Identification of Wrinkle States in Membranes Based on Dispersion of Elastic Wave

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On-orbit identification of wrinkle states in membrane structures is a key technology for the operation of space structures with high performance and high precision. This study investigates the applicability of elastic waves for the identification of wrinkle states in membranes in both theoretical and experimental aspects. Cross-sectional changes generated by the wrinkles are focused on, and the induced dispersion characteristics of elastic wave are used for the identification of wrinkle states. The identification methodology is theoretically derived, and experimental demonstration is also presented.

Key Words: Membrane, Wrinkle, Elastic Wave, Dispersion

1. Introduction

Membrane structures have been recognized as vital structural elements for large expandable space structures, such as solar sails and large reflectors. A wide application of membranes is, however, one of the major technical challenges for development and management of space structures with high performance and high precision. Wrinkles in the membrane cause the significant degradation in performance of space structures. The wrinkle states in on-orbit space membranes are generally unpredictable in advance, and it is necessary to develop the method to identify the wrinkle states and to modify the membrane conditions on orbit.

One of the viable and promising methods to identify the membrane states on orbit is the visual inspection using cameras. Static and quasi-static deformations in membranes can be well captured by cameras, and the wrinkles and defects in membranes are identified visually.

Elastic wave is one of the well-known methods to identify the stiffness of materials, damages and discontinuities in materials and structures. Damage detection using traveling Lamb wave has been also investigated for thin plates, composite plates, and joint structures. For example, Okabe et al. successfully detected damages in CFRP laminates using Lamb wave propagation. Thus, elastic wave can be considered to be a powerful tool for detection of damages and imperfections in thin membrane structures.

This study utilizes the elastic waves propagating in the membranes for wrinkle identification. Cross-sectional properties of the membranes change geometrically owing to wrinkle formation, which causes the significant change in bending stiffness along the wrinkle direction. As a result, bending waves are expected to exhibit dispersion characteristics (i.e. propagating speed depends on frequencies). Thus, the characteristics change in wave propagation is considered to be used as a measure of wrinkle states in membranes. In this paper, some theoretical aspects of elastic waves in wrinkled membranes are presented at first, and the identification methodology of wrinkle states is derived. The proposed identification method is verified by conducting experimental investigation on wave dispersion properties in wrinkled membranes.

2. Theoretical Aspects

2.1. Wrinkle geometry

It is assumed that periodic wrinkles homogeneously occur in the membrane structures subjected to the external loads. As the stress in the direction transverse to the wrinkle is considered to be zero, the membrane is subject to uniaxial stress $\sigma$ in the wrinkle direction. The elastic wave propagates along the wrinkle direction, as shown in Fig. 1. The representative element of the cross-sectional geometry of the wrinkle is shown in Fig. 2, where $\delta$ is the amplitude of the wrinkle, $\lambda$ is the length of the unit element along $x$ direction (i.e. wave length of the wrinkle), $l$ is the membrane length of the unit element, and $t$ is the membrane thickness.

![Elastic wave propagation along the wrinkle direction in a membrane.](image)
In this study, the cross-sectional shape of the wrinkle is assumed to be the sine curve, and the corresponding second moment of area, $I_x$, can be approximated as the following equation.

$$I_x = \frac{h^3}{12} \left( \frac{6\delta^2 \lambda}{t^2} + 1 \right)$$

(1)

2.2. Phase velocity of elastic wave in wrinkled membrane

The wrinkled membrane is regarded as the homogeneous beam with the second moment of area corresponding to Eq. (1) under the uniaxial tensile load $T_x$. The phase velocity of the bending wave propagating in the constantly stressed beam can be expresses as

$$a_B = \sqrt{\frac{E I_y}{\mu} \left( \frac{2\pi}{\lambda} \right)^2 + \frac{T_x}{\mu}}$$

