Relative Position and Attitude Control of a Satellite Considering Fuel Efficiency

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This study deals with the orbital motion of a follower satellite controlled by a few thrusters relative to a target satellite in a circular orbit. The thrusters of the follower are fixed to its body and generate constant unilateral forces. To generate control forces in the required directions, the follower attitude is controlled using the thruster forces only. Furthermore, the follower attitude needs to be controlled to estimate the relative position of the follower satellite from the time profile of the line-of-sight (LOS) angle. First, controllability using constant inputs in one direction is examined on the basis of modal analysis. Thereafter, the energy efficiency of controllers is discussed according to the direction of the control forces. Finally, numerical simulations are performed to verify the effectiveness of the controller and its energy efficiency.

Key Words: Relative Orbit, Fuel Efficiency, Position and Attitude Control, Unilateral Inputs

Nomenclature

- $\beta_i$: moment arm
- $C(x)$: cosine Fresnel integral
- $e_i$: eigenvectors
- $F_b$: thruster forces in a body-fixed frame
- $J_z$: moment of inertia of a follower
- $\lambda_i$: eigenvalues
- $L$: Lyapunov function candidate
- $m$: satellite mass
- $n$: orbital rate
- $R$: orbital radius of a follower
- $R_0$: orbital radius of a leader
- $S(x)$: sine of Fresnel integral
- $T_e$: external torque
- $\mu$: gravitational constant
- $u$: external acceleration
- $\psi$: attitude angle
- $\omega_x$: angular velocity of a follower
- $\xi_i$: modal variables
- $x, y$: relative position of a follower

Subscripts

- $0$: initial
- $d$: desired

1. Introduction

A key concept of future space missions involving satellite clusters is formation flying, and orbit control problems affecting the satellite formation flying have been extensively studied. The relative motions of satellites in an elliptic or a circular orbit are usually expressed by linearized Tchauner–Hempel and Hill–Clohessy–Wiltshire (HCW) equations. These equations have periodic solutions and their periodicity is useful for formation reconfiguration or rendezvous–docking missions. Palmer derives an optimal reconfiguration method of satellite formation using a Fourier expansion. The controller steers a follower satellite to a desired orbit as well as toward a leader satellite, i.e., rendezvous–docking, with a fixed time. Kumar et al. show a formation control technique using only along-track thrusters based on linear stability analysis. Despite the restriction of input direction, satellite formation is controlled within bounded errors even under the influence of nonlinear disturbances such as secular disturbances and orbit eccentricity of the leader satellite. Bando and Ichikawa exhibit the controllability of the in-plane motion of a follower satellite with a single input and derive a feedback controller based on the linear-quadratic regulator (LQR) theory.

The relative distance between two satellites must be estimated during formation or rendezvous missions. The line-of-sight (LOS) angle is used to estimate the relative position of a follower satellite in the proximity to a leader satellite. The difficulty associated with position estimation using LOS angles stems from the fact that an infinite number of relative orbits exist for the same LOS time profile. Gaia et al. discuss satellite formation control using LOS angles from the viewpoint of observability as well as filter design. Woffinden and Geller show the observability of LOS guidance on the basis of the analytic solutions to HCW equation. These studies use the LOS angle measurements on different relative orbits generated by thruster forces. Thus, the relative position of a follower satellite can be estimated using a well-calibrated thrust profile, and consequently, the satellite formation can be controlled using the state variables estimated from the LOS angle.

The purpose of this study is to realize control of both the position and attitude of a follower satellite using only thruster forces relative to a target state in a circular orbit. Our previous studies show that the position and attitude of a satellite can be controlled to a target state using only four thrusters. The
position and attitude control of a free-floating satellite based on analytical solutions is discussed in one of our previous studies. In another study, satellite relative motion control in a circular orbit is treated using LOS angles. However, this study assumes that thruster inputs are applied only in a radial direction to prevent the attitude change of the follower. The attitude of a follower satellite is maintained throughout the approach phase, which enables continuous observation of the target satellite with a camera fixed on the follower satellite. Thus, this control procedure can safely direct the follower to the leader using fewer thrusters. In the present study, we extend the control method shown in the previous study and consider the energy efficiency of the control thrusters. Numerical simulations verify the effectiveness of the proposed control method and the energy efficiency is compared.

