High Precision Shape Control Based on Surface Accuracy Analyses Using an Optical Shape Measurement System

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A novel method for controlling the shape of an antenna reflector based on surface accuracy analyses using an optical shape measurement system such as a stereo vision or photogrammetry system is developed, and its feasibility is demonstrated. In the control procedure, intentional deformations are added to a reflector surface by using surface adjustment mechanisms, and the corresponding changes in the surface accuracy of the measured surface shape are analyzed using an optical shape measurement system. Then, the control inputs for the shape correction of the deformed reflector are directly determined from the information about the changes in the surface accuracy. Some numerical simulations and experiments are performed, and the feasibility of the developed method is demonstrated. The results of the investigation show that deformations in the reflector are properly corrected by the developed method when its shape is measured at appropriate points.

Key Words: Space Structures, Surface Errors, Shape Control, Optical Shape Measurement

Nomenclature

\[ A \]: area of a reflector surface
\[ B_z \]: control parameter determined by mode shape of an intentional deformation
\[ B_{mz} \]: parameter calculated from evaluated square of the RMS surface errors
\[ \varepsilon_{ms} \]: measurement error of a reflector surface
\[ H \]: matrix of control parameters
\[ n_{ms} \]: number of measurement points on a surface
\[ S \]: sensitivity matrix that relates control inputs to corresponding surface deformations
\[ sa \]: raw vector of surface deformation caused by an intentional deformation
\[ \Delta z \]: surface error corresponding to an intentional deformation
\[ \Delta z_o \]: surface error due to disturbance
\[ \sigma_{ms} \]: standard deviation of measurement error.

1. Introduction

Recently, it has become necessary for space antenna reflectors to be large and precise to meet expanding mission requirements\(^1,2\). Shape controls for future antenna reflectors have been investigated and developed by several research groups\(^3,4\). In these reflector systems, the surface errors of the reflectors are measured, and the measured errors are corrected by surface adjustment mechanisms. As a result, the measurement accuracy affects the surface accuracy obtained by the shape control. Therefore, high-accuracy whole shape measurements are required. Furthermore, a noncontact measurement system must be used because of the flexibility of large space antenna reflectors.

Many of these studies have focused on structural shape controls and have assumed ideal shape measurement. Some studies have employed radio holographic analyses\(^7,8\) to diagnose reflector distortions. Optical shape measurement systems such as photogrammetry measurement and stereo vision systems are often used to carry out ground adjustments on space reflector antennas\(^9\) and have been investigated for shape measurements in orbit\(^10,11\).

Radio holographic analyses\(^12-14\) are very powerful methods for diagnosing and measuring the distortions in reflector antennas. The measurement accuracies of these methods rely on the radio wavelengths of the antennas. Therefore, it is more appropriate to measure the surface deformations of a reflector antenna, rather than determining the accuracy of the antenna surface based on the antenna’s wavelength. However, these methods require special equipment such as near-field scanners or other reference antennas. They also involve special procedures such as precise scanning. Therefore, it is difficult to adopt such methods for space antennas.

Optical shape measurement systems such as photogrammetry and stereo vision systems are widely used to measure the shapes of structures\(^15,16\). These are noncontact measurement methods based on triangulation, and they can simultaneously measure the three-dimensional positions of many points. Their measurement accuracies are in inverse proportion to the size of the target object and are poor for large structures. Therefore, the direct use of the shape measured by an optical shape measurement system is not appropriate for the shape control of a large high-precision antenna reflector in orbit.

In order to overcome these difficulties, we developed a
novel method for controlling the shape of a reflector based on surface accuracy analyses using an optical shape measurement system. This method is based on the theory that the estimation errors in the surface accuracy with random measurement errors will converge to a theoretically determined value when the positions on the surface are measured at many points.

In the developed control procedure, intentional deformations are added to a reflector surface by using surface adjustment mechanisms, and the corresponding changes in the surface accuracy are analyzed from the surface shape measured using an optical shape measurement system. Then, the control inputs for correcting the shape of the deformed reflector are directly determined from the information on the changes in the surface accuracy.

