Attitude Control of a Spinning Solar Sail via Spin Rate Control using Exact Linearization

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Solar sails are a form of spacecraft which deploys a large sail in space and uses solar radiation pressure (SRP) for propulsion. The spin-axis direction of a spinning solar sail is known to rotate around an equilibrium point near the Sun direction due to the influence of the SRP. This unique attitude motion can be controlled by the spin rate of a spacecraft, and may help in reducing fuel consumption for attitude control. In this paper, attitude control of a spinning solar sail via the spin rate is optimized and compared with a general attitude control method. The comparison shows a powerful effectiveness of the attitude control via the spin rate.

Key Words: Spinning Solar Sail, Attitude Control, Spin Rate Control, Exact Linearization

Nomenclature

\( \varphi \) : out-of-orbital-plane Sun angle
\( \varphi_r \) : in-orbital-plane Sun angle
\( F \) : force
\( T \) : torque
\( P_0 \) : solar constant
\( A \) : sail surface area
\( c \) : speed of light
\( R_0 \) : mean distance between the Earth and the Sun (i.e. 1 AU)
\( r \) : distance between a spacecraft and the Sun
\( s \) : spacecraft-to-sun vector
\( n \) : normal vector to a sail
\( L \) : distance between the center of mass of a spacecraft and the center of SRP
\( C_{spe} \) : specular coefficient
\( C_{dif} \) : diffusive coefficient
\( C_{abs} \) : absorption coefficient
\( H \) : angular momentum of a spacecraft
\( \tilde{\omega} \) : angular velocity of the spin-free coordinate system with respect to the sun-pointing coordinate system
\( I_S \) : moment of inertia around a spin axis
\( I_T \) : moment of inertia around an axis perpendicular to a spin axis
\( \omega_s \) : orbital angular velocity of a spacecraft with respect to the Sun
\( \Omega \) : spin rate of a spacecraft
\( I_{sp} \) : specific impulse
\( g \) : gravitational acceleration
\( l \) : lever arm
\( \theta \) : attitude control angle
\( M \) : fuel mass

Subscripts

\( \text{Sun} \) : sun-pointing coordinate system
\( \text{SF} \) : spin-free coordinate system
\( \text{SRP} \) : solar radiation pressure
\( \text{SRAC} \) : spin-rate attitude control
\( \text{GAC} \) : general attitude control
\( 0 \) : initial value

1. Introduction

Solar sails are a form of spacecraft which deploys a large sail in space and uses solar radiation pressure (SRP) for propulsion. Since they do not require any propellant for propulsion, they are expected to be a promising technology for future deep space exploration.

Fig. 1. Spinning solar sail IKAROS.

For example, ‘IKAROS’ shown in Fig. 1 is a solar-sail spacecraft developed by Japan Aerospace Exploration Agency (JAXA). It was launched on 21 May 2010 to demonstrate various novel technologies: the deployment of a 200 m² sail, power generation with thin-film solar cells on the sail, measurement of acceleration due to the SRP, and navigation and control of a solar sail. IKAROS is a spinning solar sail deploying and stretching its sail using the centrifugal force due to spinning. The major advantage of a spinning solar sail is the fact that it does not require rigid structures such as masts or booms for its sail. This leads to a large sail area per unit
mass, and a high acceleration can be obtained using the SRP.

In past Japanese interplanetary missions including IKAROS, it was discovered that the spin-axis direction of a spinning spacecraft is capable of tracking the Sun direction due to the effect of the SRP.\(^2\)\(^3\) This phenomenon is called an ‘attitude drift motion’. Although it is beneficial since it helps in tracking the Sun direction for power generation with no control, it may result in a complexity in the attitude control of a spacecraft.

![Image](image_url)  
Fig. 2. Example of attitude drift motion of IKAROS.

