Magnetohydrodynamic Analysis of Thrust Characteristics on Magneto-Plasma Sail with Plasma Magnetic Field Inflation by Low-Beta Plasma

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Magneto-Plasma Sail (MPS) is a sail space propulsion system to accelerate the spacecraft in the solar wind direction by capturing the solar wind energy. MPS's significant feature is it's a utilization of magnetic field inflation process, in which a small magnetosphere by on-board coils is inflated by low-speed plasma released from the spacecraft. In this study, a parametric survey is conducted using the axis-symmetric two-dimensional magnetohydrodynamic (MHD) simulation to investigate the relation between the thrust characteristics and the injected plasma parameters (the mass flow rate and the thermal beta value at the injection point). As a result, the thrust gain, which is defined as the ratio of the MPS spacecraft thrust force to the Magnetic Sail thrust force, and the specific impulse have the maximum value against the thermal beta value, and in addition, increases with lower mass flow rate. It is indicated that the maximum value of the thrust gain is 3.77 when the thermal beta value is 25 and the mass flow rate is 100 in the non-dimensional parameter (when a magnetic moment of the spacecraft is assumed 6.28×10^7 Wb·m, the thrust force is 60 N and the mass flow rate is 3.6 mg/s).

Key Words: Magneto-Plasma Sail, Magnetohydrodynamic Simulation, Magnetic Field Inflation

Nomenclature

B	: magnetic flux vector
C_{CFL}	: CFL condition number
c_f	: fast wave speed
E	: total energy density
F	: flux vector
I	: unit matrix
J	: current density
L	: length
M	: Mach number
m	: magnetic moment
$\dot{m}$	: mass flow rate
P	: pressure
R	: radius
R_m	: magnetic Reynolds number
S	: source term
S_{Powell}	: Powell's source term
t	: time
U	: conservative quantity
v	: velocity vector
x, y	: coordinate variable
β_{th}	: thermal beta
γ	: specific heat ratio
μ_0	: magnetic permeability in vacuum
ρ	: density
ω	: frequency

Subscripts

c	: characteristic value
e	: electron
i	: ion

IP : injected plasma

SW : solar wind

Superscripts

n : time step number

1. Introduction

Magnetic Sail and Mini-Magnetospheric Plasma Propulsion (M2P2), also known as Magneto-Plasma Sail (MPS), were proposed as a candidate for near-future space propulsion. These propulsion systems are expected to capture the solar wind energy in interplanetary space by an artificial magnetosphere to generate a thrust force (Fig. 1). In the case of Magnetic Sail, the thrust force of the propulsion system increases in proportion to the size of the magnetosphere, and it is expected that the km order magnetosphere provides the significant thrust force ¹⁻⁴⁾. The important point of the propulsion system is the development of efficient techniques to produce a large magnetosphere.

Magnetic sail was proposed by Zubrin et al. ⁵⁾ in 1991. The artificial magnetosphere is generated by on-board superconducting coils. On the other hand, with the magnetic field inflation method of the M2P2 spacecraft which was proposed by Wingree et al. ⁶⁾ in 2000, a small magnetosphere created by on-board superconducting coils can be inflated to a larger magnetosphere by injecting high-velocity plasma jets from the M2P2 spacecraft (Fig. 2). However, the previous study ⁷⁾ shows, in the inflation method using the kinetic energy of the injected plasma, the thrust force drastically decreases when the injected plasma flow velocity becomes higher than the magneto-sonic speed. To improve the situation of the M2P2 spacecraft, in the MPS spacecraft, the magnetosphere

inflation is conducted by plasma equilibrium, i.e., by injecting low-velocity plasma. The magnetosphere inflation by plasma equilibrium in Fig. 3 uses plasma currents by trapped slow plasma particles inside the magnetosphere to increase the magnetic moment of the MPS spacecraft. The inflation method is named the ring-current inflation. It is expected that the thrust force of the MPS spacecraft, by the ring-current inflation with the optimized mass flow rate, position and energy of the injected plasma, can be bigger than the M2P2 spacecraft.