In the developed method, the resultant surface accuracy under shape control relies on the accuracies of the shapes of the intentional deformations. Therefore, the intentional deformations need to be predicted accurately. On the other hand, large space reflector antennas are generally flexible and have structural nonlinearity. Accordingly, it is difficult to accurately predict the intentional deformations.

In this study, a method is also developed for calibrating the shape control parameters that are used for the developed shape control method. The effects of the intentional deformations are taken into account in these parameters. Therefore, accurate shape control is achieved by using the calibrated parameters.

Some numerical simulations and experiments are performed, and the feasibility of the developed method is demonstrated. The results of the investigation show that deformations in the reflector are corrected properly by the developed method when its shape is measured at appropriate points.

2. Shape Control Procedure Based on Surface Accuracy Analyses Using Optical Shape Measurement System

2.1. Surface accuracy analyses with measurement errors

The developed shape control method is based on surface accuracy analyses with measurement errors. The measurement errors are classified into three categories: drift errors, systematic errors, and random errors. In these errors, the drift and systematic errors can be mitigated through periodic calibration. Therefore, the random errors are of the greatest concern for high-accuracy shape control.

Let us consider the surface accuracy of a reflector surface deformed by a disturbance with random measurement errors. The square of the RMS surface errors is calculated from the measured shape as follows:

\[ \Delta z_{\text{rms}}^2 = \frac{1}{A} \int (\Delta z_a + e_{\text{mes}})^2 dA \]

\[ \approx \sum_{n_{\text{mes}}} (\Delta z_a + e_{\text{mes}})^2 / n_{\text{mes}} \]

\[ = \Delta z_{\text{rms}}^2 + 2 \sum_{n_{\text{mes}}} \Delta z_a e_{\text{mes}} / n_{\text{mes}} + e_{\text{mes},\text{rms}}^2 \]  

(1)

Here, \( \Delta z_{\text{rms}} \) denotes the RMS surface errors and \( \Delta z_a \) denotes the surface error due to disturbance. \( A \) is the area of the reflector surface. \( e_{\text{mes}} \) denotes the measurement error, and its standard deviation is \( \sigma_{\text{err, mes}} \). \( n_{\text{mes}} \) is the number of measurement points on the surface, and it is assumed that the shape of a reflector surface is measured at many points using an optical shape measurement system in this study. The subscript rms indicates the RMS value. In this study, we call the square of the RMS surface errors “SRSE.”

We can see from Eq.(1) that the calculated SRSE has an evaluation error due to the measurement errors. The average and standard deviation of the evaluation errors are obtained as follows by assuming that the measurement errors follow a normal distribution, and, accordingly, \( n_{\text{mes}} e_{\text{mes},\text{rms}}^2 / \sigma_{\text{err, mes}}^2 \) follows a chi-squared distribution:

\[ E(\Delta z_{\text{rms}}^2) = \sigma_{\text{err, mes}}^2 \]  

\[ \sigma(\Delta z_{\text{rms}}^2) = \sqrt{\frac{2}{n_{\text{mes}}}} \sigma_{\text{err, mes}}^2 \]  

\[ E(\sum_{n_{\text{mes}}} e_{\text{mes}} / n_{\text{mes}}) = 0 \]  

\[ \sigma(\sum_{n_{\text{mes}}} \Delta z_a e_{\text{mes}} / n_{\text{mes}}) = \sqrt{\frac{1}{n_{\text{mes}}}} \sigma_{\text{err, mes}}^2 \]  

(2)

(3)

(4)

(5)

It can be observed from Eqs.(4) and (5) that the variance of the evaluation errors becomes small when the reflector surface is measured at many points, and the evaluation errors converge to a theoretically determined value given by Eq.(2). Although we assume the measurement errors follow a normal distribution in the above equations, the variance of the evaluation errors with other measurement error distributions follows the same trend because of the central limit theorem.