Fig. 2 shows an attitude history of IKAROS for approximately 2 months. The spin-axis direction is seen from the Sun which is at the origin, and the horizontal axis of the figure is on the orbit plane of IKAROS. As can be seen, the spin-axis direction rotates anti-clockwise naturally although no active attitude control was performed in this period.

Since the attitude drift motion is determined by the balance of a torque induced by the SRP and the angular momentum of a solar sail, it can be controlled by the spin rate because the angular momentum varies according to the spin rate. Thus the attitude of a spinning solar sail can be controlled by the spin rate. This control method may be effective and help in reducing fuel consumption as it exploits the SRP which is freely available.

Although this is an important feature about a spinning solar sail, there are no established theories about such control. In this paper, an analytical closed-form solution of spin rate is derived.

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### 2. Coordinate Systems

Two coordinate systems used in this study are introduced as shown in Fig. 3. Firstly, a sun-pointing coordinate system is defined. The origin is defined to be the center of mass of a spacecraft. The z-axis is in the Sun direction and the y-axis is perpendicular to the orbit plane while the x-axis is defined to form a right-handed coordinate system.

Secondly, a spin-free coordinate system is introduced. While the attitude of a spinning spacecraft is not required to be described in three axes, its spin-axis direction is important. In the spin-free coordinate system, the origin is defined to be the center of mass of the spacecraft, and a coordinate transformation is executed with angles \( \varphi \), \( \psi \) from the sun-pointing coordinate system. In this study, equations of motion and the attitude of a spacecraft are expressed in this coordinate system in terms of \( \varphi \) and \( \psi \).

### 3. Attitude Drift Motion

In this section, the attitude drift motion of a spinning solar sail is introduced.

If a solar sail has a completely flat sail, the SRP force experienced by the sail can be expressed with Eq. (1).

\[
\mathbf{F}_{\text{SRP}} = -\frac{\rho_0 A}{c} \left(\frac{R}{r}\right)^2 \mathbf{s} \times \mathbf{n} \left(\cos \varphi \cos \psi \cos^2 \psi \right)
\]

If there is an offset between the center of mass of a spacecraft and the center of the SRP, a torque is induced by the SRP.

\[
\mathbf{T}_{\text{SRP}} = \frac{\rho_0 A}{c} \left(\frac{R}{r}\right)^2 \mathbf{L} \left(\cos \varphi \cos \psi \cos \psi \right)
\]

The Euler’s equation, shown in Eq. (3), in the spin-free coordinate system is used to derive equations of the attitude drift motion.

\[
\mathbf{H} + \mathbf{\dot{\omega}} \times \mathbf{H} = \mathbf{T}_{\text{SRP}}
\]

Assuming that \( \varphi \) and \( \psi \) are small enough and that the nutation of the spin-axis is negligible compared to the precession, Eq. (3) can be solved. The resulting solution shown in Eq. (4) represents a simple circular motion around an equilibrium point.

\[
\varphi = A \cos \left(\frac{p}{\Omega_n} t + B\right) - \frac{p_n}{p} \Omega_s
\]

\[
\psi = -A \sin \left(\frac{p}{\Omega_n} t + B\right)
\]

where

\[
\Omega_n = \frac{180}{\pi} \Omega_s, \quad p = \frac{\rho_0 A L}{c_v} \left(\frac{R}{r}\right)^2 (C_{abs} + C_{dir})
\]

Eq. (4) shows a rotation of the spin-axis direction of a spinning solar sail around an equilibrium point \((\Omega_s t, \psi, 0)\). This is the attitude drift motion which is caused by the SRP torque.

As can be seen, the period of the attitude drift motion and the position of the equilibrium point are dependent on the spin rate. It is, therefore, possible to control the attitude of a spinning solar sail by means of controlling the spin rate through the attitude drift motion.

Using a simplification in Eq. (6),

\[
\omega = \frac{p}{\Omega_n}
\]

the differential equations of the attitude drift motion can be written as follows:
\[ \phi = \omega \psi \]
\[ \dot{\psi} = -\omega \dot{\phi} - \omega_t \]

(7)

For ease of demonstration, the orbital motion of a spacecraft is not taken into account precisely in this paper. Hence \( \omega_t \) is assumed to be a constant.

