Analysis of Sail Deformation Based on Attitude Motion of Spinning Solar Sail

By Toshihiro CHUJO¹⁾, Yuichi TSUDA²⁾ and Junichiro KAWAGUCHI²⁾

¹⁾Department of Aeronautics and Astronautics, The University of Tokyo, Tokyo, Japan

²⁾Japan Aerospace Exploration Agency, Sagami-hara, Japan

(Received June 17th, 2013)

The attitude motion of the spinning solar sail IKAROS was unique. Its behavior is highly dependent on the deformation of the flexible sail, which is easily changed by the attitude motion. Therefore, it is important to understand the relation between the sail deformation and the attitude motion in order to obtain better knowledge of sail design and management. In this study, a consideration is made about how the sail deformation varies according to the spin rate of the solar sail and how it changes the attitude motion from the view point of the structure. As a result, it was revealed that the variations of the parameters of the sail deformation and the attitude motion are proportional to the squares of the spin rate. The theory is evaluated by using the flight data of IKAROS, its sail shape is estimated, and its equivalent Young's modulus is calculated, which is an index of how much the sail with wrinkles stretches when the force is applied to it.

Key Words: Solar Sail, Attitude Motion, Solar Radiation Pressure, Sail Deformation, Flexible Structure

Nomenclature

A	: parameter of shape and attitude motion
B	: parameter of shape and attitude motion
C	: parameter of shape and attitude motion
Ω	: spin rate
AU	: Sun distance in astronomical unit
I	: moments of inertia
I_s	: moment of inertia around spin axis
t	: time
α	: right ascension of spin axis
δ	: declination of spin axis
α_s	: right ascension of sun direction
δ_s	: declination of sun direction
σ	: stress
ν	: Poisson's ratio
ρ	: density
R	: radius
ε	: strain
E	: Young's modulus
ω	: angular velocity of s/c
$\tilde{\omega}$	: relative angular velocity of spin-free frame to inertial frame
T	: torque on s/c
C_a	: absorptive coefficient
C_s	: specular coefficient
C_d	: diffusive coefficient
B_f	: Lambertian coefficient
S_0	: solar constant
c	: light speed
s	: Sun direction vector (Sun to s/c)
n	: normal vector
r	: position vector of arbitrary point
Θ	: phase about spin axis

β	: Sun angle (angle between spin axis and Sun direction)
J	: sum of squares of residual error
η	: radial deflection
ξ	: circumferential torsion
h	: offset of center of gravity
Subscripts	
r	: radial
θ	: circumferential

1. Introduction

Solar sail is a system to use solar radiation pressure (SRP) for propulsion. Though the pressure is very weak, it can be applied to a sail continuously for a long period of time. Consequently, the solar sail can be accelerated significantly with no fuel consumption. It is also possible to generate electricity with thin-film solar cells on the sail.¹⁾ Spinning solar sail IKAROS launched by JAXA in 2010 succeeded in photon propulsion using the SRP and all of its missions. With the technology demonstrated in the IKAROS project,²⁾ another new mission to explore Jupiter and its Trojan asteroids is currently under consideration.

Fig. 1. IKAROS.

In its mission period, the attitude motion of IKAROS was

unique mainly in two ways.³⁾ Firstly, its spin axis direction was rotating spirally around a certain equilibrium point (Fig. 2). Secondly, there were an increase and a decrease in the spin rate. Since these are phenomena caused by the SRP⁴⁾ and due to the shape of the sail and its optical properties, it is important to know how the flexible sail deforms while it is spinning. Such knowledge would contribute to sail design for the next mission.

Fig. 2. The history of the direction of IKAROS spin axis. x axis and y axis are relative right ascension and declination of the spin axis direction to the Sun direction.

When the attitude motion of IKAROS was analyzed, its sail was generally dealt with as a rigid membrane because the sail vibration was not observed in its operation period. Seen in a long term history of the attitude motion, however, this assumption was not correct. The parameters of the sail which would be constant if the sail was rigid actually varied slowly.³⁾ It can be thought that there are mainly three reasons for this phenomenon. Firstly, there were variations of the sail deformation according to the spin rate.³⁾ Secondly, the optical properties of the sail surface changed in the long mission period. Thirdly, there were large scale deformations such as buckling which changed the natural shape of the sail.