2.2. Shape control procedure based on surface accuracy analyses

A shape control method based on surface accuracy analyses is derived. We now consider adding an intentional deformation to a reflector surface deformed by a disturbance. As a result of the intentional deformation, the SRSE is evaluated from the measured shape as follows:

\[ V(\Delta z_a) = \frac{\Delta z_a^2}{A} + \frac{2}{A} \sum \int (\Delta z_a + e_{\text{mes}})^2 dA - \frac{2}{A} \sum \int (\Delta z_a^2 + e_{\text{mes}}^2) dA 

+ \frac{2}{A} \sum \int (\Delta z_a^2 + e_{\text{mes}}^2) dA + e_{\text{mes, rms}}^2 \]  

(6)

Here, \( \Delta z_a \) is the surface error corresponding to the intentional deformation, and we set the evaluated SRSE as \( V(\Delta z_a) \), the function of \( \Delta z_a \). From Eqs.(1), (4) and (6), the change in the evaluated SRSE is obtained as follows:

\[ dV(\Delta z_a) = V(\Delta z_a) - V(0) \]

\[ = \frac{\Delta z_a^2}{A} dA - \frac{2}{A} \sum \int \Delta z_a^2 dA + \frac{\sigma_{\text{err, mes}}^2}{\sqrt{n_{\text{mes}}}} \]  

(7)

As can be seen from this equation, the average error for the evaluation of SRSE, Eq.(2), is eliminated, and the evaluation errors become negligibly small when the reflector surface is...
measured at numerous points.

By rearranging the expression and dividing the reflector surface into small elements, the following equation is derived from Eq.(7).

$$\sum \Delta z_{i,j} \Delta z_{a,j} = s_a \Delta Z_a \approx B_m + B_a$$  \hspace{1cm} (8)

Here,

$$B_m = \frac{n_{max}}{2} dV(\Delta z_a)$$  \hspace{1cm} (9)

$$B_a = -\frac{n_{max}}{2} \Delta z_{a,rms}^2$$  \hspace{1cm} (10)

Here, $s_a$ is the raw vector of the surface deformation caused by the intentional deformation, and $\Delta Z_a$ is the column vector of the surface deformation caused by the disturbance. $B_m$ is calculated from the evaluated SRSE, and $B_a$ is a control parameter determined by the mode shape of the intentional deformation.

In the control procedure, some modes of intentional deformations are added to a reflector surface by using surface adjustment mechanisms, and the corresponding changes in the surface accuracy are analyzed. By converting the results obtained by each intentional deformation mode into matrix form, the following equation is obtained.

$$S \Delta Z_0 = B_m + B_a$$  \hspace{1cm} (11)

Here,

$$S = \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_i \\ \vdots \\ s_s \end{bmatrix}, \quad B_m = \begin{bmatrix} B_{m1} \\ B_{m2} \\ \vdots \\ B_{mi} \\ \vdots \\ B_{ms} \end{bmatrix}, \quad B_a = \begin{bmatrix} B_{a1} \\ B_{a2} \\ \vdots \\ B_{ai} \\ \vdots \\ B_{as} \end{bmatrix}$$  \hspace{1cm} (12)

$S$ is a sensitivity matrix that relates the control inputs for the intentional deformations to the corresponding surface deformations. Therefore, the relations between the control inputs for the shape control and the surface deformations are given as follows:

$$S^T p = \Delta Z_e \approx -\Delta Z_0$$  \hspace{1cm} (13)

where $p$ is a column vector of the control inputs and $\Delta Z_e$ is a column vector of the achievable deformations caused by the shape control. The control inputs of the conventional shape control methods are usually obtained using Eq.(13). Substituting Eq.(13) into Eq.(11) yields

$$H p = S S^T p \approx -(B_m + B_a)$$  \hspace{1cm} (14)

Here, matrix $H$ is a control parameter determined by the mode shapes of the intentional deformation as $H = S S^T$. From Eq.(14), the control inputs are determined as follows:

$$p = -H^{-1} (B_m + B_a)$$  \hspace{1cm} (15)

In the shape control method, the control inputs are directly obtained from the changes in the evaluated SRSE. The procedure of the developed shape control is summarized in Fig.1.