4. Exact Linearization

If \( \omega \) in Eq. (7) is to be a control input since it is a function of the spin rate, Eq. (7) is a bilinear system. Bilinear systems are a class of nonlinear systems, in which nonlinear terms are constructed by a multiplication of state variables and a control input.

In general, the attitude control of a spacecraft is conducted with chemical thrusters, and it is desirable to minimize the amount of fuel required to realize the control. In this paper, it is assumed that the spin rate is controlled with chemical thrusters. Hence it is required to maneuver from a certain attitude to a target attitude while minimizing a change in the spin rate in order to minimize the fuel consumption. This can be treated as an optimal control problem with initial and final conditions to be satisfied.

It is, however, usually impossible to solve analytically an optimal control problem for nonlinear systems except for the simplest cases. One of the approaches to tackle such a problem is a method called 'exact linearization'. It is a method to transform a nonlinear system into an equivalent linear system using a coordinate transformation and state feedback. Since there is no approximation in this process, nonlinear characteristics of a nonlinear system are exactly preserved.

Using the exact linearization, Eq. (7) can be linearized as follows:

\[ \begin{align*}
\dot{\xi}_1 &= \xi_2 \\
\dot{\xi}_2 &= v
\end{align*} \]

(8)

where

\[ \begin{pmatrix}
\xi_1 \\
\xi_2
\end{pmatrix} = \begin{pmatrix}
\phi^2 + \psi^2 \\
-2\omega \psi
\end{pmatrix} \]

(9)

\[ v = 2\omega^2 \psi + 2\omega \omega_t \phi \]

(10)

\( \xi_1 \) and \( \xi_2 \) are new state variables, and \( v \) is a new control input. Eqs. (9) and (10) correspond to a coordinate transformation and state feedback in Eqs. (9) and (10). Although the exact linearization is effective as it linearizes a nonlinear system, it may impose such a constraint. It is, however, important that an optimal control problem of a nonlinear system is solved analytically in a closed-form by grace of the exact linearization. Such an approximate solution gives an idea as to an exact optimum solution of the nonlinear system.

The Hamiltonian can be expressed with adjoint variables, \( \lambda \).

\[ H = \frac{1}{2} \dot{\psi}^2 + \lambda_{\xi_1} \xi_2 + \lambda_{\xi_2} \xi_3 + \lambda_{\xi_3} v \]

(15)

The optimal control input can be, therefore, derived as follows.

\[ \frac{\partial H}{\partial \psi} = v + \lambda_{\xi_3} = 0 \]

(16)

\[ \dot{\psi} = -\lambda_{\xi_3} \]

(17)

Hence simultaneous differential equations to be solved are obtained using Eqs. (12), (16) and (17).

\[ \begin{align*}
\dot{\xi}_1 &= \xi_2 \\
\dot{\xi}_2 &= \xi_3 \\
\dot{\xi}_3 &= -\lambda_{\xi_3} \\
\lambda_{\xi_1} &= 0 \\
\lambda_{\xi_2} &= -\lambda_{\xi_1} \\
\lambda_{\xi_3} &= -\lambda_{\xi_2}
\end{align*} \]

(18)