In this study, the variations of the sail deformation according to the spin rate are focused on and their relation to the attitude motion is investigated from the viewpoint of the structure.

2. Evaluation with Flight Data by GSSM

In this research, Generalized Spinning Sail Model (GSSM, see Appendix) is used to analyze the attitude motion of the solar sail. This is a method in which the shape of the spacecraft (s/c) and the attitude motion with the SRP is related through the parameters A , B , and C .

Here this GSSM is applied to IKAROS and its parameters A , B , and C are estimated using the flight data. In this paper, the flight data between June 11th and October 30th 2010 (Figs. 3 and 4) was chosen because there was less RCS (Reaction Control System) active attitude control⁵⁾ in this period.

Fig. 3. The history of the direction of IKAROS spin axis between June 11th and October 30th 2010.

Fig. 4. The history of the spin rate of IKAROS between June 11th and October 30th 2010. x axis stands for time.

Fig. 4 shows that the spin rate of IKAROS decreased naturally, which means parameter C was negative (Eq. (B-16)), and attitude control with RCS was conducted at the spin-up periods. These control periods were excluded in the estimation.

To estimate the parameters, assumptions are made that parameters and Sun distance were constant between any two passes. At any two passes, the attitude data of the first pass is used as the initial condition, the attitude motion is calculated step by step with Runge-Kutta methods, and convergent calculation is conducted to fit it to the attitude data of the second pass at the period.

Here the standardized parameters of A , B , and C are defined as follows.

$$\tilde{A}\Omega = \frac{AU^2}{I_s} A, \quad \tilde{B}\Omega = \frac{AU^2}{I_s} B, \quad \tilde{C} = \frac{AU^2}{I_s} C \quad (1)$$

Using these parameters, Eqs. (B-15) and (B-16) can be simplified as

$$\frac{d}{dt} \begin{bmatrix} \alpha \\ \delta \end{bmatrix} = \frac{1}{AU^2} \begin{bmatrix} \frac{\tilde{A}}{\cos \delta} & -\frac{\tilde{B}}{\cos \delta} \\ \cos \delta_s \sin(\alpha - \alpha_s) & -m[\sin(\delta - \delta_s) - \sin \delta \cos \delta_s (1 - \cos(\alpha - \alpha_s))] \end{bmatrix} \quad (2)$$

$$\frac{d}{dt}\Omega = \frac{\tilde{C}}{AU^2} \quad (3)$$

These standardized parameters $\tilde{A}\Omega$, $\tilde{B}\Omega$, and $\tilde{C}$ are the functions of the Sun angle if the sail deformation is invariable, and when the Sun angle is sufficiently small, they can be seen as constants.

5. Sail Deformation Caused by Spin

Fig. 5 is the result of the estimation. $\tilde{A}\Omega$, $\tilde{B}\Omega$, and $\tilde{C}$ are plotted with the spin rate on x axis. Although these parameters must be constant if the sail is perfectly flat, Fig. 5 shows that they vary according to the spin rate. It can be seen that the absolute values of the parameters tend to decrease as the spin rate increases. This is because the sail shape becomes flatter due to the increased centrifugal force. The figure also shows that the parameter $\tilde{B}\Omega$ is positive, which means that the sail deforms towards the Sun direction (Eq. (B-14)). That is contrary to the fact that the sail is under the SRP and tends to deform towards the opposite direction to the Sun. It can be postulated, therefore, that the SRP is much weaker compared to the stiffness of the sail, which may be brought by devices on the sail, for instance. This had not been anticipated before IKAROS was launched. Moreover, it can be said that the sail deforms upwards from the beginning without any force, which may be caused by the manufacturing process.

Fig. 5. Standardized parameters derived from IKAROS flight data as functions of spin rate.