2.3. Method for calibrating shape control parameters

As can be seen from Eqs.(10) and (14), vector $B_i$ and matrix $H$ contain terms related to the intentional deformations. Therefore, errors in the prediction of the intentional deformations lead to inaccurate shape control. In order to achieve high-accuracy shape control, accurate values have to be obtained for vector $B_i$ and matrix $H$, even if the shapes of the intentional deformations are unknown. A method to calibrate matrix $H$ and vector $B_i$ in orbit is developed. The calibrations are carried out as follows.

In the first step, deformations that are $\alpha_i$ times the intentional deformations are added. As a consequence of these deformations, the following equation is obtained from Eq.(8).

$$\alpha_s \Delta Z_a = \frac{n_{max}}{2} (dV(\alpha_i \Delta z_a) + \alpha_i^2 \Delta z_{a,rms}^2)$$  \hspace{1cm} (16)

By evaluating the SRSEs for many scales of the intentional deformation and putting the results in a matrix form, the following equation is obtained, and the element of vector $B_a$ is calculated from this equation.

$$\begin{bmatrix} 1 \\ \alpha_1^2 \\ \alpha_1 \\ \alpha_2^2 \\ \alpha_2 \\ \vdots \\ \alpha_s^2 \\ \alpha_s \end{bmatrix} = \frac{n_{max}}{2} \begin{bmatrix} dV(\Delta Z_e) \\ dV(\alpha_1 \Delta Z_a) \\ dV(\alpha_1 \Delta Z_a) \\ dV(\alpha_2 \Delta Z_a) \\ \vdots \\ \alpha_s \Delta Z_a \end{bmatrix}$$  \hspace{1cm} (17)

In the next step, the i1-th and i2-th intentional deformations are simultaneously added to the reflector. The corresponding change in the evaluated SRSE is obtained as follows:
By dividing the reflector surface into small elements, the \(i_1, i_2\)-th element of matrix \(H\) is given as follows:

\[
H_{i_1, i_2} = \frac{n_{\text{meas}}}{2} \left( dV(\Delta z_{i_1} \pm \Delta z_{i_2}) - dV(\Delta z_{i_1}) - dV(\Delta z_{i_2}) \right)
\]

(19)

3. Numerical Simulations

3.1. Model for analysis

In order to clarify the feasibility of the developed method, some numerical simulations were performed. We developed a proof of concept model equipped with surface adjustment mechanisms. This model was employed for experiments to clarify the feasibility of the developed shape control method; the results of the experiments are discussed in the next section. A schematic representation of the proof of concept model is shown in Fig.2. Figure 3 shows photographs of the model. The specifications of the proof of concept model are summarized in Table 1.

The proof of concept model had a plate fixed to a central fixture, and this plate was assumed to be a reflector surface. The model was equipped with 6 linear actuators, which were used as surface adjustment mechanisms by changing the shape of the plate. The corresponding surface deformations were considered to be the intentional deformations. Therefore, the 6 intentional deformations were considered to be the control modes.

The corresponding numerical model of the proof of concept model was employed for the numerical simulations. The numerical model was developed and its stiffness matrix was obtained using the commercial FEM code ANSYS. Numerical simulations were performed in MATLAB using the stiffness matrix. In the simulations, it was assumed that the surface errors were simultaneously measured at many points with random measurement errors.

3.2. Results of numerical simulations

In the simulations, the control parameters were calibrated, and the deformations of the plate caused by disturbance were corrected using the developed method. The initial position errors of the surface adjustment mechanisms were assumed to be disturbances. Thus, the corresponding surface deformations were correctable. We set the surface deformations caused by the disturbances to have surface errors of 1 mm RMS. In order to investigate the efficiency of the developed method, numerical simulations were performed while changing the measurement conditions such as the number of measurement points and the standard deviations of the measurement errors. The numerical simulations were carried out 2000 times for each measurement condition. The conventional shape control method, which uses a sensitivity matrix, was also investigated to compare the efficiency.

