Analysis of Fuel-Efficient Airliner Flight via Dynamic Programming Trajectory Optimization

By Yuto MIYAMOTO,¹⁾ Navinda Kithmal WICKRAMASINGHE,¹⁾ Akinori HARADA,¹⁾
Yoshikazu MIYAZAWA²⁾ and Kozo FUNABIKI³⁾

¹⁾Graduate School of Engineering, Kyushu University, Fukuoka, Japan

²⁾Department of Aeronautics and Astronautics, Kyushu University, Fukuoka, Japan

³⁾Japan Airlines Co., Ltd., Tokyo, Japan

(Received February 28th, 2013)

Fuel-efficient flight is in strong demand due to environmental concerns and skyrocketing fuel prices. This paper analyzes airliner flight trajectory from the perspective of fuel efficiency through comparison with the optimal trajectory. Since the optimal trajectory gives the maximum achievable performance without any constraint of air traffic control, it introduces reference data for quantitative merit of the ideal flight. Dynamic programming is adopted to calculate the optimal trajectory. Examples show that considerable reduction in the fuel consumption from the estimated values for actual flights could be achieved by the trajectory optimization, where wind conditions are taken into account as well.

Key Words: Aircraft, Trajectory Optimization, Dynamic Programming, Free Flight, Air Traffic Management

Nomenclature

ϕ	: latitude
θ	: longitude
R_0	: radius of the Earth
H	: geo potential altitude
V_{TAS}	: true air speed
V_{ES}	: Earth speed
V_{CAS}	: calibrated air speed
T	: thrust
D	: drag
γ	: path angle
ψ	: azimuth angle
W_x	: west wind
W_y	: south wind
m	: mass of aircraft
g	: gravity acceleration
ξ	: down range angle
η	: cross range angle
t	: time
S	: wing area

Subscripts

a	: respect to aerodynamic
0	: initial
f	: final
$\min$	: minimum
$\max$	: maximum
ref	: reference
mo	: maximum operational

1. Introduction

Major problems of recent air traffic include the remarkable rise of fuel prices, environmental concerns and delays due to the congestion of airspace. In Japan, various measures are implemented to accommodate demands, such as the development of multi-functional transport satellite (MTSAT), area navigation (RNAV) air routes and reduced vertical separation minima (RVSM).¹⁾ These are possible measures in the current air traffic control (ATC) system, but more flexible operations are expected to be possible by introducing a new concept. Trajectory-based operations (TBO) is a concept of new operations based on an optimized trajectory from gate to gate where the whole of Japan is considered as a single airspace. In fact, NextGen (United States), SESAR (Europe) and CARATS (Japan) are defined as long-term visions for a futuristic air transportation system. It is important in TBO to optimize the flight trajectory of each airplane. If a trajectory is selectable which is not bound to various limitations in the current air traffic control, numerous merits could be expected such as the reduction of fuel consumption, flight time management and performance increase in air traffic. The flight without limitations is suggested in the concept called “free flight” as well. Free flight is a proposed system which improves the efficiency of airspace by selecting an optimal flight path and velocity while considering safe separation and prevention of exceeding the airport capacity.

The objective of this paper is to clarify the merits with the assumption that the free flight concept is utilized in the air traffic system. If plausible merits could be gained,

the need for a new air traffic system to be deployed would be inevitable and would eventually lead to the acceleration of research and development. In addition, it would provide a positive impact towards the development of a future flight management system (FMS). In this study, the trajectory is optimized to minimize the total fuel consumption with dynamic programming²⁾ and a quantitative evaluation is carried out to observe the gained benefits through comparison with jet passenger aircraft flight trajectory.

The current airliner flight trajectories are constrained by airspace, air routes, flight levels and descent velocities, in general, and they are further restricted in the case of vector control in and around congested terminal areas. This paper compares fuel consumption and flight time between the actual flight trajectory and the fuel optimal flight trajectory. It analyzes 18 flight cases, where the Haneda airport terminal area is included in the departure or arrival in all cases, and it studies statistical properties. The base of aircraft data (BADA) performance model is used in both cases to calculate the fuel consumption. For the fuel-efficient flight trajectory, three parameters; that is, altitude, velocity and lateral deviation, are considered free for the optimization.