(2)

where $\mu$ is the mass of the beam per unit length along $y$ direction, and $\lambda$ is the wave length of the propagating elastic wave. When the density of the membrane is set to be $\rho$, $\mu = \rho \ell t$ holds. In addition, consideration of the relationship between the wave length and the wave frequency ($\lambda \approx \beta f$, and the relationship between the tensile load and the stress ($T_x = \sigma \ell t$) results in the following equation.

$$a_B = \sqrt{\frac{\sigma}{2\rho} + \frac{E t^2}{12\rho} \left( \frac{6\delta^2 \lambda}{t^2} + \frac{2\pi f}{\lambda} \right)^2 + \frac{\sigma}{2\rho}}$$

(3)

Considering the typical geometry of space membrane structures and the observed wrinkle amplitude, $\lambda / l$ can be approximated as 1. For example, numerical simulation and experimental results by Wong and Pellegrino\(^{(10,11)}\) suggested that the wrinkle amplitude ($\delta$) and wrinkle wavelength ($\lambda$) are in the order of 0.5 mm and 10 mm, respectively. In the experimental and numerical results by Su et al.\(^{(12)}\) and Kogiso et al.\(^{(13)}\), wrinkle amplitudes were also significantly lower than wrinkle wavelengths. These results suggest that the approximation, $\lambda / l = 1$, is valid in the typical wrinkles in membranes. Thus, the following equation can be derived.

$$a_B = \sqrt{\frac{\sigma}{2\rho} + \frac{E t^2}{12\rho} \left( \frac{6\delta^2 \lambda}{t^2} + \frac{2\pi f}{\lambda} \right)^2 + \frac{\sigma}{2\rho}}$$

(4)

2.3. Wave dispersion in wrinkled membrane

The derived Eq. (4) suggests that phase velocity is almost independent of wave frequency when the membrane is free from wrinkles (i.e. $\delta = 0$) because the membrane is very thin (i.e. $t << 1$), and the following phase velocity is obtained.

$$a_B |_{\delta=0} \approx \sqrt{\frac{\sigma}{\rho}}$$

(5)

However, when the wrinkles appear in the membrane, the phase velocity depends on the wave frequency, $f$, as shown in Eq. (4). In other words, the bending wave in the wrinkled membrane shows the dispersion characteristics. Therefore, the authors propose that the existence of wrinkles and the wrinkle amplitude, $\delta$, are identified by the measurement of phase velocities of bending waves with different frequencies. It should be noted that the applied stress, $\sigma$, may be also identified.

For the numerical demonstration, the phase velocities are plotted as a function of the wave frequency when the wrinkle amplitude and the applied stress vary (see Fig. 3). The membrane is assumed to be a Kapton® film with thickness of 12.5 $\mu$m, Young’s modulus of 5.8 GPa, and density of 1450 kg/m$^3$. The uniaxial stress is assumed to be applied in the membrane along the wrinkle direction. The solid lines in Fig. 3 correspond to the cases of the membrane without wrinkles, and the dashed lines show the cases of the wrinkled membranes ($\delta = 1$ mm). When the membrane is free from wrinkles, the phase velocity is almost independent of wave frequency. In contrast, the phase velocity in the wrinkled membrane increases with the increase in the wave frequency. The applied stress influences the phase velocity, but this effect is not related to the wave dispersion. This result suggests that the applied stress can be estimated when the phase velocity of the bending wave with low frequency is measured.

The wave dispersion curve of the wrinkled membrane with $\delta = 0$ mm, 0.5 mm, and 1.0 mm is shown in Fig. 4 when the applied stress is 10 MPa. When the wrinkle amplitude increases, the inclination of the dispersion curve increases.

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*Fig. 2. Representative unit element of the wrinkled membrane.*

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*Fig. 3. Dispersion characteristics of bending wave in a wrinkled membrane.*
The above-described results indicate that the wrinkle amplitude and the applied stress can be simultaneously identified by the measurement of the phase velocities of broadband bending waves (or bending waves with different wave frequencies). Wrinkles in membranes can be identified based on the wave dispersion characteristics. Note that the propagating wave length should be selected to be enough long compared to the wrinkle amplitude so that the propagating wave captures the cross-sectional changes or the wrinkles.

