Centroid Calculation Algorithm Using Weight Table to Increase Accuracy of Center Position Detection

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In this paper, a new algorithm that measures the centroid position of a spot in the image obtained from an image sensor with sub-pixel accuracy was presented. The lens forms a spot on the image sensor, which spreads over a few pixels on the image. The algorithm is used to carry out centroid calculation using the brightness values of the spread pixels, and it increases the accuracy of the center detection of the spot image. In the calculation, a weight table is used, which has a circular form with a blurred border. The weight value is multiplied by the corresponding brightness value, which further increases the accuracy. An image containing a spot and random noise is produced in the simulation. Centroid calculation is carried out using the produced image, and the accuracy of the centroid position is evaluated. It is confirmed that the presented algorithm can detect the centroid position with an accuracy of 0.02 = 1/50 pixel when the spot is defocused. It is 1.5 times more accurate comparing with one of the traditional algorithms and the presented algorithm proves to be quite effective.

Key Words: Free-Space Laser Communication, Optical Antenna, Acquisition, Centroid Estimation, Tracking and Pointing

1. Introduction

In the future, earth orbiters, including observation satellites and geostationary satellites, would be required to transmit a large amount of data at a high speed rate. On the other hand, a conventional radio frequency (RF) system requires a larger communication device for rapid transmission, and the devices developed so far are approaching the speed-up limitation. To solve this problem, laser communication between two satellites, for example, a low Earth orbit (LEO) satellite and a geostationary Earth orbit (GEO) satellite shown in Figure 1 can meet the increased demand of high data volume. The advantages of laser communication include wide bandwidth, small antenna and swept volume, low weight and power, and secure communication. However, laser communication requires 10 or 100 times more precise pointing of the optical line than the RF system because of its narrow optical beamwidth. Therefore, one of the most important technologies of laser communication is to precisely point the transmit beam to the narrow receive field of view in the receiving satellite, with an accuracy of around 10 μrad or less. Using a spot image is one of the approaches for ensuring precise pointing. The received beam passes through a lens and forms a spot image. By accurately measuring the center position of the spot, we obtain the accurate direction of the beam. This accurate direction is used to achieve precise pointing and fine tracking.

Figure 2 shows one of the concepts of the configuration of the devices equipped with optical antenna and steering mirrors for laser communication. The configuration is supposed to be equipped on a LEO satellite. A GEO satellite transmits a laser beam, and the coarse pointing mechanism (CPM) acquires it. Then, the beam enters the fine pointing mechanism (FPM), which consists of a steering mirror and a controller. In the FPM, the movable mirror reflects the beam and the mirror rotates by a small angle to correct the disturbed direction of the beam generated by local disturbances such as satellite oscillation. Then, the beam passes through the beam splitter, which combines the transmitted beam and the received beam. The beam passes through another beam splitter where it is divided into two. One of the two beams enters the acquisition and tracking image sensor, where the lens forms a spot image of the beam on the image sensor. By accurately measuring the center position of the spot, we obtain the beam direction,
which is necessary for the correction of the mirror rotation in the FPM. The other beam enters the light receiver, which reads the communication data. On the other hand, the transmitter produces a laser beam, which enters the point ahead mechanism (PAM). In the PAM, the movable mirror reflects the beam and the mirror rotates by a small angle to control the direction of the beam. The beam then passes through the beam splitter that combines the received beam, before it passes through the FPM and the CPM. Finally, the beam is transmitted toward the GEO satellite. The GEO satellite has basically the same mechanism.

In this paper, a new centroid calculation algorithm is presented, which accurately measures the center position of the spot image. The lens of the acquisition and tracking image sensor aggregates the received laser beam to form a spot on the image sensor, and the spot spreads over a few pixels on the image. The algorithm is used to carry out centroid calculation using the brightness values of the spread pixels, and it increases the accuracy of the center detection of the spot image. In the calculation, a weight table is used, which has a circular form with a blurred border. Each weight value is set in the range from 0 to 1, and the weight value is multiplied by the corresponding brightness value. The weight table is designed such that the weight values inside the circle are taken as 1, those outside the circle are taken as 0, and those along the circumference of the circle are taken as decimal numbers between 0 and 1. This weight table further increases the accuracy.