This paper is organized as follows. Section 2 presents the relative equations of motion of a follower satellite in a circular orbit and their analytical solutions. Section 3 applies modal analysis to the state equations because the modal variables simplify the interpretation of controllability for a system with unilateral inputs. Thereafter, the controllability and energy efficiency with both along-track thrusts and radial thrusts are considered. Sections 2 and 3 present the method used to control the satellite’s position and attitude using constant thrusts. Finally, numerical simulation results are demonstrated in Section 5. We provide the conclusions in Section 6.

2. Equations of Motion

This study discusses only the in-plane motion control of a follower satellite because an out-of-plane motion is decoupled from the in-plane motion. The control procedure for the out-of-plane motion is shown in our previous study.

2.1. Hill–Clohessy–Wiltshire equation

The equations of motion for formation flying in a circular orbit have been studied on the basis of the linearized HCW equation. HCW equation describes the relative motion of a follower satellite in a leader-fixed coordinate. In the leader-fixed frame, the x-axis lies in the radial direction from the Earth, the z-axis is in the direction of the orbital momentum vector of the leader, and the y-axis completes the right-handed coordinates. The equations of motion of the follower satellite are written as follows:

\[ \ddot{x} - 2n\dot{y} - n^2(x_0 + x) = -\frac{\mu}{R^3}((R_0 + x) + u_x) \]  
\[ \ddot{y} + 2n\dot{x} - n^2y = -\frac{\mu}{R^3}y + u_y \]  

Assuming that the orbital radius of the leader is much larger than the distance between the leader and follower, we obtain the linearized equation as

\[ \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ \psi \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ u_r \end{bmatrix} \]

\[ \Rightarrow \dot{x} = Ax + Bu \]  

The analytical solution of the HCW equation with no external forces is obtained as follows:

\[ x(t) = \begin{bmatrix} 4 - 3c \\ 6(s - nt) \\ 3ns \end{bmatrix} \begin{bmatrix} 0 \\ -2(1 - c)/n \\ 0 \end{bmatrix} x_0 \]

where \( c := \cos nt \) and \( s := \sin nt \). The equation is further simplified as follows:

\[ x(t) = a\cos(nt + \Phi) + (2b/n) \]  
\[ y(t) = 2\sin(nt + \Phi) - 3bt + d \]  
\[ \dot{x}(t) = nasin(nt + \Phi) \]  
\[ \dot{y}(t) = 2ncos(nt + \Phi) - 3b \]

where

\[ a := \sqrt{(3x_0 + (2\gamma_0/n)^2 + (\dot{x}_0/n)^2)} \]  
\[ b := 2nx_0 + \dot{y}_0 \]  
\[ d := \gamma_0 + (2\dot{x}_0/n) \]  
\[ \Phi := \arctan(x_0 - \frac{\dot{x}_0}{n(3x_0 + (2\gamma_0/n))}) \]

The parameters, \( a, b, d, \) and \( \Phi \) denote the size of the relative orbit, drift velocity, distance of the center of the ellipse from the leader satellite, and initial phase angle, respectively. Because the position of the follower satellite is written as

\[ \left( x - \frac{2bn}{a} \right)^2 + \left( y + \frac{3bt - d}{2a} \right)^2 = 1 \]

the relative motion becomes an elliptic orbit when \( b = 0 \); furthermore, it becomes a leader-centered ellipse when \( b = d = 0 \).

2.2. Rotational equation

The rotational equation of motion of the follower satellite with respect to the leader-fixed frame can be considered to be a single rotational motion, which is given as follows:

\[ J_2\dot{\Omega}_2 = T_2 \]

The attitude kinematics is described as follows:

\[ \psi' = \omega_x - n \]

The last term of the right-hand side represents the orbital rate in a circular orbit due to the rotating frame.

2.3. Thruster configuration

The minimum thruster configuration required to control the satellite position and attitude has not been specified because the input constraints of thruster mechanisms complicate the mathematical discussion. However, the previous studies show that the satellite attitude and position can be controlled using four thrusters that are parallel to the principal axis of the
satellite. Thus, this study assumes that the follower is equipped with two thrusters that are parallel to the $x$-axis for in-plane motion control (Fig. 1). The thrusters generate forces with the same magnitude and have moment arms that have the same lengths about the $z_b$ axis.