To verify the availability of the ring-current inflation with the optimized parameter of the injected plasma, both experimental studies and numerical analyses⁸⁻¹²⁾ need to be conducted, and especially, the numerical simulation is an effective research tool because it is easy to conduct a parameter survey. In the MPS spacecraft, the interaction between the plasma and magnetic field (ring-current inflation and solar wind-magnetosphere) has the phenomenon of wide range of interaction scale. Therefore, the numerical analysis need to be comprehensively conducted by a number of numerical approaches including the kinetic theory of the particle (Full-PIC analysis, Hybrid analysis) and the magnetohydrodynamic (MHD) analysis. The MHD assumption is applicable when the characteristic length scale is much larger than the ion Larmor radius; in the flow field around the MPS, the MHD assumption is valid when the magnetosphere size is much larger than the ion Larmor radius of the solar wind (several tens kilometers). The features of the MHD analysis are as follows: (1) simulation cost is relatively low, and therefore a comprehensive parameter survey can be conducted, (2) the magnetosphere can be analysed in large scales.

The objective of this study is to investigate the relation between the thrust characteristics and the injected plasma parameters (the mass flow rate and the thermal beta value). For this purpose, a parametric survey is conducted using axis-symmetric two-dimensional ideal MHD simulation.

Fig. 1. Concept of Magnetic Sail.

Fig. 2. Magnetic field inflation of M2P2.

Fig. 3. Magnetic field inflation of MPS (ring-current inflation).

2. Numerical Simulation Model

2.1. Governing equations

Governing equations are ideal MHD equations which describe the motion of a perfectly conducting fluid interacting with magnetic fields. The normalized ideal MHD equations are written as follows:

$$\frac{\partial \mathbf{U}}{\partial t} + \nabla \cdot \mathbf{F} = \mathbf{S}, \quad \mathbf{U} = \begin{pmatrix} \rho \\ \rho \mathbf{v} \\ \mathbf{B} \\ E \end{pmatrix}, \quad (1)$$

$$\mathbf{F} = \begin{pmatrix} \rho \mathbf{v} \\ \rho \mathbf{v} \mathbf{v} + \left(p + \frac{\mathbf{B} \cdot \mathbf{B}}{2} \right) \mathbf{I} - \mathbf{B} \mathbf{B} \\ \mathbf{v} \mathbf{B} - \mathbf{B} \mathbf{v} \\ \mathbf{v} \left(E + p + \frac{\mathbf{B} \cdot \mathbf{B}}{2} \right) - (\mathbf{v} \cdot \mathbf{B}) \mathbf{B} \end{pmatrix}, \quad (2)$$

$$\nabla \cdot \mathbf{B} = 0, \quad (3)$$

here, $\mathbf{S}$ is a source term which will be denoted in detail later.

The energy density E is

$$E = \frac{\rho v^2}{2} + \frac{p}{\gamma - 1} + \frac{B^2}{2}. \quad (4)$$

Here, the specific heat ratio $\gamma (= 5/3)$ is set for the monoatomic ideal gas because the solar wind is mainly composed of protons and electrons.

Normalization of the governing equations is conducted using characteristic physical values in the flow-field (Table 1). The characteristic length is taken as the size of the spacecraft, and the characteristic density and magnetic flux are in the same order as in the solar wind. Other characteristic values are determined by the following relations:

$$v_c = \frac{B_c}{\sqrt{\mu_0 \rho_c}}, \quad p_c = \rho_c v_c^2, \quad t_c = \frac{R_c}{v_c}, \quad J_c = \frac{B_c}{\mu_0 L_c}. \quad (5)$$

Table 1. Characteristic physical values.

R_c [km] (radius of the spacecraft)	10
ρ_c [kg/m ³]	1.67×10^{-21}
B_c [T]	10^{-8}
v_c [km/s]	218
p_c [Pa]	7.96×10^{-11}

In usual hydrodynamics, the local Maxwellian velocity distribution function is accomplished by particle collisions. However, in the plasma, velocity distribution functions are

relaxed to the Maxwellian within a short time-scale of plasma instability. The relaxation time to the Maxwellian is determined from ω_{pe}^{-1} , ω_{pi}^{-1} and ω_{Li}^{-1} , which are much shorter than the collisional relaxation time in many situations. These frequencies are described as below:

$$\omega_{pe} = \sqrt{\frac{n_e e^2}{\epsilon_0 m_e}}, \quad \omega_{pi} = \sqrt{\frac{n_i e^2}{\epsilon_0 m_i}} \quad (5)$$

$$\omega_{Li} = \frac{eB}{m_i}. \quad (6)$$

The ion cyclotron has the most longest time scale, and therefore in the phenomena whose time scale is much shorter than ω_{Li}^{-1} , a fluid approximation is well justified even in a collisionless plasma. This condition can be rewritten in more convenient form as follows:

$$L_c \gg r_{Li}. \quad (7)$$

Here, r_{Li} is the Larmor radius of ions. When the characteristic length L_c , which is the size of the magnetic field, is much larger than the ion Larmor radius (of the solar wind), the fluid approximation is valid. In the flow field around the MPS spacecraft, the magnetosphere size should be in an order of several tens of kilometers.