Some of the other methods for measuring the center position of the spot image by centroid calculation are known, such as the method reported by Sung-Hoon Baik et al. and the method reported by Shinhak Lee. The method reported by Sung-Hoon Baik et al. squares each brightness value in the centroid calculation to reduce the noise impact. In this paper, the method reported by Sung-Hoon Baik et al. and the presented algorithm are compared in terms of the accuracy of detection of the center position of the spot. Other applications that require centroid calculation vary from the centroid measurement for the Shack-Hartmann wavefront sensor (SHWS) to the centroid measurement of individual fluorescent particles and molecules for accurate localization and tracking in light microscopes, noncontact three-dimensional (3-D) point acquisition for the optical triangulation, star tracker position measurement, and so on.

2. Centroid Calculation Algorithm

The new centroid calculation algorithm presented in this paper consists of the following steps:

1. Find the maximum brightness value $p_{\text{u}}$ in the image that has a spot.
2. Let $p_u$ be $p_{\text{u}}$ multiplied by $\alpha$ ($\alpha \cdot (p_u = \alpha p_{\text{u}}$).
3. Set zero to the brightness values that are lower than $p_u$ in the image.
4. After truncation, carry out centroid calculation using Eq. (1). Let us call the obtained position the first centroid position.
5. Make a circle with radius $r$ around the first centroid position.
6. Calculate the ratio of how much the $i$th pixel overlaps the circle using Eq. (2), and let the obtained ratio be $w_i$.
7. Truncate $w_i$ between 0 and 1.
8. Carry out centroid calculation using Eq. (3), multiply the obtained value by $w_i$, and obtain the final centroid position.

Step 4 is calculated using Eq. (1), where $p_i$ denotes the brightness value of the $i$th pixel, $x_i$ and $y_i$ denote the location of the $i$th pixel, and $c_x$ and $c_y$ denote the first centroid position.

$$c_x = \sum_i p_i x_i / \sum_i p_i$$
$$c_y = \sum_i p_i y_i / \sum_i p_i$$

(1)

Step 6 is calculated using Eq. (2), where $a_i$ denotes the distance between the first centroid position and the $i$th pixel, and $W$ is a parameter that decides the width of the circle border. A set of $w_i$ values is the weight table.

$$w_i = \frac{r - a_i}{W} + 0.5$$

(2)

Step 8 is calculated using Eq. (3), where $w_i$ is calculated using Eq. (2), $p_i$ denotes the brightness value of the $i$th pixel, $x_i$ and $y_i$ denote the location of the $i$th pixel, and $c_x$ and $c_y$ denote the final centroid position.

$$c_x = \sum_i w_i p_i x_i / \sum_i w_i p_i$$
$$c_y = \sum_i w_i p_i y_i / \sum_i w_i p_i$$

(3)

Let us call the presented algorithm [BC]-[BW] (bottom cutting and blurred weighted method). The three key points of this algorithm are as follows:
The execution of step 3, in which we set the brightness values that are lower than $p_\alpha$ in the image as zero, makes the algorithm exclude noise, which the pixels outside the spot generate.

The execution of step 4, in which we calculate the first centroid position, increases the accuracy of the final centroid position.

The execution of step 8, in which we carry out the centroid calculation by multiplying $w_i$, further increases the accuracy of the centroid position, especially when the center of the spot is located in and around the boundary between two pixels.

Figures 3 and 4 show examples of the forms of the weight tables obtained in steps 6 and 7. The weight values $w_i$ of each pixel are calculated over the image and are shown in grayscale images, where black indicates that the weight value is 0 and white indicates that the weight value is 1. Figure 3 shows the weight tables when radius $r$ is changed from 2.0 to 4.0 and Figure 4 shows those when width $W$ is changed from 0.5 to 2.5. When $r$ is changed, the size of the circular form increases, and when $W$ is changed, the border of the circular form becomes more blurred.