Eq. (18) can be solved analytically as follows:
\[ \xi_1 = -\frac{1}{12\alpha} C_1 t^5 + \frac{1}{24} C_2 t^4 - \frac{1}{6} C_3 t^3 + \frac{1}{2} \xi_{30} t^2 + \xi_{20} t + \xi_{10} \]
\[ \xi_2 = -\frac{1}{2} C_1 t^5 + \frac{1}{6} C_2 t^4 - \frac{1}{2} C_3 t^3 + \xi_{30} t + \xi_{20} \]
\[ \xi_3 = -\frac{1}{6} C_1 t^5 + \frac{1}{2} C_2 t^4 - C_3 t + \xi_{30} \]
\[ \lambda_{\xi_1} = C_1 \]
\[ \lambda_{\xi_2} = -C_1 t + C_2 \]
\[ \lambda_{\xi_3} = \frac{1}{2} C_1 t^2 - C_3 t + C_3 \]
\[ \xi_{20} = C_1, C_2, \text{ and } C_3 \text{ are integration constants. The optimal control input is, therefore,} \]
\[ \psi = -\frac{1}{2} C_1 t^2 + C_2 t - C_3 \]
\[ \text{To perform attitude control with specified initial and final conditions, final state constraints must be imposed as shown in Eq. (21).} \]
\[ \phi(\xi(t_f), t_f) = 0 \]
\[ \text{The adjoint variables at the final time are defined in Eq. (22) with undetermined multipliers, } \nu. \]
\[ \lambda^T(t_f) = [\nu]^T \frac{\partial \phi}{\partial \xi} \bigg|_{t=t_f} \]
\[ \text{Using Eq. (19), the integration constants can be defined in terms of } \nu. \]
\[ \lambda_{\xi_1}(t_f) = C_1 = v_1 \]
\[ \lambda_{\xi_2}(t_f) = -C_1 t + C_2 = v_2 \]
\[ \lambda_{\xi_3}(t_f) = \frac{1}{2} C_1 t^2 - C_3 t + C_3 = v_3 \]
\[ \text{Substituting Eq. (23) into the first three lines in Eq. (19), the equations can be solved as simultaneous equations in terms of the undetermined multipliers. Consequently, the undetermined multipliers can be expressed in terms of initial and final conditions and a final time. Thus the integration constants can be determined by substituting the solutions into Eq. (23). The history of state variables in Eq. (19) and the optimal control input in Eq. (20) can be, therefore, derived if initial and final conditions and a time taken for control are given.} \]
\[ \text{The new state variables and control input can be transformed back into the original frame of } \phi, \psi, \omega \text{ and } \dot{\omega}. \]
\[ \phi = \sqrt{\xi_1 - \left(\frac{\xi_3}{2a_0}\right)^2} \]
\[ \psi = -\frac{\xi_3}{2a_0} \]
\[ \omega = \frac{\xi_3 - \frac{2a_0^2}{2a_0} \left(1 - \left(\frac{\xi_3}{2a_0}\right)^2\right)}{\sqrt{\xi_1 - \left(\frac{\xi_3}{2a_0}\right)^2}} \]
\[ \dot{\omega} = \frac{\xi_3 - \frac{2a_0^2}{2a_0} \left(1 - \left(\frac{\xi_3}{2a_0}\right)^2\right)}{2a_0 \sqrt{\xi_1 - \left(\frac{\xi_3}{2a_0}\right)^2}} \]

In this paper, a time required for a transfer between initial and final attitude is varied in a certain range in order to find a solution with a minimum spin rate change. This approach is effective since a time required for long-term attitude control is usually not specified strictly in real-life operations of a spacecraft.

6. Fuel Consumption Comparison

The attitude control of a spinning solar sail via the spin rate is optimized analytically in the previous section. This control method is named ‘Spin-Rate Attitude Control’ (SRAC). In this section, the analytical solution is compared with two other methods.

The first is a numerical solution of SRAC. The optimal control problem of minimizing the fuel consumption for the spin rate change is optimized numerically using the direct collocation with nonlinear programing method. This is conducted in the original bilinear system in order to verify the validity of the analytical solution including its derivation using the exact linearization.

The second is a general attitude control method in which the attitude is controlled directly with chemical thrusters. This control method is based on the control performed in the operation of IKAROS and described in detail in a subsequent subsection. This control is named ‘General Attitude Control’ (GAC) for ease of reference.

The comparison of the analytical solution with these two methods is made in terms of fuel consumption.

