and the smooth detonation wave front can be maintained consistently in the stable mode. One can also see in Fig. 5(b) that the detonation cell in the vicinity of the inner wall is enlarged soon after the detonation wave enters the curved section of the curved channel due to the expansion waves from the inner wall. Tsutboi et al. also showed in their two-dimensional numerical simulation that the same phenomenon can be observed when a detonation wave propagates through a 90-degree bent tube\(^{10}\).

In Fig. 5(b), new detonation cells are generated smoothly within the enlarged cells in the vicinity of the inner wall and the cell structure is maintained. Since the detonation cell width is sufficiently small compared to the inner radius of curved channel under this condition, the increasing rate of the detonation cell width is also sufficiently small. Consequently, transverse waves of sufficient strength are maintained, instabilities grow, and new cells are generated\(^{12}\). It is possible that this smooth cell generation maintains the consistently smooth detonation wave front in the stable mode as shown in Fig. 5.

### 4.3. Condition of the stable mode of a detonation wave in a curved channel

The relation between \(r_i\) and \(\lambda\) is shown in Fig. 6 in terms of the propagation mode of a detonation wave propagating through a curved channel. The values of \(\lambda\) are obtained using the relation \(\lambda = 72.020 p_0^{-1.127}\), where the units of \(\lambda\) and \(p_0\) are mm and kPa, respectively. This relation is obtained by applying the least-square method to the data set of \(\lambda\) of a stoichiometric ethylene-oxygen mixture gas extracted from the Detonation Database of the California Institute of Technology\(^{13}\) and the experimental result of Nakayama et al.\(^{9}\) At a given \(\lambda\) (or \(p_0\)), a larger \(r_i\) makes the detonation wave more stable. A smaller \(\lambda\) (or higher \(p_0\)) also makes the detonation wave more stable at a given \(r_i\).

In the present study, the unstable mode was not observed above the line \(r_i/\lambda = 14\), and the stable mode was consistently attained above the line \(r_i/\lambda = 26\). From this result, therefore, the propagation mode of a detonation wave propagating through a curved channel is considered to transition to the stable mode from the unstable mode within \(14 \leq r_i/\lambda \leq 26\) under the conditions of the present study.

![Fig. 5](image-url)

**Fig. 5.** Typical images of detonation waves in the stable mode \((r_i = 20 \, \text{mm}, p_0 = 60.8 \, \text{kPa} \text{ (Shadowgraph/60.3 kPa (MSOP))})\).

![Fig. 6](image-url)

**Fig. 6.** Relation between the inner radius of a curved channel and the normal detonation cell width in terms of the propagation mode.

It was experimentally shown in the present study that the fundamental propagation characteristics of detonation waves under the conditions of different \(r_i\) and/or \(p_0\) are similar if the detonation waves were in the stable mode. Therefore, the front shock behavior and reconstruction of the front shock shapes of detonation waves in the stable mode will be discussed in sections 4.4 and 4.5 by employing the detonation wave corresponding to the stable mode shown in Fig. 4 \((r_i = 20 \, \text{mm}, p_0 = 60.8 \, \text{kPa})\) as an example.

### 4.4. Front shock behavior of a detonation wave in the stable mode

Figure 7 shows an example of the moment-to-moment variation in the front shock shape of a detonation wave in the stable mode. The front shock shape is expressed by the \(r-\theta\) relation. The value of \(r\) is nondimensionalized using \(r_i\). The front shock shape eventually becomes a specific curved shape where the curvature increases toward the inner wall as the detonation wave propagates through the curved channel. Once the specific curved shape is attained, the detonation wave...
propagates while maintaining the specific curved shape. In the case of Fig. 7, the detonation wave propagates while maintaining the specific curved shape after about $\theta = 100$ deg.

The relations between $D_n$ and $\kappa$ of curved detonation waves propagating stably through the curved channels while maintaining a specific curved shape are investigated. Since the present study.

4.5. Reconstruction of the front shock shape of a detonation wave in the stable mode

Figure 9 shows an example of the experimental and approximated $D_n$ of a detonation wave in the stable mode. The symbols represent the experimental result obtained from Eq. (7) and the solid line represents the approximated $D_n$ given by Eq. (5). Both the experimental and approximated $D_n$ become the slowest on the inner wall of the curved channel and increase as if to approach asymptotically to a certain value with increasing $r/r_o$. The reconstructed $D_n$ is in good agreement with the experimental $D_n$.

4.6. $D_n$-$\kappa$ relation of detonation waves in the stable mode

From the results shown in Fig. 9 and Fig. 10, we can consider Eq. (5) to be an appropriate formula which gives approximately the $D_n$ distribution of a detonation wave in the stable mode at a given $r_i$ and $\lambda$ (or $p_0$) under the conditions of the present study.