Fig. 3. Weight tables when radius $r$ is changed from 2.0 to 4.0.

Fig. 4. Weight tables when width $W$ is changed from 0.5 to 2.5.

3. Simulation Configuration

Figure 5 shows the configuration of the optical elements, which is used in the simulation reported in this paper. A parallel laser beam passes through the circular aperture and enters the lens. We assume that the profile of the spot should be the Airy disc pattern. The Airy disc pattern is a description of the best focused spot of light that a perfect lens with a circular aperture can form, caused by the diffraction of light. Based on this assumption, an image containing a spot identical to that shown in Figure 6 is produced in the simulation. Random noise is added to the image. Then, the centroid position of the produced spot image is calculated with sub-pixel accuracy using the centroid calculation algorithm, and the centroid error between the calculated position and the true position is obtained. This is a single cycle for the calculation of centroid error. We repeat this cycle several times to analyze centroid error statistically. In each cycle during the repetition, the position of the spot is moved within 1 pixel in a sub-pixel and in a uniformly random manner to obtain a number of centroid errors. These centroid errors have a distribution similar to the normal distribution. Therefore, we can calculate the standard deviation ($\sigma$) of these errors and obtain the value of $3\sigma$. We have evaluated the centroid accuracy using the magnitude of $3\sigma$ in this paper.

Fig. 5. Configuration of the optical elements, which is used in the simulation of this paper.

Fig. 6. Image containing a spot. (The image on the left is the original image produced in the simulation by drawing in the range of 0–4095 [DN], whereas the image on the right is redrawn in the range of 0–1000 [DN]. Both are produced with the condition in which the focal length is 168 [mm], the lens position is 168 [mm] and the energy injected to the image sensor is 3.0 [pJ].)

4. Simulation Conditions

There are many factors that influence the accuracy of the centroid calculation. These factors and their relationships with each other are shown in Figure 7.

The factors that have the most influence are noise characteristic, light energy, and defocus distance. The noise characteristic of the amount of random noise in the spot image reduces the accuracy of the centroid calculation. The accuracy also depends on the light energy. When the light energy decreases, the brightness value of the spot in the image decreases and the size of the noise becomes relatively large;
thus, it reduces the accuracy of the centroid calculation. The defocus distance significantly changes the profile of the spot, and the accuracy of the centroid calculation greatly depends on the profile of the spot.

Fig. 7. Factors and their relationships with each other. (Parameter dependency graph)

An important case that we have to take into consideration while producing a spot image by simulation is a case where almost all of the energy of a spot goes into a single pixel and the spot does not spread to any other pixel around the center pixel, as shown in Figure 8, depending on the pixel pitch width of the image sensor and the sharpness of the spot. In this case, it is difficult to increase the accuracy of the centroid position by centroid calculation because the calculation uses the spread pixels. In order to avoid such a case, an intentionally designed defocus is necessary, where the lens is deliberately displaced slightly from the focal position. In this paper, we also consider the case where defocus is necessary and compare the accuracy of the centroid positions in the two cases, i.e., one in which the defocus is used and the other where it is not used.

In this paper, we use a single wavelength laser, which is used in laser communications system as well. When a single wavelength laser is collected by a lens, the profile of the spot should ideally be the Airy disc pattern. The non-defocus line in Figure 9 shows the Airy disc pattern under the conditions in Table 1 with case 1. In contrast, when a single wavelength laser is defocused, the pattern changes like that of the defocus line shown in Figure 9. It has a bump in the middle of the spot, indicated by A in the figure. Using the two patterns, we create the images in the simulation, which are supposed to be obtained from the image sensor. Figures 10, 11, and 12 show the images produced by the two patterns. The brightness values are expressed in the unit of DN, which is the digital output value from the image sensors. It is a nondimensional value of pixel brightness voltage [V] normalized by its sensor dynamic range [V]. We assume the use of an image sensor with 12 bits output dynamic range and it outputs 0–4095 [DN].