2. Flight Data

The flight data recorded by airliner onboard quick access recorder (QAR) are used to perform a quantitative evaluation of fuel consumption. The 18 flight cases analyzed for the optimization are shown in Table 1. The aircraft type is the same in all flight cases, where a heavy, wide-body twin-engine jet passenger aircraft is selected. The three alphabetical characters in the departure/destination column are airport codes defined by the International Air Transport Association (IATA) and

are as follows: HND: Tokyo Haneda; FUK: Fukuoka; CTS: New Chitose; and SFO: San Francisco. Initial and terminal times are selected to cover the clean configuration flight of each case. The flight data includes various parameters representing the flight condition; e.g., position, altitude, velocity, true airspeed, mass, etc., and they are used for calculation of the fuel consumption with the BADA performance model.

3. Optimal Trajectory

3.1. Equations of motion

Generally, the motion of aircraft is described by three degrees of freedom (3DOF) equations using point mass approximation. Furthermore, the velocity change which is directly related to energy is considered, but velocity direction change is neglected. This assumption reduces the number of state variables from six to four, which is suitable for applying the dynamic programming optimization. Control variables are path angle, azimuth angle and thrust. Equations of motion are given as follows.³⁻⁶⁾

$$\frac{d\theta}{dt} = \frac{1}{(R_0 + H)\cos\phi} \{V_{TAS} \cos\gamma_a \sin\psi_a + W_x\} \quad (1)$$

$$\frac{d\phi}{dt} = \frac{1}{R_0 + H} \{V_{TAS} \cos\gamma_a \cos\psi_a + W_y\} \quad (2)$$

$$\frac{dH}{dt} = V_{TAS} \sin\gamma_a \quad (3)$$

$$m \frac{dV_{ES}}{dt} \cos(\gamma_a - \gamma) \cos(\psi_a - \psi) = T - D - mg \sin\gamma_a \quad (4)$$

Fig. 1. Relationship between two grid points.

Southerly and westerly winds are defined as positive, and the vertical wind component is neglected. The aircraft position is given by longitude, latitude and altitude. True air speed and Earth speed are used as the aircraft speed. Earth speed is defined as the inertial velocity relative to the Earth. The reason for using Earth speed is that wind affects the aircraft acceleration which is an important factor in calculating the thrust. These two velocity directions are decided by γ_a , γ , ψ_a and ψ , respectively. As shown in Fig.1, this system's acceleration direction is along the Earth speed axis but the thrust force is along the direction of air speed. Therefore, the left-hand side of Eq.(4) includes cosines of differences of air-to-ground vectors.

Table 1. Flight Data.

	Year/Month	Departure / Destination	Data record time (UTC)	
			Initial time	Terminal time
01	2011/8	HND / FUK	5:07	6:17
02	2011/8	FUK / HND	7:28	8:37
03	2011/10	HND / FUK	8:23	9:45
04	2011/10	FUK / HND	10:57	12:08
05	2011/12	HND / FUK	1:32	3:03
06	2011/12	FUK / HND	4:12	5:07
07	2011/12	HND / FUK	9:37	11:08
08	2011/12	FUK / HND	12:12	13:13
09	2011/8	HND / CTS	8:01	9:07
10	2011/8	CTS / HND	5:21	6:26
11	2011/10	HND / CTS	6:57	7:53
12	2011/10	CTS / HND	9:16	10:27
13	2011/8	HND / OKA	4:37	11:11
14	2011/8	OKA / HND	8:05	10:00
15	2011/10	HND / OKA	4:40	7:05
16	2011/10	OKA / HND	8:17	10:04
17	2011/10	HND / SFO	15:23	23:28
18	2011/10	SFO / HND	1:59	12:24

3.2. Aircraft performance model

The BADA model developed by the European Organization for the Safety of Air Navigation (EUROCONTROL) is used for calculating the aircraft performance.⁷⁾ The BADA model consists of parametric models such as the aerodynamic model, maximum thrust model and fuel flow model which are used for aircraft performance calculations.^{8,9)} It provides concrete parameters for almost all types of aircraft. According to another analysis for the flight data, the numerical error of fuel consumption calculated from the flight variables is $\pm 5\%$ compared to the onboard measurement.¹⁰⁾ Since the same BADA model is used to calculate the fuel consumption for both real flight data and proposed optimal trajectory, the model error does not directly affect the difference of the fuel consumptions.