The control thrusters cause the translational and rotational motion of the follower. The control thrust in the leader-fixed frame is rewritten as

$$u = \frac{1}{m} R_\alpha F_b$$

where

$$R_\alpha = \begin{bmatrix} \cos \psi & -\sin \psi \\ \sin \psi & \cos \psi \end{bmatrix}$$

$$F_b = \begin{bmatrix} f_x + f_z & 0 \end{bmatrix}^T$$

The control torque generated by the thrusters is described as

$$T_z = \beta_x f_x + \beta_y f_z$$

where the moment arms $\beta_i$ $(i=1,2)$ take a negative value when the thruster generates a clockwise directional torque.

3. Modal Analysis

3.1. Variable transformation

The system matrix $A$ for the in-plane motion of HCW equation has three eigenvectors and eigenvalues although the order of $A$ is four; thus, it is defective. The eigenvectors and eigenvalues are obtained as

$$\begin{bmatrix} e_1 & e_2 & e_3 & e_4 \end{bmatrix} = \begin{bmatrix} 0 & -1/(2n) & -1/(2n) & 1 \\ 1 & i/n & -i/n & 0 \\ 0 & i/2 & -i/2 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\lambda_1 = 0, \lambda_2 = -in, \lambda_3 = in$$

where $i$ is an imaginary number. The following calculations are conducted to form two real eigenvectors;

$$e_1' = n(e_1 - e_3)/i = \begin{bmatrix} 0 & 2 & n & 0 \end{bmatrix}^T$$

$$e_2' = n(e_3 + e_1)/i = \begin{bmatrix} -1 & 0 & 0 & 2n \end{bmatrix}^T$$

A generalized eigenvector $e_3$ is obtained as follows:

$$(A - \lambda_2 I)e_3 = e_1$$

$$\Rightarrow e_3 = \begin{bmatrix} -2/(3n) & \alpha & 0 & 1 \end{bmatrix}^T$$

where $\alpha$ is an arbitrary value. Therefore the case when $\alpha = 0$ is considered for simplicity.

The variable transformation using the eigenvectors simplifies the interpretation of relative orbit control. Here modal variables are defined as

$$\xi = \begin{bmatrix} \xi_1 & \xi_2 & \xi_3 & \xi_4 \end{bmatrix}^T = P \mathbf{x}$$

where

$$P = E^{-1} = \begin{bmatrix} e_1 & e_2 & e_3' & e_4' \end{bmatrix}^{-1}$$

The components of the modal variables are written as follows:

$$\xi_1 = y - (2\dot{x} / n)$$

$$\xi_2 = -3(2nx + \dot{y})$$

$$\xi_3 = \dot{x} / n$$

$$\xi_4 = 3x + (2\dot{y} / n)$$

The modal variable $\xi_3$ indicates the distance between the leader and center of the relative elliptic orbit and $\xi_2$ indicates the drift velocity, whereas $\xi_1$ and $\xi_4$ denote the oscillatory modes. In fact, the initial values of $\xi_1$ and $\xi_2$ are equivalent to the parameters $d$ and $-3b$ defined in Eqs. (11) and (10), respectively. The Euclidean norm of $\xi_i$ and $\xi_i$ has the same form as $\alpha$ shown in Eq. (9). The differential equations of the modal variables are described as follows:

$$\dot{\xi} = PAP^{-1}\xi + PBU$$

or equivalently as follows:

$$\dot{\xi}_1 = \xi_2 - (2u_x / n)$$

$$\dot{\xi}_2 = -3u_x$$

$$\dot{\xi}_3 = n\xi_1 + (u_x / n)$$

$$\dot{\xi}_4 = n\xi_2 + (2u_x / n)$$

3.2. Controllability and energy consumption

The state equations of the modal variables are useful when considering controllability and energy efficiency, despite control inputs are constrained to be constant in one direction. Thus, the controllability and energy consumption in terms of the modal variables are considered for two cases: positive along-track thrusts and positive radial thrusts.