The system parameter condition shown in Table 1 is used in the simulation. Let case 1 be the case where the non-defocus pattern in Figure 9 is used, and case 2 be the case where the defocus pattern is used. The simulation is performed in the two cases. In case 2, the length to displace the lens for defocus is found by determining the appropriate location that gives the most accurate centroid calculation. The unit of e- in Table 1 means the number of electron. The noise to be added in the simulation is defined by Eq. (4), where $p_i$ represents the brightness value of the $i$th pixel and both $n_a$ and $n_b$ are the parameters that decide the noise characteristic. The obtained $n_y$ represents the 3σ of the random noise that has normal distribution. The parameters $n_a$ and $n_b$ are chosen as in Table 1.

$$n_y = n_a p_i + n_b$$

5. Other Algorithms to Compare

The centroid calculation algorithm [BC]-[BW] was presented in Section 2. In order to compare the effectiveness of [BC]-[BW], we perform simulations using other algorithms of [BC], [BW], and [EW] in Section 6. In this section, we briefly explain those algorithms.

[BC] and [BW] comes from [BC]-[BW], which consists of two stages of centroid calculation. We separate [BC]-[BW] into single stages named [BC] and [BW] and perform simulations using those two algorithms as well. One of the other effective methods of centroid calculation is the method reported by Sung-Hoon Baik et al. It squares each brightness value to reduce the noise impact. Let us call the method [EW] (exponential weighted method) in this paper.
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Fig. 9. Profile of the spot. (The non-defocus line is the Airy disc pattern and the defocus line is the pattern when the lens is defocused by 0.3 [mm]. The non-defocus line is computed with the simulation condition in which the focal length is 84 [mm], the lens position is 84 [mm] and the energy injected to the image sensor is 3.0 [pJ], whereas the defocus line is done with the same condition except the lens position is 84.3 [mm].)

Fig. 10. Distribution of the brightness values of the image produced by the two patterns in Figure 9. (The defocus line is in the case where the lens is defocused by 0.3 [mm].)

Fig. 11. Spot image in the non-defocus case. (The image on the left is the original image produced in the simulation by drawing in the range of 0–4095 [DN], whereas the image on the right is redrawn in the range of 0–1000 [DN]. Both are produced with the non-defocus condition in Figure 9.)

Fig. 12. Spot image in the defocus case. (The image on the left is the original image produced in the simulation by drawing in the range of 0–4095 [DN], whereas the image on the right is redrawn in the range of 0–1000 [DN]. Both are produced with the defocus condition in Figure 9.)

Table 1. System parameters used in the simulation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>wavelength of light</td>
<td>1064 [nm]</td>
</tr>
<tr>
<td>energy injected to sensor</td>
<td>2.0 [pJ]</td>
</tr>
<tr>
<td>focal length</td>
<td>84 [mm]</td>
</tr>
<tr>
<td>lens position (Case 1)</td>
<td>84 [mm]</td>
</tr>
<tr>
<td>lens position (Case 2)</td>
<td>84.3 [mm]</td>
</tr>
<tr>
<td>aperture radius</td>
<td>10 [mm]</td>
</tr>
<tr>
<td>pixel pitch width</td>
<td>8 [um]</td>
</tr>
<tr>
<td>saturation number of electron</td>
<td>140000 [e-]</td>
</tr>
<tr>
<td>quantum efficiency</td>
<td>0.09 [-]</td>
</tr>
<tr>
<td>range of brightness</td>
<td>0 - 4095 [DN]</td>
</tr>
<tr>
<td>noise parameter n_a</td>
<td>0.005 [-]</td>
</tr>
<tr>
<td>noise parameter n_b</td>
<td>15 [DN]</td>
</tr>
</tbody>
</table>

5.1. Algorithm [BC]

The centroid calculation algorithm [BC] consists of the following steps:

1. Find the maximum brightness value \( p_M \) in the image that has a spot.
2. Let \( p_\alpha \) be \( p_M \) multiplied by \( \alpha \). \( p_\alpha = \alpha \cdot p_M \).
3. Set zero to the brightness values that are lower than \( p_\alpha \) in the image.
4. After truncation, carry out centroid calculation using Eq. (1), and obtain the final centroid position.