3.3. Meteorological data

Numerical weather prediction grid point value (GPV) models provided by the Japan Meteorological Agency show objective analysis data and numerical weather prediction data.¹¹⁾ According to an evaluation analysis using the flight measurement data, the global spectral model (GSM) numerical data are more accurate than meticulous meso scale model (MSM) numerical data.¹²⁾ Therefore, the GSM model data for the Japan region is used in this paper. In the GSM model, information such as geo potential altitude, wind velocities and temperature are included in lattices of latitude and longitude for each of the 16 barometric surfaces. Fundamental properties are given in Table 2.

Table 2. GSM for the Japan region.

Time	00, 06, 12, 18 (UTC)
Region	Longitude : 120 ~ 150 [deg] Latitude : 20 ~ 50 [deg]
Resolution	Longitude direction : 0.25 [deg] Latitude direction : 0.20 [deg]
Physical variable data	Altitude, wind velocity, temperature, vertical flow
Barometric surfaces value	1000, 975, 950, 925, 900, 850, 800, 700, 600, 500, 400, 300, 250, 200, 150, 100 [hPa]

3.4. Dynamic programming

As the optimization method, dynamic programming is used in this study. It has the following advantages and disadvantages.

Advantages

- It is easy to understand the algorithm and easy to build up the calculation program.
- It is easy to consider constraint conditions.
- It gives the global optimum solution.
- It is possible to predict the calculation time in advance.

Disadvantages

- Computational complexity increases extremely in the case when the dimensions of state variables increase, which is called “the curse of dimensionality.”
- Discretization error occurs inevitably because of discretization of state variables and control variables.

The independent variable and state variables are

discretized by an equidistant orthogonal grid system. All the combinations of transition between grid points are searched and the optimal solution is chosen. The optimal trajectory of minimum fuel consumption is calculated as follows.

Down range angle (ξ) is defined to be an independent variable, and cross range angle (η), altitude and calibrated air speed are defined as state variables. As shown in Fig. 2, down range angle is taken on the great-circle course, which is the shortest route from the initial point to the terminal point, and cross range angle presents lateral deviation from the great-circle course.

The performance index is given as follows.

$$J = \int_{t_0}^{t_f} FF dt \quad (5)$$

Fuel flow (FF) provided by the BADA model is the fuel consumption per unit time and the total fuel consumption is expressed as a time integral of FF .

Fig. 2. Definition of down range angle and cross range angle.

3.5. Calculation conditions

For performance calculation, the specific model given by BADA is utilized. The flight envelopes and parameters for the objective aircraft are given as follows.

$$\begin{aligned} S &= 427.82 [\text{m}^2] \\ m_{ref} &= 208.7 \times 10^3 [\text{kg}] \\ V_{CAS, min, ref} &= 1.3 \times V_{stall} \cong 100 [\text{m/s}] \\ V_{CAS, mo} &\cong 168 [\text{m/s}] \\ M_{mo} &= 0.84 \\ H_{mo} &\cong 13,100 [\text{m}] \end{aligned}$$

The initial and terminal conditions are identical to the given flight data; i.e., the parameters conforming to flight data at the initial and terminal points are longitude, latitude, altitude and calibrated air speed. Additionally, aircraft mass conforms to flight data at the initial point. The dynamic programming grid resolutions of cross range angle, altitude and velocity in the calculation are as follows.

$$\begin{aligned} \Delta \eta &= 0.05 [\text{deg}] (= 5.5 [\text{km}]) \\ \Delta H &= 100 [\text{m}] \\ \Delta V_{CAS} &= 1 [\text{m/s}] \end{aligned}$$

Although the flight data include fuel flow, fuel flow is calculated from the flight variables with the BADA model

and Eqs.(1)~(4) in order to reduce the error using the performance model. The calculated total fuel consumption is compared with that of the optimal trajectory with the same BADA model and meteorological data.