2.2. Numerical method

The governing equations are solved by the finite volume method. The numerical flux of the convective term is evaluated by the TVD Lax-Friedrich scheme¹³⁾ in which the highly accurate analysis is achieved by the MUSCL method with MINMOD limiter adopted as the limited function.

This simulation procedure consists of two steps. In the first step of the simulation, the flow field without the injected plasma (the flow field around Magnetic Sail) is simulated. In the second step, the flow field of MPS with the injected plasma flow is simulated using the flow field obtained in the first step as the initial condition. The time integration is conducted by the second-order Runge-Kutta scheme with the local time stepping in the first step simulation and the ADI-SGS scheme is used in the second step simulation¹⁴⁾.

To eliminate numerical errors in $\nabla \cdot \mathbf{B}$, the Powell's source term and the projection scheme are used¹⁵⁾.

2.3. Numerical settings

The interaction between the solar wind and the magnetosphere is simulated in the axis-symmetric two-dimensional space (x-y plane).

Figure 4 shows the computational grid system (100 x 100) and boundary conditions. A sphere with a unit radius is located at the origin and represents the spacecraft. The inflow boundary condition of the solar wind is implemented by fixing all physical states, and the outflow boundary condition is implemented by extrapolating all physical states. In the second step, the plasma flow with zero velocity is injected in the range of one diameter (non-dimensional) from $(x, y) = (0, 3)$. The effects of the plasma injection to the flow field is added to the right hand side of Eq. (1) as source terms; the source terms $\mathbf{S}$ in the right hand side of Eq. (1) include a mass source and an energy source arising from the plasma injection and are

expressed as in Eq. (8):

$$\mathbf{S} = \mathbf{S}_{Powell} + \mathbf{S}_{IP} = - \begin{pmatrix} 0 \\ \mathbf{B} \\ \mathbf{v} \\ \mathbf{v} \cdot \mathbf{B} \end{pmatrix} \nabla \cdot \mathbf{B} + \begin{pmatrix} \rho_{IP} \\ \mathbf{0} \\ \mathbf{0} \\ \frac{p_{IP}}{\gamma - 1} + \frac{B_{IP}^2}{2} \end{pmatrix}. \quad (8)$$

The plasma continues to be injected until the simulation is found as the stationary solution. At the spacecraft surface, the boundary condition implemented is the perfect-conducting slip wall.

Figure 5 shows the MPS spacecraft attitude and the coordinate definition of the thrust vector. The magnetic moment of the spacecraft is set to be parallel to the solar wind flow direction (the x axis). The thrust force is calculated from the momentum change in the plasma flow; the plasma flow includes not only the solar wind but also the injected plasma flow. The momentum change in the plasma flow is calculated by numerically integrating the momentum flux over a control volume surrounding the spacecraft. In this study, this thrust force is denoted by $\mathbf{F}_{all}$. The normalized $\mathbf{F}_{all}$ is written as follows:

$$\mathbf{F}_{all} = - \oint_S (\rho \mathbf{v} \mathbf{v} + p \mathbf{I}) \cdot \mathbf{n} dS - \oint_S \left(\frac{\mathbf{B} \cdot \mathbf{B}}{2} \mathbf{I} - \mathbf{B} \mathbf{B} \right) \cdot \mathbf{n} dS. \quad (9)$$

Here, the first term of the right hand side is the momentum change as the fluid and the second term of the right hand side is the Maxwell stress.

Fig. 4. Computational grid system and boundary condition.

Fig. 5. Definition of thrust.

2.4. Computational conditions

Table 2 shows the computational conditions of the solar wind. These conditions are the typical parameter around the earth. Table 3 shows the mass flow rate and the thermal beta value of the injected plasma. The thermal beta value is defined at the plasma injection point and the equation is

$$\beta_{th} = \frac{P_{IP}}{B_{IP}^2/2}. \quad (10)$$

The magnetic moment of the spacecraft is assumed the dipole magnetic moment. The value of the magnetic moment m is set to 500 in non-dimensional quantity ($6.28 \times 10^7 \text{ Wb} \cdot \text{m}$).