5.2. Algorithm [BW]

The centroid calculation algorithm [BW] consists of the following steps:

1. Find the maximum brightness value \( p_M \) in the image that has a spot.
2. Make a circle with radius \( r \) around the pixel which is found in Step 1.
3. Calculate the ratio of how much the \( i \)th pixel overlaps the circle using Eq. (2), and let the obtained ratio be \( w_i \).
4. Truncate \( w_i \) between 0 and 1.
5. Carry out centroid calculation using Eq. (3), multiply the obtained value by \( w_i \), and obtain the final centroid position.

5.3. Algorithm [EW]

The centroid calculation algorithm [EW] consists of the following steps:

1. Find the maximum brightness value \( p_M \) in the image that has a spot.
2. Let $p_a$ be $p_m$ multiplied by $\alpha$. ($p_a = \alpha p_m$).
3. Set zero to the brightness values that are lower than $p_a$ in the image.
4. After truncation, carry out centroid calculation using Eq. (5), and obtain the final centroid position.

Step 4 is calculated using Eq. (5), where $p_i$ denotes the brightness value of the $i$th pixel, $\beta$ is a parameter that enhances the brightness values near the center of the spot, $x_i$ and $y_i$ denote the location of the $i$th pixel, and $c_x$ and $c_y$ denote the first centroid position.

$$
c_x = \frac{\sum_i p_i^\beta x_i}{\sum_i p_i^\beta}
$$
$$
c_y = \frac{\sum_i p_i^\beta y_i}{\sum_i p_i^\beta}
$$

5. Simulation and Results

We perform simulations using the presented centroid calculation algorithm [BC]-[BW]. In order to compare the effectiveness of [BC]-[BW], other algorithms of [BC], [BW], and [EW] introduced in Section 5 are used in the simulation as well.

Each algorithm contains several parameters and those shown in Table 2 are used in the simulation. They are determined by iterative simulation and the optimum values that give the most accurate centroid calculation are used. The results of centroid calculation are shown in Figures 13 and 14. In the figures, the vertical axes list the names of the methods of centroid calculation and the horizontal axes list the centroid errors in the $3\sigma$, which are expressed in the unit of pixel.

Figure 13 is the result of case 1 and Figure 14 is the result of case 2.

Table 2. Algorithm parameters used in the simulation.

<table>
<thead>
<tr>
<th>Case</th>
<th>Algorithm</th>
<th>$\alpha$</th>
<th>$r$</th>
<th>$\beta$</th>
<th>$W$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>[BC]</td>
<td>0.038</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>[BW]</td>
<td>-</td>
<td>1.6</td>
<td>-</td>
<td>1.05</td>
</tr>
<tr>
<td></td>
<td>[BC]-[BW]</td>
<td>0.048</td>
<td>6.2</td>
<td>-</td>
<td>1.05</td>
</tr>
<tr>
<td></td>
<td>[EW]</td>
<td>0.038</td>
<td>-</td>
<td>1.02</td>
<td>-</td>
</tr>
<tr>
<td>Case 2</td>
<td>[BC]</td>
<td>0.018</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>[BW]</td>
<td>-</td>
<td>5.0</td>
<td>-</td>
<td>1.05</td>
</tr>
<tr>
<td></td>
<td>[BC]-[BW]</td>
<td>0.092</td>
<td>3.2</td>
<td>-</td>
<td>1.05</td>
</tr>
<tr>
<td></td>
<td>[EW]</td>
<td>0.018</td>
<td>-</td>
<td>1.10</td>
<td>-</td>
</tr>
</tbody>
</table>