To minimize the influence of discretization error, fuel consumption calculation between two grid points are divided by two equidistant sections. Furthermore, meteorological data such as wind velocities, air pressure, air density and atmospheric temperature are interpolated for the center points of each section. Similarly, velocity is interpolated for the same points. These interpolated values are used for calculations as representative values because the flight time of those separated sections is short. In this calculation, the aircraft mass is not treated as a state variable but a given parameter because the influence of weight variation on the optimization is assumed negligible.

4. Results

At first, a typical example from 18 flight cases is shown. This is the flight case in August from Tokyo to Fukuoka (data01). The jet stream doesn't exist over the Japan airspace during this season. Flight data and variables calculated from flight data are plotted with the black line

Fig. 3. Altitude and velocity (data01).

Fig. 4. Flight time and path (data01).

and the variables obtained from the optimal trajectory are plotted with the blue line.

As shown in Fig. 3, the optimal altitude and velocity selection at the climb phase are similar to the flight data. This is because a climb with the maximum thrust is efficient as shown in Fig. 5. A different point from the flight data is that the optimal trajectory performs a high L/D continuous descent. This decent profile is seen in almost all other optimal trajectories. As shown in Fig. 5, Lift-to-Drag ratio with lower velocity is bigger than the flight data at the descent phase. As the loss of energy is the minimum, long distance gliding flight is achieved with idle thrust conditions. It is clear that fuel consumption is reduced at the phase of descent with lower fuel flow. The flight time is slightly longer because the descent speed is lower compared to the flight data. Due to weak wind conditions, the great-circle course of the shortest distance is selected as the optimal trajectory, where the RNAV route used by the actual flight from Tokyo to Fukuoka is close to the great circle.

Secondly, a unique example in December (data05) is shown. In this case, a strong jet stream wind exists over the Japan airspace. Wind shear at 250 hPa near the cruising altitude of aircraft is shown in Fig. 6. The

Fig. 5. Other flight variables (data01).

Fig. 6. Wind profile at 250 hPa (data05).

optimal flight takes a higher altitude to achieve better performance, while the actual flight takes a lower altitude to avoid a strong head wind. Almost all other variables give similar results as shown in the previous case except for the lateral flight profile. The flight path deviates towards the north from the great-circle course to avoid strong headwinds as shown in Fig. 9. Both flight time and flight range increase but the fuel consumption decreases. Thus, it could be said that reduction of total fuel consumption is possible with a lateral deviation by considering the effect of wind profile.

To check the validity of flight path change, altitude and velocity are optimized on the great-circle course with the same meteorological data. The result is shown in Figs.10 and 11. The subjected parameters are almost identical to previous results. The optimal fuel consumption is estimated as 8,542.2 [kg] and flight data fuel consumption is 9,913.6 [kg]. It is possible to reduce the fuel consumption of 1,371.4 [kg], 13.8[%] with respect to flight data. However, if lateral deviation is also considered as shown in Fig. 8, fuel consumption is estimated as 8,314.8 [kg] which is 1,598.8 [kg], 16.1[%] less than flight data.

Therefore it is shown that fuel consumption can be reduced 227.4 [kg], 2.3 points by considering the lateral deviation of flight path.

Fig. 7. Altitude and velocity (data05).

Fig. 8. Flight time and path (data05).

The results of all 18 cases are shown in Fig. 12. Flight time, flight range and fuel consumption of the optimized trajectories are compared with each calculated data from the flight data. The top figure shows the optimal trajectory has a trend of longer flight time and shorter flight range than the flight data. The reason for flight range reduction is that most optimal trajectories choose the great-circle course while the actual flights take predetermined airways or longer flight paths by vector control. The fuel consumption reduction is achieved by the optimal trajectory with the longer descent phase. For the 18 cases, total fuel consumption could be reduced from 3 to 20%. On the other hand, most of the actual flights perform less flight time than the respective optimal trajectories. This is due to the consideration of not only fuel consumption, but also flight time in conventional flight planning.