Table 2. Condition of the solar wind.

	Real dimension	Non-dimensional
ρ_{SW}	$8.35 \times 10^{-21} \text{ kg/m}^3$	5
B_{SW}	0 T	0
v_{SW}	400 km/s	1.83
p_{SW}	$3.20 \times 10^{-11} \text{ Pa}$	0.4
M	-	5

Table 3. Condition of the injected plasma (non-dimensional).

Mass flow rate (real dimension)	Thermal beta
100 (3.6 mg/s)	1, 2, 3, 4, 5, 10, 15, 18, 25, 30, 35
500 (18 mg/s)	2, 4, 5, 7
700 (25.2 mg/s)	1, 2, 3, 5, 10, 15
1000 (36 mg/s)	1, 2, 3, 4, 10

3. Simulation Results and Discussion

Figure 6 shows the flow field around the Magnetic Sail and the MPS spacecraft (the mass flow rate is 1000 in non-dimensional value and the thermal beta value is 2). The solar wind at supersonic velocity is interrupted by the spacecraft magnetic field and a bow shock is formed. The magnetic field is stretched by the solar wind and turns into the magnetosphere. The magnetosphere size (the projected area indicated by white arrows in the figure) of the Magnetic Sail spacecraft is about 10 (non-dimensional) and the magnetosphere size of the MPS spacecraft is about 27. Therefore, the magnetosphere size is enlarged by 2.7 times larger with the injected plasma.

Figure 7 shows the plasma current distribution around the Magnetic Sail spacecraft and the MPS spacecraft (the mass flow rate is 1000 and the thermal beta value is 2). The injected plasma induces plasma currents which enhance the magnetic field strength and expands the magnetosphere size. The injected plasma flows downstream crossing the magnetic field lines. The steady solution is obtained when the injected plasma mass per unit time is balanced with the plasma mass escaping from the outflow boundary per unit time. In the ideal MHD flow field, the magnetic field lines are frozen into the plasma flow, i.e., in theory, the plasma cannot be slipped from the magnetic field lines. However, in this simulation, the plasma slips from and flows across the magnetic field lines, leading to the break down the frozen-in condition. The plasma slip in the simulation is caused by the numerical dissipation. The effects of the numerical dissipation will be analytically discussed at a later section in detail.

Figure 8 shows the x directional velocity distribution around the MPS spacecraft (a: the mass flow rate is 1000 and the thermal beta value is 2, b: the mass flow rate is 1000 and the thermal beta value is 4). Due to the strong numerical dissipation, it is considered that the part of the injected plasma

crosses the magnetic field lines, and flow into the cusp region of the spacecraft magnetic field. The plasma flowing into the magnetic cusp is ejected downstream through the divergent magnetic field; the divergent magnetic field acts as a magnetic nozzle accelerating the plasma jet. The jet generated by the magnetic nozzle produces a thrust force in the windward direction. Therefore, the jet by the magnetic nozzle reduces the thrust force of the MPS spacecraft; the generation of the jet gives negative effects on the thrust performance of the MPS spacecraft (Fig. 9).

Figure 10 shows a thrust gain (red line) as a function of the thermal beta value of the injected plasma; the thrust gain is the ratio of the MPS spacecraft thrust force to the Magnetic Sail thrust force. In addition, a magnetosphere gain (blue line), which is defined as the ratio of the magnetosphere size of the MPS to that of the Magnetic Sail, and the thrust force produced by the plasma jet through the magnetic nozzle (green line) are also plotted against the thermal beta value of the injected plasma in Fig. 10. Note that, Fig. 10 shows the results in the case of $\dot{m} = 1000$. The magnetosphere gain becomes larger with the thermal beta value. However, the thrust gain becomes the maximum value at $\beta_{th} = 2$, because the thrust generated by the magnetic nozzle jet is also becomes larger with the thermal beta value of the injected plasma.

Figure 11 shows the effect of the mass flow rate on the thrust gain and table 4 shows the local maximum value of the thrust gain for each mass flow rate condition. From the figure and the table, it is indicated that the maximum value of the thrust gain is 3.77 when the thermal beta value is 25 and the mass flow rate is 100. In the lower mass flow rate and the higher thermal beta value, the thrust gain tends to be higher. In addition, it is also indicated that a specific impulse tends to be higher with the lower mass flow rate (Fig. 12).