Figure 13 shows that [BW] is the most accurate method with an accuracy of 0.07 [px] = 1/15 [px] for the condition of case 1, and Figure 14 shows that [BC]-[BW] is the most accurate method with an accuracy of 0.02 [px] = 1/50 [px] for the condition of case 2. It means that the presented algorithm [BC]-[BW] can calculate the most accurate centroid position when the spot is defocused. It was found by doing several simulations that the accuracy of the centroid calculation is likely to increase when the form of the weight table is similar to the profile of the spot. In this case, the profile of case 2 has a characteristic bump in the middle of the spot indicated by A in Figure 9. Meanwhile the weight table of [BC]-[BW] has the blurred border of the circular form as seen in Figures 3 and 4. That blurred border of the weight table fits the position where the bump exists, and it increases the accuracy of the centroid calculation. In addition, we can calculate the centroid position with higher accuracy by combining the two stages as in [BC]-[BW] than by using only a single stage like [BC] or [BW]. In [BW], the center position of the brightest pixel is used to make a circle in Step 2 of [BW]. Meanwhile in [BC]-[BW], the first centroid position which is calculated by [BC] in the first stage is used to make a circle in Step 5 of the second stage of [BC]-[BW]. Thus [BC]-[BW] is able to make a circle at a more precise position, and it increases the accuracy of the centroid calculation higher.

Table 3 shows the processing performance of each algorithm. The middle column in the table shows the time necessary for a single process. The cost time was measured by carrying out each algorithm which was implemented in C++ one million times on Windows PC (Intel® CoreTM2 Duo CPU 2.1GHz) and the time was calculated by dividing the measured time by one million. Although [BC]-[BW] takes about 35 times longer than [BC] and about 10 times longer than [EW], it finishes the centroid calculation less than 150 [ns], which means it is short enough for real-time processing.
7. Parameter Optimization

The presented algorithm [BC]-[BW] has three parameters, namely the ratio $\alpha$, the radius $r$, and the width $W$. In this section, we examine the influence of the parameters by varying each parameter within a particular range. Figures 15, 16, and 17 are obtained using [BC]-[BW] under the system conditions of case 2 as shown in Table 1 and the algorithm parameters of case 2 as shown in Table 2.

In Figure 15, the ratio $\alpha = 0.09$ minimizes the centroid error but its variation becomes small, i.e., in the range from 0.02 to 0.025 [px], and the influence of the parameter is considered to be weak. Instead, by using two stages for calculating the first and then the final centroid position brings an effect. In Figure 16, the radius $r = 3$ [px] minimizes the centroid error and its variation is in the range from 0.02 to over 0.07 [px]. Its influence is large and the radius $r$ should be well adjusted during design. In Figure 17, the width $W = 2$ [px] minimizes the centroid error but it generates almost the same centroid errors in the range from 1 to 3 [px], and its influence is considered to be weak.

8. Conclusion

In this paper, a new algorithm [BC]-[BW] was presented, which measures the centroid position of the spot in the image obtained from an image sensor with sub-pixel accuracy. The lens forms a spot on the image sensor and it spreads over a few pixels on the image. [BC]-[BW] performs the centroid calculation using the brightness values of these spread pixels and it increases the accuracy of the center detection of the spot image. In the calculation, a weight table is used, which has a circular form with a blurred border. The weight value is multiplied by the corresponding brightness value and this increases the accuracy further.

An image containing a spot and random noise is produced in the simulation. Centroid calculation is performed using the produced image and the accuracy of the centroid position is evaluated. It is confirmed that [BC]-[BW] can detect the centroid position with an accuracy of 0.02 [px] = 1/50 [px] when the spot is defocused. It is 1.5 times more accurate than [EW], which is one of the traditional algorithms and [BC]-[BW] proves to be quite effective.

References