First, the controllability and energy consumption with positive along-track thrusts, i.e., $u = [0 \ u_x \ y \ (u_z \geq 0)]$, are examined. The modal equations show that a positive along-track input can control all modal variables to zero only when the initial states satisfy the following conditions. The initial value of $\xi_3$ must be positive because Eq. (33) indicates that $\dot{\xi}_3$ monotonically decreases with positive acceleration. Subsequently, the time derivative of $\xi_4$ becomes positive as shown in Eq. (32), and thus, the initial value of $\xi_4$, $\xi_4(0)$, must be negative. On the other hand, the initial states of $\xi_1$ and $\xi_4$ have no restrictions because of their oscillatory motion. Thus, the acceleration required to steer all variables to zero is uniquely determined by the initial
value of $\xi_3$. Equation (33) is integrated and the analytical solution provides the minimum required acceleration as $u_{ad} := u_a \Delta t = - (\xi_2 - \xi_{2,b}) / 3$, where $\Delta t$ is the input time.

Second, the relative motion control with radial thrusts is considered. For an input $u_a (\geq 0)$, Eq. (33) shows that the variable $\xi_2$ is uncontrollable, whereas the other modal variables are controllable. Thus, in this case, the drift velocity $\xi_2$ must be controlled beforehand to a desired value $\xi_{2,b}$. In addition, the energy consumption is uniquely determined according to the initial value of $\xi_i$ for the positive input. The analytical solution of Eq. (32) provides the required acceleration as $u_{ad} = - m (\xi_1 - \xi_{1,b}) / 2 + n \xi_{2,b} \Delta t / 2$.

Therefore the relative orbit control with along-track thrusts is more energy efficient than that with radial thrusts. This is because for $\xi_{2,b} = 0$, the necessary acceleration in the along-track direction is only $u_{ad}$, whereas in radial direction, the required input to control $\xi_2$ in advance is equal to $u_{ad}$ and additional $u_{ad}$ for $\xi_2$ control is necessary. Thus the following section shows a control method using a drift motion generated with the along-track thrusts and compares the energy consumption of the two control procedures.

4. Control Procedure

This section derives the analytical solutions of the translational and rotational motion of the satellite in the x-y plane. The analytical solutions provide input timings that drive the satellite to a target state. On the basis of the analytical solutions, the energy efficiency of two control procedures are compared.

4.1. Analytical solution

The satellite’s angular velocity monotonically increases or decreases because of the constant inputs. Therefore, this study uses the angular rate as the independent variable instead of time. For example, the time derivative of an arbitrary parameter $\eta$ is transformed as follows:

$$\dot{\eta} = \frac{d\eta}{dt} = \frac{d\eta}{d\omega_i} \frac{d\omega_i}{dt} \Rightarrow \eta' = \gamma \eta$$  \hspace{1cm} (36)

where the prime denotes the derivative with respect to the angular rate and $\gamma := J_i / T_i$. Consequently, the analytical solution for the attitude angle is obtained as follows:

$$\psi' = \gamma (\omega_t - n)$$  \hspace{1cm} (37)

$$\Rightarrow \psi = \frac{\gamma}{2} (\omega_t^2 - \omega_t \omega_0) - \gamma n (\omega_t - \omega_0) + \psi_0$$

This analytical solution includes no terms of time because of the integration along the angular velocity.

The analytical solutions of the modal equations are also integrated along the angular rate using Fresnel integrals. For example, the equation of $\xi_2$ is rewritten as follows:

$$\dot{\xi}_2 = - 3u_r = - \frac{3F_b}{m} \sin \psi$$

$$\Rightarrow \xi_2 = - \frac{3y F_b}{m} \sin \psi$$

$$= - \frac{3y F_b}{m} \sin \left( \frac{\gamma}{2} (\omega_t^2 - \omega_t \omega_0) - \gamma n (\omega_t - \omega_0) + \psi_0 \right)$$

$$= - \frac{3y F_b}{m} \sin \left( \frac{\gamma}{2} (\omega_t - \omega_0) \right)$$

$$= \gamma n + \psi_0$$  \hspace{1cm} (38)

where

$$\xi_0 = - \frac{\gamma}{2} (\omega_t - \omega_0) + \psi_0$$  \hspace{1cm} (39)

