Accuracy Evaluation of an Aircraft Performance Model with Airliner Flight Data

By Akinori HARADA,1) Yuto MIYAMOTO,1) Yoshikazu MIYAZAWA2) and Kozo FUNABIKI3)

1)Graduate School of Engineering, Kyushu University, Fukuoka, Japan
2)Department of Aeronautics and Astronautics, Kyushu University, Fukuoka, Japan
3)Japan Airlines Co. Ltd., Tokyo, Japan

(Received February 22nd, 2013)

This paper quantitatively evaluates the accuracy of the base of aircraft data (BADA) model by comparing calculated fuel flow and total fuel consumption with flight data recorded by airliners’ quick access recorder systems. For future more efficient air transportation system research, it is necessary to derive fuel consumption for an arbitrary flight trajectory with good accuracy. BADA, the European Organization for the Safety of Air Navigation (EUROCONTROL) aircraft performance model is widely used for such purposes, but there is only a small number of papers which discuss its accuracy with actual flight data. The purpose of this paper is to investigate whether the BADA model has sufficient accuracy to deal with the study of air transportation systems. As a result of comparison conducted for a single fleet of twin engine wide-body jet passenger aircraft, calculated fuel flow showed good agreement with the flight data in the cruise phase and it was found that the error between the BADA model and flight data falls within plus or minus 5 [%] for a whole flight.

Key Words: BADA Model, Flight Data, Fuel Flow, Total Fuel Consumption

Nomenclature

\[ D \] : drag
\[ F \] : thrust
\[ H \] : geopotential altitude
\[ L \] : lift
\[ R \] : gas constant
\[ R_0 \] : radius of the Earth
\[ S \] : wing area
\[ T \] : temperature
\[ V \] : velocity
\[ W \] : velocity of wind
\[ b \] : temperature lapse rate
\[ f \] : fuel flow
\[ g \] : gravity acceleration
\[ m \] : mass of aircraft
\[ p \] : pressure
\[ \gamma \] : flight path angle
\[ \eta \] : thrust specific fuel consumption
\[ \theta \] : longitude
\[ \rho \] : air density
\[ \phi \] : latitude
\[ \psi \] : azimuth angle

GS : ground speed
IAS : indicated air speed
TAS : true air speed
\[ a \] : relative to airflow
\[ s \] : static

1. Introduction

Reduction of fuel consumption for passenger aircraft has been studied extensively because of the skyrocketing price of fossil fuel and environmental concerns in recent years. Various measures are implemented in air traffic management (ATM) and airliners’ flight operations to save the fuel consumption and flight time, such as area navigation (RNAV), reduced vertical separation minima (RVSM), continuous descent and air traffic flow management.1) Although these measures are considered realizable under current air traffic control (ATC) systems, it is also important to clarify an efficient flight without any constraints for future air traffic management research. This unconstrained flight is called “free flight.” It was proposed long ago as an ideal concept in the future air transportation system.2) Currently, NextGen in the United States, SESAR in Europe and CARATS in Japan have been planned as long-term visions for the ideal future air transportation system. Trajectory optimization is a key element of the free flight concept, because it calculates not only each aircraft’s best performance, but it also generates ideal performance of the total air transportation system. For trajectory optimization, a performance model of aircraft must be accurate in order to
make the analysis meaningful. The BADA model developed and maintained by a research group of EUROCONTROL is widely used in the research community of air traffic management.\textsuperscript{13} The accuracy of the BADA model is investigated in some papers,\textsuperscript{12,13} and one of them suggests that fuel consumption derived with the BADA model has high accuracy in the cruise phase.\textsuperscript{12} This paper evaluates the accuracy of the aircraft performance model by comparing calculated fuel flow with the actual flight data recorded by airliners’ quick access recorder (QAR). Total fuel consumption (TFC), which is a time integral of the fuel flow, is also compared.