As the stress-dependent phase velocity of bending wave in strings and thin plates are widely recognized, frequency-dependent phase velocity in wrinkled membranes is the major concern in this study. In the following demonstrative experiments, wave dispersion properties in a wrinkled membrane are measured, and we try to indentify the wrinkle states based on the proposed method.

3. Experiment

3.1. Experimental procedure

In order to demonstrate the proposed identification method of wrinkles in membranes, experimental measurement of phase velocities of traveling waves in a wrinkled membrane is performed. Uniaxial tensile loads are applied to a Kapton® film (EN50, Du Pont-Toray) with thickness of 12.5 μm as shown in Fig. 5. A MFC (Macro Fiber Composite) actuator and a FBG (Fiber Bragg Grating) sensor are attached on the membrane. Elastic waves are induced by the MFC actuator and the traveling waves are sensed by the FBG. The actuation/sensing system is synchronized, and the details of the MFC/FBG system are referred to the previous reference 9). Narrow band sine waves with three periods modified by a hamming window are induced by the MFC actuator. Input waves are excited using the function generator and amplified by the bi-polar amplifier. The received FBG signals are analyzed using the high-speed optical wavelength interrogation system9). The selected central wave frequencies are 2 kHz, 4 kHz, 6 kHz, 8 kHz, and 10 kHz. The applied stress is set to be 7.5 MPa. A same membrane with three different wrinkle states is subjected to the experiment; case(1): δ=0mm, case(2): δ=0.4mm, and case(3): δ=0.9mm. The detailed procedure is written below.

The membrane is carefully clamped by two plastic plates so as not to induce any wrinkles. In order to prevent sliding of the membrane, thin rubber sheets are placed between the membrane and plastic plates. Tensile stress is applied to the membrane by suspended weights. At this stage, there is no wrinkle in the membrane, and the wave propagation test using the above-described five central frequencies is performed (corresponding to case (1)). Then, slits which split the membrane width evenly into thirds are introduced, and then, wrinkles with wrinkle amplitude of about 0.4mm occur in the membrane. The wave propagation test is carried out using the wrinkled membrane (corresponding to case (2)). Finally, slight shear movement is applied to the fixtures, resulting in wrinkle formation with larger amplitude of about 0.9 mm. The wave propagation test is conducted to the membrane (corresponding to case (3)). The wrinkle amplitude is measured by the side-view image of the wrinkle. Note that the measured accuracy ranges from 0.1 mm to 0.2 mm.
3.2. Results and discussions

The input waves and received waves are analyzed using the Wavelet transformation. The transformed Wavelet signals are shown in Fig. 6 when the membrane is free from wrinkles and the central wave frequency of 2 kHz is used. In this figure, Wavelet coefficients (corresponding to wave amplitude) are plotted, and “red” indicates high coefficients while “blue” shows low coefficients. Difference of the time at the maximum wavelet coefficient between the input and the received waves is regarded as the traveling time of the bending wave at the selected central frequency. The phase velocity is calculated using the measured traveling time and the distance between the MFC and the FBG. In a similar manner, the phase velocities at different frequencies are also experimentally obtained.

![Wavelet contour of (a) the input wave and (b) the received wave](image)

(a) input wave

(b) received wave

Fig. 6. Wavelet contour of (a) the input wave and (b) the received wave ($\delta=0$mm, 2kHz).