Their results are summarized in Fig.4, which shows the relation between the number of measurement points and the RMS of the residual deformations from the nominal shape. The error bars indicate the standard deviations of the surface errors. As can be seen in this figure, the developed method achieved better control performance when the number of measurement points was above 1000. Figure 5 shows the relation between the standard deviations of the measurement errors and the RMS of the residual deformations when the number of measurement points was 31397 (the corresponding grid size was 1.5 mm). This figure indicates that a high-accuracy surface was obtained by the developed method when the surface shape was measured at appropriate points, even though the measurement system had large measurement errors.
Table 1. Specifications of proof of concept model.

| Surface plate | diameter: 300 mm, thickness: 1 mm, material: aluminum alloy (A2024) |
| Surface adjustment mechanisms | 6 linear actuators (SUS XA28-50) |
| Positions of surface adjustment mechanisms | Fixed point Positions of surface adjustment mechanism |
| Measurement system | number of measurement points: 65–31397 (corresponding grid size: 30 mm–1.5 mm) standard deviations of measurement errors: 0–0.5 mm |

Table 2. Specifications of measurement system.

| Type of measurement system | Stereo vision |
| Camera type | Nikon D80 |
| Number of cameras | 2 |
| Modification of the camera image | Correction of distortion using DxO Optics software. |
| Baseline distance between cameras | 500 mm |
| Corresponding point detection | Correlation-based template matching analysis of the random dot patterns |
| Number of measurement points | 343, 621, 1389, 5557 |

4. Proof of Concept Experiments

4.1. Experimental setup

The experiments were performed using the proof of concept model and an actual optical shape measurement system. In the numerical simulations, it was assumed that the measurement system had ideal random errors. However, an actual measurement system has some systematic errors, which may affect the control performance.

A stereo vision system was employed as the shape measurement system in the experiments, and the feasibility of the developed method was experimentally clarified. Figure 6 shows the experimental setup, and the specifications of the measurement system are summarized in Table 2.

4.2. Calibration of shape control parameters

In order to achieve high-accuracy shape control, matrix $H$ and vector $B_a$ were calibrated using the developed method, as described in the previous section. The elements of matrix $H$ calculated by the numerical model and those calibrated experimentally are shown in Figs.7 and 8, respectively. Figure 9 shows the elements of vector $B_a$. It can be observed from these figures that the control parameters were calibrated with some differences as a result of the measurement errors and the difference between the numerical model and the developed proof of concept model. The effects of these differences are investigated in the next section.
4.3. Experimental results

In the experiments, the initial position errors of the surface adjustment mechanisms were assumed to be disturbances. Shape controls for 4 sets of initial position errors were investigated. These position errors were random errors with a standard deviation of 0.5 mm. The initial position errors are summarized in Table 3, and one example of the surface deformation caused by these errors is shown in Fig.10.

The results are summarized in Fig.11, which shows the relation between the number of measurement points and the residual position errors of the surface adjustment mechanisms. The best and worst results are indicated by the error bars in the figure. As can be seen in this figure, almost the same trend as seen in the numerical simulations was observed, and the developed method achieved better control performance when the number of measurement points was above 1000 by using the calibrated control parameters.

Table 3. Initial position errors for experiments.

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<th>3</th>
<th>4</th>
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<td>0.64</td>
<td>-0.39</td>
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<tr>
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<td>-0.54</td>
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</tr>
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5. Conclusion

A novel method for controlling the shape of an antenna reflector based on surface accuracy analyses using an optical shape measurement system was developed. In the control procedure, intentional deformations are added to a reflector surface by using surface adjustment mechanisms, and the corresponding changes in the surface accuracy are analyzed from the surface shape measured using an optical shape measurement system. Then, the control inputs for correcting
the shape of the deformed reflector are directly determined from the information on the surface accuracy.

A method was also developed for calibrating the shape control parameters that are used for the developed shape control method. The effects of the intentional deformations are taken into account in these parameters. Therefore, accurate shape control is achieved by using the estimated parameters.

Numerical simulations and experiments were performed, and the feasibility of the developed method was demonstrated. The results of the investigation showed that deformations in the reflector are properly corrected by the developed method when its shape is measured at appropriate points.

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