The analytical solution of Eq. (38) can be obtained with Fresnel integrals, although it cannot be integrated with fundamental functions. The normalized Fresnel integrals are defined as follows:

$$C(x) := \int_0^x \cos \frac{\pi}{2} r^2 dr$$  \hspace{1cm} (40)

$$S(x) := \int_0^x \sin \frac{\pi}{2} r^2 dr$$  \hspace{1cm} (41)

Using the Fresnel integrals, we derive the analytical solution as follows:

$$\xi_2 = - \frac{3F_b}{m} \sqrt{\pi \gamma} \cos x \left[ S \left( \frac{\gamma}{\pi} (\omega_t - n) \right) - S \left( \frac{\gamma}{\pi} (\omega_0 - n) \right) \right]$$

$$- \frac{3F_b}{m} \sqrt{\pi \gamma} \sin x \left[ C \left( \frac{\gamma}{\pi} (\omega_t - n) \right) - C \left( \frac{\gamma}{\pi} (\omega_0 - n) \right) \right]$$

$$= - \frac{3F_b}{m} \sqrt{\pi \gamma} \left[ C \left( \frac{\gamma}{2} (\omega_t - \omega_0) \right) - C \left( \frac{\gamma}{2} (\omega_0 - \omega_0) \right) \right]$$

$$\Rightarrow \chi := \frac{\gamma}{2} n^2$$  \hspace{1cm} (42)

The analytical solutions for the other modal variables can be obtained in the same manner.

4.2. Rendezvous maneuver

The relative motion control shown in the previous study is reviewed using the modal variables. In the study, the satellite attitude is controlled so that the thrusters are aligned in the radial direction and the orbit is changed while keeping the attitude angle constant. As discussed in Section 3, the radial directional inputs can drive the follower to the leader only when $\xi_{1,b} = 0$. Thus, a control maneuver is required to eliminate the drift velocity, i.e., $\xi_2 \rightarrow 0$, in advance. After the maneuver, the relative orbital motion of the follower becomes an ellipse and the thruster direction is oriented to the radial direction as shown in Fig. 2.

When $\xi_{2,b} = 0$ and no along-track inputs are applied, Eqs. (32) and (33) indicate that the positive radial thrust monotonically decreases $\xi_1$, and the distance of the follower from the leader can be easily controlled to zero. We introduce a Lyapunov function candidate to control $\xi_3$ and $\xi_4$ as

\[ V(x) = \frac{1}{2} (x_1^2 + x_2^2 + x_3^2 + x_4^2) \]
The control procedure using the drift velocity consists of the rendezvous with the leader with no external forces, less energy is required for the two control methods is due to the drift velocity of the relative radial direction. The difference in the energy efficiency of the along-track direction is more fuel-efficient than that in the control the relative motion of the follower, the control input in using the satellite drift motion. As shown in Section 3, to control the relative motion of the follower, the control input in repeating this maneuver in each relative orbit, the maximum LOS angle does not exceed a specified LOS angle becomes negative. The thruster is turned off so that the thrust force is zero. For example, a negative drift velocity can be generated by turning off the thruster. The numerical simulation results for the two cases (referred to as case (a) and case (b)) are summarized in Table 1 and 2, respectively.

The simulation results for case (a) are shown in Figs. 3–7. The time histories of the follower position and the LOS angle are shown in Figs. 3 and 4, respectively. They describe the follower satellite approaching the target satellite without changing its attitude. Consequently, the maximum value of the LOS angle is obtained as

\[ \theta_{\text{max}} = \arctan \left( \frac{a}{d \sqrt{1 - 4a^2/d^2}} \right) \] (49)

The thruster generates a constant force during Eq. (45) becomes negative. The thrusters are turned off so that the maximum LOS angle does not exceed a specified LOS angle because the maximum value of the LOS angle is calculated as Eq. (49). Repeating this maneuver in each relative orbit, the follower satellite can approach the leader satellite while keeping the LOS angle less than the desired angle.