2. Flight Data and BADA Model

2.1. Flight data

As shown in Table 1, 18 cases of flight data are analyzed. The flight data are from the same type of modern large passenger aircraft, for three domestic routes and one international route. HND, FUK, CTS, OKA, SFO mean each airport name under the International Air Transport Association (IATA) code and correspond to Haneda (Tokyo), Fukuoka, New Chitose (Sapporo), Naha (Okinawa) and San Francisco, respectively. The 18 flight cases consist of: eight cases between Haneda and Fukuoka, four cases between Haneda and Sapporo, four cases between Haneda and Okinawa and two cases between Haneda and San Francisco.

<table>
<thead>
<tr>
<th>Year/Month</th>
<th>Departure / Destination</th>
<th>Initial time</th>
<th>Terminal time</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2011/8 HND / FUK</td>
<td>5:07</td>
<td>6:17</td>
</tr>
<tr>
<td>2</td>
<td>2011/8 FUK / HND</td>
<td>7:28</td>
<td>8:37</td>
</tr>
<tr>
<td>3</td>
<td>2011/10 HND / FUK</td>
<td>8:23</td>
<td>9:45</td>
</tr>
<tr>
<td>4</td>
<td>2011/10 FUK / HND</td>
<td>10:57</td>
<td>12:08</td>
</tr>
<tr>
<td>5</td>
<td>2011/12 HND / FUK</td>
<td>1:32</td>
<td>3:03</td>
</tr>
<tr>
<td>6</td>
<td>2011/12 FUK / HND</td>
<td>4:12</td>
<td>5:07</td>
</tr>
<tr>
<td>7</td>
<td>2011/12 HND / FUK</td>
<td>9:37</td>
<td>11:08</td>
</tr>
<tr>
<td>8</td>
<td>2011/12 FUK / HND</td>
<td>12:12</td>
<td>13:13</td>
</tr>
<tr>
<td>9</td>
<td>2011/8 HND / CTS</td>
<td>8:01</td>
<td>9:07</td>
</tr>
<tr>
<td>10</td>
<td>2011/8 CTS / HND</td>
<td>5:21</td>
<td>6:26</td>
</tr>
<tr>
<td>11</td>
<td>2011/10 HND / CTS</td>
<td>6:57</td>
<td>7:53</td>
</tr>
<tr>
<td>12</td>
<td>2011/10 CTS / HND</td>
<td>9:16</td>
<td>10:27</td>
</tr>
<tr>
<td>13</td>
<td>2011/8 HND / OKA</td>
<td>4:37</td>
<td>11:11</td>
</tr>
<tr>
<td>14</td>
<td>2011/8 OKA / HND</td>
<td>8:05</td>
<td>10:00</td>
</tr>
<tr>
<td>15</td>
<td>2011/10 HND / OKA</td>
<td>4:40</td>
<td>7:05</td>
</tr>
<tr>
<td>16</td>
<td>2011/10 OKA / HND</td>
<td>8:17</td>
<td>10:04</td>
</tr>
<tr>
<td>17</td>
<td>2011/10 HND / SFO</td>
<td>15:23</td>
<td>23:28</td>
</tr>
<tr>
<td>18</td>
<td>2011/10 SFO / HND</td>
<td>1:59</td>
<td>12:24</td>
</tr>
</tbody>
</table>

Initial and terminal times are determined in order to limit the scope of evaluation to a clean configuration; that is, take-off and landing configurations are excluded from the analysis. Each flight data consist of 13 items such as time, calculated gross weight of the aircraft, longitude, latitude, wind direction and speed, true air speed $V_{TAS}$, indicated air speed $V_{IAS}$, vertical velocity $V_{V}$, Mach number, QNE pressure altitude, static air temperature and fuel flow. Each recorded variable is used for calculation every second.

2.2. BADA model

The BADA model is an aircraft performance model developed and maintained by the European Organization for the Safety of Air Navigation (EUROCONTROL) through active cooperation with aircraft manufacturers and operating airlines. The information and data contained in BADA is designed for use in aircraft trajectory simulations and predictions.

BADA consists of two components:\textsuperscript{13} (1) Model specifications: Theoretical fundamentals are provided in form of generic polynomial expressions to calculate aircraft performance parameters. The motion model which is used within BADA is a so-called total energy model (TEM). It can be considered as a reduced point-mass model. (2) Datasets: A dataset for a given aircraft contains specific values of the coefficients in the model specification, which particularize the BADA model for a specific aircraft type.