![Fig. 7. Typical variation in the front shock shape of a detonation wave in the stable mode ($r_i = 20$ mm, $p_0 = 60.8$ kPa).](image1)

![Fig. 8. Typical distribution of the angular velocity of a detonation wave in the stable mode ($r_i = 20$ mm, $p_0 = 60.8$ kPa).](image2)

![Fig. 9. Typical reconstructed front shock shape of a detonation wave in the stable mode ($r_i = 20$ mm, $p_0 = 60.8$ kPa).](image3)

![Fig. 10. Typical distribution of the normal detonation velocity of a detonation wave in the stable mode ($r_i = 20$ mm, $p_0 = 60.8$ kPa).](image4)
front shock shapes of the detonation waves in the stable mode are successfully reconstructed in the present study, the values of \( \kappa \) can be determined applying Eq. (10) to the front shock shapes of the detonation waves reconstructed by using Eqs. (4) and (5). The ten cases of detonation waves at different \( r_i \) and/or \( \lambda \) (or \( p_0 \)) are selected for this investigation. In all the investigation cases, their propagation modes are consistently stable. Then, the stable zone shown in Fig. 6 can be covered by these investigation cases.

The relations between \( D_o \) and \( \kappa \) obtained from this investigation are shown in Fig. 11. The values of \( D_o \) and \( \kappa \) in this figure are nondimensionalized by \( D_{CJ} \) and \( \lambda \), respectively. One can see that \( D_o/D_{CJ} \) becomes a function of \( \lambda \kappa \) and decreases with increasing \( \lambda \kappa \). This result shows that \( D_o/D_{CJ} \) of a curved detonation wave propagating stably in a curved channel is determined by \( \lambda \kappa \) independently of \( r_i \) under the conditions of the present study. The \( D_o/\lambda \) relation, in which \( D_o \) and \( \kappa \) are nondimensionalized by \( D_{CJ} \) and the induction zone length scale respectively, has been examined theoretically by steady or quasi-steady one-dimensional analyses in which the chemical reaction is assumed to be the first-order exothermic reaction\(^{16-17}\). These analyses have shown that \( D_o \) is a function of \( \kappa \) in the one-dimensional Zel’dovich-von Neumann-Döring (ZND) model. The present study has experimentally shown that a similar relation can also be found in the cellular detonation wave of a stoichiometric ethylene-oxygen mixture gas propagating stably in a curved channel.

![Fig. 11. Relations between the normal detonation velocity and the local curvature of detonation waves in the stable mode.](image)

**4.7. Self-similarity of detonation waves in the stable mode**

Figure 12 shows the typical front shock shapes of detonation waves in the stable mode with constant \( r/\lambda = 36.3 \). One can see that the front shock shapes of the detonation waves are almost similar to each other under this condition. Therefore, one can suppose that the front shock shapes of the detonation waves propagating through the curved channels become almost similar to each other under constant \( r/\lambda \) conditions if their propagations are in the stable mode.

The stable mode boundary seems to be on the line \( r/\lambda = 26 \) in Fig. 6. A critical curved shape or curvature of detonation wave front permitting the stable propagation would exist on this boundary since the front shock shapes of the detonation waves are considered to be similar on this boundary. The propagation phenomena of the detonation waves might also be similar on the line \( r/\lambda = 14 \) determining the unstable mode boundary as in the case of the stable mode boundary. However, the self-similarity on this boundary has not yet been sufficiently elucidated in the present study since the propagation phenomena are considerably unsteady. Further experimental investigation is required in order to elucidate this problem.

![Fig. 12. Typical front shock shapes of detonation waves in the stable mode with constant \( r/\lambda = 36.3 \).](image)

**5. Conclusions**

The fundamental propagation characteristics of curved detonation waves stabilized in curved channels were investigated in the visualization experiment using the shadowgraph method and a high-speed video camera. A stoichiometric ethylene-oxygen mixture gas and five types of rectangular cross-section curved channels with different inner radii of curvature were used. The following results were obtained in the present study.

The ratio of the inner radius of curved channel (\( r_i \)) to the normal detonation cell width (\( \lambda \)) was an important factor determining the stability of detonation propagation in a curved channel. The critical condition under which the propagation mode of detonation wave transitioned from unstable to stable was having \( r_i \) equivalent to 14 to 26 times \( \lambda \).

The distribution of the normal detonation velocity (\( D_o \)) of a curved detonation wave in the stable mode was approximately formulated. The approximated \( D_o \) given by the formula agreed well with the experimental results. The wave front shape of the detonation wave could be reconstructed accurately using the formula.

In the stable mode, the value of \( D_o \) nondimensionalized by the Chapman-Jouguet detonation velocity became a function of the local curvature of the detonation wave (\( \kappa \)) nondimensionalized by \( \lambda \).

The front shock shapes of the detonation waves in the stable mode became almost similar to each other under constant \( r/\lambda \) conditions. A critical curved shape or curvature of detonation wave front permitting the stable propagation would exist on
the line $r/\lambda = 26$ determining the stable mode boundary since the front shock shapes of the detonation waves were considered to be similar on this line.

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