The experimental wrinkle states corresponding to cases (1)-(3) are presented in Fig. 7. The measured dispersion curve (i.e. phase velocities as a function of the wave frequency) is shown in Fig. 8. The case (1) corresponds to the membrane without any wrinkles, and the measured phase velocities are almost independent of the wave frequency. The result of case (2) (i.e. wrinkled membrane with $\delta=0.4$mm) shows slight dispersion characteristics. The case (3) (i.e. wrinkled membrane with $\delta=0.9$mm) exhibits clear dependence of phase velocity on the wave frequency. These results indicate that the traveling wave in membranes show dispersion characteristics when wrinkles appear in the membrane. It is demonstrated that wrinkles in membranes can be identified based on the measurement of dispersion characteristics of traveling waves along the wrinkle direction.

![Wrinkle states in the membrane for three cases](image)

(a) Case(1): $\delta=0$mm

(b) Case (2): $\delta=0.4$mm

(c) Case (3): $\delta=0.9$mm

Fig. 7. Wrinkle states in the membrane for three cases.

Using the obtained dispersion curve (Fig.8) and the theoretical formulation (Eq. (4)), the wrinkle amplitude ($\delta$) and the applied stress ($\sigma$) are estimated. The identified curves are also shown in Fig. 8 by lines. The estimated results are
shown below.

Case (1): $\delta=0.0\,[\text{mm}], \ \sigma=4.0\,[\text{MPa}]$

Case (2): $\delta=0.033\,[\text{mm}], \ \sigma=3.5\,[\text{MPa}]$

Case (3): $\delta=0.150\,[\text{mm}], \ \sigma=10.0\,[\text{MPa}]$

![Fig. 8. Measured dispersion curve.](image)

Although the identified results coincide with the experimental observation qualitatively, there is the quantitative discrepancy between the identified values and the observed values. As the identified stress differs from the applied stress even in the case without wrinkles, the authors consider that it is necessary to re-investigate how to apply tensile stresses and how to measure the phase velocity of the membranes. It should be noticed that wrinkle amplitude varies along the longitudinal direction (i.e. low amplitude near the edges, and high amplitude in the middle), while the wrinkled amplitude was measured at the length-wise middle position of the membrane. Therefore, the identified wrinkle amplitudes may exhibit lower values than the measured values, which is also considered to be a possible reason to explain the above-mentioned discrepancy. In addition, in case (3), the identified stress is quite different from the other cases. This may come from the experimental method to induce shear deformation in the membrane, resulting in different stress states.

It is necessary further to investigate whether the equation, eq. (4), derived using the beam model and the approximation of $rL=1$, is valid quantitatively. Further experiments on wave propagation under various tensile stresses will be also useful to validate the derived equation. In the present study, wrinkle identification method using wave dispersion characteristics is proposed, and wave dispersion in a wrinkled membrane is experimentally demonstrated. This study suggests the possibility of use of elastic wave dispersion for the identification of wrinkles in membrane structures.

The present formulation (e.g. equation 4) indicates the phase velocities in the wrinkled membranes when the wrinkle geometry is assumed. Therefore, it should be kept in mind that wrinkle states depend on the applied stress when we consider the mechanical equilibrium of the membranes.

If we use elastic wave with much higher frequencies, the frequency term in Eq. (4) become dominant. This suggests that there might be also interesting to investigate the identification of the stiffness (i.e. Young’s modulus) of the membrane using elastic waves.

4. Conclusions

This study proposed the identification method of wrinkle states in membrane structures based on the dispersion of traveling elastic waves as a possible way of on-orbit identification of the wrinkles. The dispersion properties of bending waves along the wrinkle direction are theoretically derived focusing on the geometrical change owing to wrinkle formation in the membrane. It was shown that bending wave in the wrinkled membrane exhibits dispersion characteristics, while bending wave in membrane without any wrinkles shows almost no dispersion. Experimental demonstration of the proposed method was also presented using the MFC/FBG experimental system. Although quantitative discrepancy was found between the experimental observations and the estimated results, the proposed method was qualitatively verified by the present experiment. This study demonstrated the feasibility of use of elastic wave dispersion for the identification of wrinkles in membrane structures.

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