3. Fuel Flow Calculation

Fuel flow is calculated using the aerodynamic model and fuel flow model defined in BADA 3.9. Thrust is calculated by substituting flight data variables into equations of the aircraft motion.

3.1. Thrust

Thrust is derived from acceleration along the wind axis. Only the velocity changes which are directly related to energy are considered, but the velocity direction change is assumed to be negligible. The equation of motion with point mass approximations is given as follows.

\[
m \frac{dV_{ES}}{dt} \cos(\alpha - \gamma) \cos(\psi_a - \psi) = F - D - mg \sin \gamma_a \tag{1}\]

Two kinds of velocity $V_{TAS}$ and $V_{ES}$ are used in this equation. $V_{ES}$ represents an inertial velocity considered in the Earth-centered Earth-fixed (ECEF) coordinate system. This inertial velocity is required to derive the acceleration of aircraft which excludes the influence of wind changes. The direction of each velocity is given by the path angle $\gamma$ and $\gamma_a$, and azimuth angle $\psi$ and $\psi_a$ as shown in Fig. 1.

![Fig. 1. Relationship of $\gamma$, $\psi$ and $V$.](image)

Though an aircraft has the acceleration along the direction of $V_{ES}$, the thrust force is in the direction of $V_{TAS}$. Therefore, the left-hand side of Eq. (1) includes cosines of $(\gamma_a - \gamma)$ and $(\psi_a - \psi)$. Path angle $\gamma$, $\gamma_a$ and the inertial velocity $V_{ES}$ can be calculated from vertical velocity $V_{V}$.\textsuperscript{80}
A. HARADA et al. : Accuracy Evaluation of an Aircraft Performance Model with Airliner Flight Data

$V_{GS}$ and $V_{TAS}$ included in the flight data.

$$\gamma = \tan^{-1} \frac{V}{V_{GS}} \tag{2}$$

$$\gamma_a = \tan^{-1} \frac{V}{V_{TAS}} \tag{3}$$

$$V_{ES} = \frac{V_{GS}}{\cos \gamma} \tag{4}$$

A central difference approximation is applied to gain the inertial acceleration $dV_{ES}/dt$.

$$\frac{dV_{ES}}{dt} \bigg|_{(t_1)} \approx \frac{V_{ES}(t_{1+1}) - V_{ES}(t_{1-1})}{2 \Delta t} \tag{5}$$

3.2. Atmospheric model

Atmospheric pressure is obtained from the QNE pressure altitude $H$ in the flight data using the following equation defined in the International Standard Atmosphere model.

$$p = p_1 \exp \left[ -\frac{g}{RT} \left( H - \frac{R_H H_1}{R_0} \right) \right] \tag{6}$$

$$T = T_1 \tag{7}$$

$$p = p_0 \left( \frac{T}{T_0} \right)^{g/\gamma} \tag{8}$$

$$T = T_0 + bH \tag{9}$$

Air density is gained by substituting this atmospheric pressure and static air temperature $T_s$ of the flight data into the following equation.

$$\rho = \frac{p}{RT_s} \tag{10}$$

3.3. BADA aerodynamic model

The lift and drag forces which act on the body are expressed with the coefficients $C_L$ and $C_D$ as follows.

$$L = \frac{1}{2} \rho V^2 SC_L \tag{11}$$

$$D = \frac{1}{2} \rho V^2 SC_D \tag{12}$$

The lift coefficient is derived by assuming the aircraft in steady flight with small bank angle.

$$C_L = \frac{2mg \cos \gamma_a}{\rho V^2 S} \tag{13}$$

The relationship between $C_L$ and $C_D$ is defined with two coefficients as a mathematical model.

$$C_D = C_{D0} + C_{D2} \times C_L^2 \tag{14}$$

$C_{D0}$ and $C_{D2}$ are parasite drag and induced drag coefficients. These coefficients are modeled independent from Mach number and angle of attack.