Fig. 6. Density distribution and solar wind lines around the spacecraft.

Fig. 7. Current distribution and magnetic field line around the spacecraft.

Fig. 8. Velocity distribution and magnetic field line around the spacecraft.

Fig. 9. Analogy of the jet generation.

Fig. 10. Injected plasma effect on thrust characteristics of the MPS.

Fig. 11. Injected plasma effect on thrust characteristics of the MPS.

Table 4. The maximum value of the thrust gain (non-dimensional).

Mass flow rate	Thermal beta	Thrust gain
100	25	3.77
500	4	3.45
700	2	3.39
1000	2	3.26

Fig. 12. Specific impulse v.s. thermal beta (single logarithm).

4. Magnetic Reynolds Number and Effective Reynolds Number

In this chapter, the effect of the numerical dissipation is discussed. Here, the numerical dissipation of the Lax-Friedrich scheme is discussed in the one-dimensional space for simplicity. The finite difference equation of the TVD Lax-Friedrich scheme with the MUSCL method is evaluated as below:

$$\frac{U_i^{n+1} - U_i^n}{\Delta t} = - \frac{-F_{i+2}^n + 8F_{i+1}^n - 8F_{i-1}^n + F_{i-2}^n}{12\Delta x} + |v + c_f| \Delta x^3 \cdot \frac{-U_{i+2}^n + 16U_{i+1}^n - 30U_i^n + 16U_{i-1}^n - U_{i-2}^n}{12\Delta x^4}. \quad (11)$$

The induction equation of magnetic fields in the resistive MHD is expressed including the magnetic Reynolds number term (the term by the plasma resistivity; it generates dissipation of the magnetic field) as follows:

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{v} \times \mathbf{B} + \frac{1}{R_m} \nabla \times \nabla \times \mathbf{B}. \quad (12)$$

From simply comparing the Eq. (11) and the Eq. (12), it is found that the effects of the numerical dissipation are similar to the magnetic Reynolds number term (the effects of the plasma resistivity). The plasma resistivity causes plasma slip

from magnetic field lines, and therefore it is concluded that the plasma slip in the MPS simulation is due to the numerical dissipation.

When the second order differential term of Eq. (12) is in the same order of the fourth order finite differential term of Eq. (11), an effective Reynolds number by the numerical dissipation is evaluated as follows:

$$\frac{1}{|\nu + c_f| \Delta x^3} = \frac{\Delta t}{C_{CFL} \Delta x^4}. \quad (13)$$

In the simulation, the value of the effective magnetic Reynolds number is estimated about 100, i.e., the solution obtained by the simulation is resistive MHD flow field. It is expected that the maximum performance of the MPS system is achieved when the flow field around the MPS is the ideal MHD flow field. To investigate the effect of the finite magnetic Reynolds number on the MHD flow field, the low numerical dissipation scheme needs to be used. In the future works, we have to reveal the range of the magnetic Reynolds number in which the MPS system generates the sufficient performance.

5. Conclusions

In this study, we investigated the thrust characteristics of the MPS spacecraft which adopt the magnetosphere inflation by the zero velocity plasma injection. A parametric survey is conducted using the axis-symmetric two-dimensional MHD simulation. As a result, the thrust gain and the specific impulse tend to be higher with the lower mass flow rate and the higher thermal beta value. However, when the local thermal beta value becomes higher than unity, the injected plasma moves downstream with a higher speed through the divergent magnetic field around the spacecraft. In such a case, as a result of the increased momentum of the plasma jet from the spacecraft, the net thrust by MPS, i.e., force exerting on the coil, will decrease when the thrust reduction exceeds the thrust enhancement by the magnetic field inflation. Therefore, the maximum value of the thrust gain (and the specific impulse) appears for a certain value of thermal beta. It is indicated that the maximum value of the thrust gain is 3.77 when the thermal beta value is 25 and the mass flow rate is 100. From this result, it is concluded that higher- β and the mass flow rate plasma injection does not always increase the MPS thrust, and the optimum parameters should be chosen.

In this simulation, the ideal MHD assumption is assumed in the flow field. However, the effect of the numerical dissipation is large and the frozen-in condition between the plasma and magnetic field is broken; in other words, the flow

field obtained in this study is the resistive MHD flow field. The effective Reynolds number by the numerical dissipation is estimated to be about 100.

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