The control procedure with radial thrusts is improved by using the satellite drift motion. As shown in Section 3, to control the relative motion of the follower, the control input in the along-track direction is more fuel-efficient than that in the radial direction. The difference in the energy efficiency of the two control methods is due to the drift velocity of the relative orbital motion. Because the drift motion steers the follower to the leader with no external forces, less energy is required for the rendezvous.

The control procedure using the drift velocity consists of the following steps:

1) Attitude control to generate the drift velocity
2) Attitude control to track the leader satellite
3) Attenuation of the drift velocity
4) Final maneuver without attitude change

In the first step, the follower changes the attitude angle to generate the drift velocity. For example, a negative drift velocity approaching the origin is necessary for the initial state shown in Fig. 2. This drift velocity is generated with a positive along-track acceleration, as described in Eq. (33), and is analytically calculated using Eq. (42). Thereafter, the follower attitude is controlled to track the leader with a camera. After this approach with the drift motion, the drift velocity is decreased using the analytical solution shown in Eq. (42). In the final step, the follower satellite approaches the target satellite without changing its attitude.

In the maneuver using the drift motion, the maximum LOS angle is calculated for including a specified acceptable attitude angle \( \psi \) into \( d\theta/dt = 0 \). Although the analytical solution of the maximum LOS angle cannot be obtained because of the drift velocity, the solution of \( d\theta/dt = 0 \) can be easily obtained using the Newton’s method because Eq. (46) provides a good initial estimation.

### 5. Numerical Examples

The numerical simulation results for the two cases demonstrate the effectiveness of the proposed control method and enable a comparison of energy consumption. The simulation parameters and initial condition for the two cases (referred to as case (a) and case (b)) are summarized in Table 1 and 2, respectively.

The simulation results for case (a) are shown in Figs. 3–7. The time histories of the follower position and the LOS angle for the maneuver with radial thrusts are shown in Figs. 3 and 4, respectively. They describe the follower satellite approaching the leader satellite while keeping the LOS angle within the field of view of the camera. The thrusters are fired for 62.5[s] in total and the energy consumption in this maneuver, \( J_f = f_1 + f_2 + f_3 + f_4 \Delta t \), is equal to \( -n m(\xi_{\text{d}} - \xi_{\text{a}})/2 = 2500\text{[Ns]} \).

#### Table 1. Simulation parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
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<tbody>
<tr>
<td>Mass ( m ) [kg]</td>
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<td>Position ( x_{1y} ) [m]</td>
<td>case (a): -10000.0, 10000.0, case (b): 100.0, -3500.0</td>
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<td>Translational velocity</td>
<td>case (a): 0.0, 2.0, case (b): 0.0, -0.2</td>
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</tbody>
</table>
For along-track thrusts, the time histories of the relative position of the follower and the LOS angle are described in Figs. 5 and 6, respectively. The follower satellite is successfully controlled relative to the leader using the drift motion. Figure 7 shows the time history of the attitude angle of the follower. The follower attitude is changed to generate the drift velocity; thereafter, it is controlled relative to the leader satellite while keeping the LOS angle less than the field of view of the camera. This maneuver fires the thrusters for 1204.8 [Ns], and is more fuel-efficient than the result obtained using only the radial thrusts.

The simulation results for case(b) are shown in Figs. 8–12. Similarly, the follower satellite is successfully controlled to the target satellite for the different initial state and camera angle. The required acceleration for the maneuver with the drift motion is 11.3 % smaller than that without the drift motion.
6. Conclusions

This study dealt with the relative position and attitude control of a follower satellite equipped with relatively few thrusters. The modal analysis using variable transformation simplifies the interpretation of controllability with positive constant unilateral inputs. An along-track thrust can control all modal variables, whereas a radial directional thrust requires a preliminary control maneuver to remove the drift velocity relative to a target satellite. Moreover, the difference in energy consumption was considered with the modal variables. On the basis of the modal analysis stipulated, a control maneuver using an along-track thrust is fuel-efficient and proper input timing is obtained from the analytic solutions. The numerical simulation results compare the energy efficiency of the two control procedures and verify the effectiveness of the proposed control method. In the future, this study will be extended to realize formation control in an elliptic orbit under a variety of disturbances.

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References