Thrust of the aircraft can be calculated by substituting drag into Eq. (1).

3.4. BADA fuel flow model

Nominal fuel flow

The fuel flow for nominal flight conditions $f_{nom}$ is defined as the product of the thrust $F$ and coefficient of thrust specific fuel consumption $\eta$ as in Eq. (15).

$$f_{nom} = \eta \times F \tag{15}$$

$\eta$ is a function of true airspeed, $V_{TAS}$.

$$\eta = C_{f1} \times \left( 1 + \frac{V_{TAS}}{C_{f2}} \right) \tag{16}$$

Cruise fuel flow

Cruise fuel flow $f_{cr}$ is given with a cruise fuel flow factor, $C_{fcr}$.

$$f_{cr} = \eta \times F \times C_{fcr} \tag{17}$$

Minimum fuel flow

Minimum fuel flow $f_{min}$, corresponding to idle thrust descent conditions, is specified as a function of altitude above sea level, $H$.

$$f_{min} = C_{f3} \left( 1 - \frac{H}{C_{f4}} \right) \tag{18}$$

The constant parameters for the type of aircraft are given in the operations performance files (OPF).

Fuel flow in the climb and cruise phase is obtained from Eqs. (15) and (17), respectively. For the descent phase, including idle thrust conditions, the larger value of Eqs. (15) and (18) is selected.

3.5. Filtering

Since ground speed $V_{GS}$ in the flight data contains measurement noise, the fuel flow calculated by time derivative of the velocity amplifies high frequency noise. Figure 2 is an example of calculated fuel flow and flight data comparison for flight case 1. It can be seen that the calculated fuel flow contains high frequency noise due to time derivative of $V_{GS}$.

A zero phase finite impulse response (FIR) low pass filter is applied to eliminate the high frequency noise of $V_{GS}$, where the cutoff frequency is set to 0.025 [Hz]. The gain and phase frequency characteristics of the filter are shown in Fig. 3. The higher frequency signal is reduced by one-thousandth and the lower signal passes through as it is. The $x$-axis is normalized by the Nyquist frequency defined as half of sampling frequency. The flight data are recorded every second, hence the Nyquist frequency is 0.5 [Hz]. Therefore, the normalized cutoff frequency is 0.05.
4. Evaluation of Model Error

4.1. Calculated fuel flow error and flight data

The fuel flow calculated using the BADA model is compared with the flight data. The four figures, from Figs. 4 to 7, show time histories of the fuel flow for the flight cases 1, 9, 13 and 17 as representative cases. Calculated fuel flow is in close accordance with the flight data in the cruise phase for all flight cases. On the other hand, the flight data fuel flow tends to be larger than that of the climb phase, and some biased error remains in the descent phase.

4.2. Analysis of fuel flow error

The error between calculated fuel flow and flight data is evaluated by the mean value and standard deviation. If the error of fuel flow is defined as follows, the mean value and standard deviation are expressed as Eqs. (20) and (21), respectively. The statistical values are derived from every second of data for each phase; i.e., climb, cruise and descent in all flight cases.

\[
\Delta f(t) = f_{BADA}(t) - f(t) \quad (19)
\]

\[
\bar{\Delta f} = \frac{1}{n} \sum_{t=1}^{n} \Delta f(t) \quad (20)
\]

\[
\sigma = \sqrt{\frac{1}{n-1} \sum_{t=1}^{n} (\Delta f(t) - \bar{\Delta f})^2} \quad (21)
\]

These statistical variables are plotted in Figs. 8 to 11, where the standard deviation and the mean value of fuel flow error are shown. The mean value of fuel flow error takes a large negative value in Fig. 9, because calculated fuel flow is smaller than the flight data in the climb phase in most cases. In contrast, since the calculated fuel flow varies around the flight data in the cruise phase, the mean value is close to zero as shown in Fig. 10.
Furthermore, the mean value takes a positive value in the descent phase as shown in Fig. 11. In this phase, the standard deviation is smaller than that of other phases because the calculated fuel flow takes the minimum value which is independent from the time derivative of velocity; i.e., the main cause of high frequency noise.