6.1. Fuel consumption of SRAC

The conservation of angular momentum around the spin axis of a spacecraft and the thrust of thrusters can be expressed as follows:
\[ \frac{dM}{dt} = I_s \frac{d\phi}{dt} = F l_{SRAC} \]
\[ F = I_s \theta \frac{dM}{dt} \]

Using Eqs. (25) and (26), the amount of fuel required for spin rate control can be derived.
\[ \Delta M_{SRAC} = \frac{I_s \theta}{I_s \theta_{SRAC}} \]

The fuel consumption in the analytical and numerical SRAC is both calculated with Eq. (27).

6.2. Fuel consumption of GAC

With IKAROS, the attitude control via the spin rate was not conducted actively. Its attitude was generally fixed with respect to the Sun cancelling a few effects that influence its attitude, and was controlled with thrusters by rhumb-line control only when necessary. In this study, the SRAC is compared with such an attitude control method.

In theory, it is possible to exploit the SRP in the GAC, too. Instead of cancelling the influence of the SRP torque, the attitude can be left drifted for a certain period of time until it approaches a target attitude, and then controlled with thrusters. This is likely to reduce the amount of fuel required to perform an attitude control maneuver. Such control method, however, needs to be optimized in term of the timing to use the thrusters after letting the attitude drifted by the SRP torque. The optimization of the attitude control using a combination of the attitude drift motion and the thrusters has not been studied in the past, and it is expected to be complex. Thus the SRAC is compared with the simple and straightforward control.
performed with IKAROS. Furthermore, this comparison is fair in the sense that the transfer time taken between initial and final attitude is the same.

In the GAC, there are three factors to be considered. Firstly, the amount of fuel required for a rhumb-line attitude maneuver is derived. Fig. 4 shows an impression of such control.

Assuming $\Delta \theta$ is small enough and Eq. (28) holds true, the amount of fuel required can be written with Eq. (29).

$$\tan \Delta \theta = \frac{\Delta H}{\Delta t}$$  \hspace{1cm} (28)

$$\Delta M_{GAC1} = \frac{I_{SRP} \Delta \theta}{I_{SRP} \theta_{GAC}}$$  \hspace{1cm} (29)

Secondly, the SRP torque that causes the attitude drift motion shown in Eq. (2) must be cancelled. The amount of fuel required to cancel this torque can be written with Eq. (30).

$$\Delta M_{GAC2} = \frac{T_{SRP} \Delta t}{I_{SRP} \theta_{GAC}}$$  \hspace{1cm} (30)

Thirdly, it is required to cancel the effect of an orbital motion which results in the change in the attitude with respect to the Sun. Since such an influence is dependent on the orbital angular velocity of the spacecraft, the amount of fuel required is as follows.

$$\Delta M_{GAC3} = \frac{I_{SRP} \Delta \omega_{SRP} \Delta t}{I_{SRP} \theta_{GAC}}$$  \hspace{1cm} (31)

The total fuel consumption for the GAC is, therefore, the sum of these three factors.

$$\Delta M_{GAC} = \Delta M_{GAC1} + \Delta M_{GAC2} + \Delta M_{GAC3}$$  \hspace{1cm} (32)

Eqs. (30) and (31) show that the GAC is unfavorable in general when a time taken for a maneuver is long since $\Delta M_{GAC2}$ and $\Delta M_{GAC3}$ are proportional to the time. Moreover, $T_{SRP}$ in Eq. (30) is a function of initial attitude. Thus the performance of the GAC varies depending on initial attitude.

### 6.3. Comparison results

A few arbitrary examples of attitude control are given, and the analytical and numerical SRAC and the GAC are compared in terms of the fuel consumption. Simulation conditions, which are based on a design of IKAROS, are as shown in Table 1.