Fig. 9. Other flight variables (data05).

Fig. 10. Altitude and Velocity in the no lateral deviation case (data05).

Fig. 11. Other flight variables in the no lateral deviation case (data05).

Fig. 12. Comparison with the flight data.

5. Conclusion

In this paper, trajectory optimization for a jet passenger aircraft was studied to minimize the fuel consumption. Airliner's flight trajectory data were used in order to compare the optimal trajectory with the actual flight. GSM weather prediction data, the BADA model and the dynamic programming optimization method were employed in the analysis. The calculation results for 18 flight cases were as follows.

1. In the climb phase, both optimal trajectory and flight data used the maximum thrust. In the cruise phase, both of them used the maximum operating Mach

number.

2. In the descent phase, the optimal trajectory employed an efficient Lift-to-Drag ratio with idle thrust. The fuel consumption could be reduced because the flight range of descent phase was longer, and the cruise phase was shorter.
3. In a case of strong jet stream, the optimal trajectory changed the flight path from the minimum range great-circle to avoid strong headwinds. Although the flight time and flight range were increased, the fuel consumption was reduced.
4. In all 18 flight cases, a 3 to 20% reduction of fuel consumption could be achieved by the trajectory optimization.

Finally, dynamic programming was demonstrated to be an efficient method to solve the trajectory optimization problem, which has various inequality constraints and different kinds of parameters in the aircraft performance model and meteorological data. Trajectory optimization studied in this paper is useful for the future air transportation system research.

References

- 1) Study Group for the Future Air Traffic Systems: Long Term Vision for the Future Air Traffic Systems, CARATS, Collaborative Actions for Renovation of Air Traffic Systems, <http://www.mlit.go.jp/common/000128185.pdf>, 2010.
- 2) Bellman R. E.: *Dynamic Programming*, Princeton University Press, Princeton, 1957.
- 3) Miyazawa, Y. and Wickramasinghe, N. K.: A Study on Benefits of Free Flight, Proceedings of the 49th Aircraft Symposium, JSASS, Kanazawa, 2011, pp. 346-352, in Japanese.
- 4) Miyazawa, Y., Wickramasinghe, N. K., Harada, A. and Miyamoto, Y.: Passenger Aircraft Optimal Trajectory with an Arrival Time Constraint, *Aerospace Technology Japan JSASS*, **12**(2013), pp.31-38, in Japanese.
- 5) Miyamoto, Y., Wickramasinghe, N. K., Harada, A. and Miyazawa, Y.: Computational Time Reduction for Dynamic Programming Trajectory Optimization Applied to Passenger Aircraft, Proceedings of the JSASS Western Branch Conference, 2012.
- 6) Wickramasinghe, N. K., Harada, A. and Miyazawa, Y.: Flight Trajectory Optimization for an Efficient Air Transportation System, ICAS2012, Brisbane, 2012.
- 7) Eurocontrol Experimental Center: User Manual For the Base of Aircraft Data (BADA) Revision 3.9, EEC Technical/Scientific Report No.11/03/08-08, 2011
- 8) Shirakawa, M., Fukuda, Y. and Senoguchi, A.: Trajectory Predictions with the Aircraft Performance Models, Technical Report, IEICE, 2008, pp. 19-24, in Japanese.
- 9) Shirakawa, M., Fukuda, Y. and Senoguchi, A.: Aircraft Trajectory Prediction Error Analysis, Technical Report, IEICE, 2009, pp. 47-52, in Japanese.
- 10) Harada, A., Miyazawa, Y. and Funabiki, K.: Accuracy Evaluation of an Aircraft Performance Model with Airliner Flight Data, *Trans. JSASS Aerospace Technology Japan*, **11**(2013), pp.79-85.
- 11) Japan Meteorological Business Support Center Online Data Service, <http://www.jmbssc.or.jp/hp/online/f-online0.html>
- 12) Totoki, H., Kozuka, T., Miyazawa, Y. and Funabiki, K.: Comparison of JMA Numerical Prediction GPV Meteorological Data and Airliner Flight Data, *Aerospace Technology Japan JSASS*, **12**(2013), pp.57-63, in Japanese.