4.3. TFC error

The TFC is obtained by integrating the fuel flow with time. The following numerical approximation for every second is applied to the integration.

\[ TFC = \sum_{t=0}^{t_f} f(t) \Delta t \]  

Each TFC obtained from calculated fuel flow and flight data is compared for the whole flight time and three phases. Figures 12 to 15 show the comparison of TFC obtained from calculated fuel flow with the BADA model and the real fuel flow in the flight data. They are in good agreement with the cruise phase. TFC calculated from the BADA model is smaller than that from flight data for the whole flight time in many cases because of large error in the climb phase. On the other hand, calculated TFC is larger than flight data in the international route in case 17. This is because the calculated fuel flow in the cruise phase which occupies a major part of the international flight is slightly larger than the flight data as shown in Fig. 7.

Table 2 shows the error of calculated TFC from flight data. The error value is defined in Eq. (23), and proportion of the error to the flight data is calculated by Eq. (24), where TFC means the actual value obtained from flight data in the equations.

\[ \Delta TFC = TFC_{BADA} - TFC \]  

\[ \frac{\Delta TFC}{TFC} = \frac{TFC_{BADA} - TFC}{TFC} \]  

Although the calculated TFC fits in well with the TFC obtained from flight data in the cruise phase as the percentage of error shows from -4.7 [%] to +4.2 [%], the accuracy reduces in the climb and descent phase as the percentage is -12.8 [%] to +1.5 [%] and -4.6 [%] to +21.2 [%], respectively. However, if the cruise phase occupies a large part of the flight as in these 18 cases, the error for whole flight time falls within plus or minus 5 [%]. It can be said that sufficient accuracy is obtained overall.

5. Conclusion

The accuracy of the BADA model was evaluated by comparing calculated fuel flow and TFC with the flight data stored in the QAR. The results of comparison show that the
error between the BADA model and flight data is within plus or minus 5 [%]. Although the accuracy reduces relatively in the climb and descent phases, the influence on the error for whole flight time is not very significant because the cruise phase, which has high accuracy, constitutes a large part of the flight. In conclusion, this paper revealed that the BADA model has sufficient accuracy to calculate the fuel consumption for general flight trajectory and it is of great use to various analyses for the research of future air transportation systems.

References

Table 2. Error of calculated TFC from flight data.

<table>
<thead>
<tr>
<th></th>
<th>F01</th>
<th>F02</th>
<th>F03</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Climb</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cruise</td>
<td>-77[kg]</td>
<td>-2.59[%]</td>
<td>1[kg]</td>
</tr>
<tr>
<td>Descent</td>
<td>58[kg]</td>
<td>14.94[%]</td>
<td>43[kg]</td>
</tr>
<tr>
<td><strong>Cruise</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-256[kg]</td>
<td>-7.51[%]</td>
<td>-111[kg]</td>
</tr>
<tr>
<td>Cruise</td>
<td>-58[kg]</td>
<td>-1.71[%]</td>
<td>-54[kg]</td>
</tr>
<tr>
<td>Descent</td>
<td>63[kg]</td>
<td>7.39[%]</td>
<td>12[kg]</td>
</tr>
<tr>
<td>Whole</td>
<td>-251[kg]</td>
<td>-3.29[%]</td>
<td>-153[kg]</td>
</tr>
<tr>
<td><strong>Descent</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-185[kg]</td>
<td>-6.23[%]</td>
<td>-270[kg]</td>
</tr>
<tr>
<td>Cruise</td>
<td>-110[kg]</td>
<td>-1.89[%]</td>
<td>-75[kg]</td>
</tr>
<tr>
<td>Descent</td>
<td>64[kg]</td>
<td>6.08[%]</td>
<td>84[kg]</td>
</tr>
<tr>
<td>Whole</td>
<td>-232[kg]</td>
<td>-2.36[%]</td>
<td>-260[kg]</td>
</tr>
<tr>
<td><strong>Whole</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-232[kg]</td>
<td>-2.36[%]</td>
<td>-260[kg]</td>
</tr>
</tbody>
</table>

85