<table>
<thead>
<tr>
<th>Case</th>
<th>Initial attitude $(\psi, \phi)$ [deg]</th>
<th>Final attitude $(\psi, \phi)$ [deg]</th>
<th>Time [day]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>(30, 10)</td>
<td>(5, 30)</td>
<td>2.08</td>
</tr>
<tr>
<td>Case 2</td>
<td>(20, 15)</td>
<td>(-30, 5)</td>
<td>18.40</td>
</tr>
<tr>
<td>Case 3</td>
<td>(-10, -10)</td>
<td>(-5, -25)</td>
<td>21.45</td>
</tr>
<tr>
<td>Case 4</td>
<td>(5, -15)</td>
<td>(5, -5)</td>
<td>15.25</td>
</tr>
<tr>
<td>Case 5</td>
<td>(-40, -10)</td>
<td>(30, -5)</td>
<td>57.30</td>
</tr>
</tbody>
</table>

In the SRAC, the initial and final spin rates are defined to be 1 rpm, which is the nominal spin rate of IKAROS. In the GAC, the spin rate is assumed to be constant at 1 rpm. These conditions may have an influence on the total fuel consumption. For instance, the factor in the GAC expressed in Eq. (29) is proportional to the spin rate. In the SRAC, although the spin rate is not directly related to the fuel consumption, it may have an impact on the spin rate control law. In fairness to the two methods, the nominal spin rate is defined to be 1 rpm according to IKAROS.

The examples of attitude control maneuvers to be considered are summarized in Table 2. Attitude histories of the SRAC and fuel consumption results are shown in Figs. 5 and 6, respectively. Since attitude histories of the GAC would be simply straight lines connecting the initial and final attitude, a figure showing them is omitted.

<table>
<thead>
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</table>
Two important points are shown in Figs. 5 and 6. Firstly, the analytical and numerical solutions of SRAC are identical. In Fig. 5, attitude histories are plotted for both solutions, and they overlap each other exactly. The fuel consumption is also identical as shown in Fig. 6 because the same spin rate control is performed. The validity of the analytical solution derived in this study is, therefore, proved.

Secondly, the fuel consumption results shown in Fig. 6 demonstrate a powerful effectiveness of the SRAC. Although it is important to note that the GAC, to which the SRAC is compared, is not optimized, the comparison suggests that the attitude drift motion must be exploited and controlled via the spin rate in the attitude control of a spinning solar sail.

The fuel consumption of the SRAC in Case 3 is, however, slightly greater than that of the GAC. This is because the attitude is changed in the $-\phi$ direction, and the spin rate is forced to be raised high since the distance between the Sun and the equilibrium point is proportional to the spin rate. The spin rate history is shown in Fig. 7.

Thus this type of attitude control is unfavorable for the SRAC if the nominal spin rate is low as with IKAROS.

A major disadvantage of the SRAC is that a possible attitude control maneuver is strictly restricted. If the spin direction of a spinning solar sail is unchanging, the equilibrium point of the attitude drift motion can exist only in the $-\phi$ direction and the rotational direction of the attitude drift motion is consistent. Since the spin direction of a spacecraft cannot be reversed in general, this constraint results in certain maneuvers that are physically impossible. Another disadvantage of the SRAC is that it takes a certain period of time since it uses the SRP torque which is relatively weak. It cannot react to a requirement of a quick maneuver. Hence although the SRAC must be used with another alternative such as chemical thrusters, it is a powerful control method in a long-term attitude planning.

7. Conclusions

In this study, the optimal attitude control problem of a spinning solar sail via the spin rate is solved. The validity of the analytical solution is verified with the numerical solution. A powerful effectiveness of the SRAC is proved in terms of fuel consumption. The attitude drift motion must be, therefore, exploited and controlled by the spin rate actively in a long-term attitude planning.

The attitude control of a spinning solar sail via the spin rate is an original concept proposed by the authors. As demonstrated in this study, the innovative control method is effective and expected to be adopted in future missions of a spinning solar sail. The formulization established in this paper is important because it contributes to studies of spinning solar sails and future deep space missions.

References