Assuming that the deformation is small, the parameters should be expressed as

$$\begin{bmatrix} \tilde{A}\Omega \\ \tilde{B}\Omega \\ \tilde{C} \end{bmatrix} = \begin{bmatrix} \tilde{A}_0 + \Delta\tilde{A}(\Omega) \\ \tilde{B}_0 + \Delta\tilde{B}(\Omega) \\ \tilde{C}_0 + \Delta\tilde{C}(\Omega) \end{bmatrix} \quad (4)$$

where $\tilde{A}_0$, $\tilde{B}_0$, and $\tilde{C}_0$ are the parameters of the natural sail shape without spin motion and $\Delta\tilde{A}$, $\Delta\tilde{B}$, and $\Delta\tilde{C}$ are differences caused by the centrifugal force of spinning. Here it is investigated that how the sail deforms with spin motion and how $\Delta\tilde{A}$, $\Delta\tilde{B}$, and $\Delta\tilde{C}$ must be expressed from the view point of the structure.

To simplify this, an approximation is put that the sail is axisymmetric, and that the optical and the material property

are constant. Assuming that the sail is under the plane stress condition, they are expressed as follows.⁶⁾

$$\sigma_r = \frac{(3+\nu)\rho\Omega^2 R^2}{8} \left(1 - \frac{r^2}{R^2}\right) \quad (5)$$

$$\sigma_\theta = \frac{(3+\nu)\rho\Omega^2 R^2}{8} \left(1 - \frac{1+3\nu}{3+\nu} \frac{r^2}{R^2}\right) \quad (6)$$

Under these stress conditions, the strain can be written by Hooke's law as

$$\varepsilon_r = \frac{1}{E}(\sigma_r - \nu\sigma_\theta) = \frac{(3+\nu)(1-\nu)\rho\Omega^2 R^2}{8E} \left\{1 - \frac{3(1+\nu)}{3+\nu} \frac{r^2}{R^2}\right\} \quad (7)$$

$$\varepsilon_\theta = \frac{1}{E}(\sigma_\theta - \nu\sigma_r) = \frac{(3+\nu)(1-\nu)\rho\Omega^2 R^2}{8E} \left(1 - \frac{1+\nu}{3+\nu} \frac{r^2}{R^2}\right) \quad (8)$$

These are how much the sail stretches to respective directions.

Defining that the deformation in a state of rest is expressed as $w(r, \theta)$, that in a state of spin is $w'(r, \theta)$, and the equivalent Poisson's ratio (the ratio between geometrical z strain and geometrical r or θ strain considering the stretch of wrinkles) is ν_z , and assuming that the geometrical strain is proportional to the material strain (the constant of the proportionality is k), the following equations are obtained (Fig. 6).

$$-\frac{(1+\varepsilon_r)\frac{\partial w'}{\partial r} - \frac{\partial w}{\partial r}}{\frac{\partial w}{\partial r}} = \nu_z \varepsilon_r \quad \therefore \frac{\partial w'}{\partial r} = (1 - \nu_z k \varepsilon_r) \frac{\partial w}{\partial r} \quad (9)$$

$$-\frac{(1+\varepsilon_\theta)\frac{\partial w'}{\partial \theta} - \frac{\partial w}{\partial \theta}}{\frac{\partial w}{\partial \theta}} = \nu_z \varepsilon_\theta \quad \therefore \frac{\partial w'}{\partial \theta} = (1 - \nu_z k \varepsilon_\theta) \frac{\partial w}{\partial \theta} \quad (10)$$

$$w' \approx \frac{\partial w'}{\partial r} w \approx (1 - \nu_z k \varepsilon_r) w \quad (11)$$

Fig. 6. Deformation in a state of rest (black line) and in a state of spin (red line) which is stretched by centrifugal force, seen in r - z coordinates.

Using Eqs. (9) – (11), the standardized parameters can be expressed as follows.

$$\begin{aligned}\tilde{A}\Omega &= \frac{AU^2}{I_s} \cdot \frac{1}{2} (2p_{n1} \cos \beta + p_{n2} + p_s \cos \beta) \overline{\iint r \frac{\partial w}{\partial \theta} d\theta dr} \\ &+ \frac{AU^2}{I_s} \left[-\frac{1}{2} (2p_{n1} \cos \beta + p_{n2} + p_s \cos \beta) \right. \\ &\quad \left. \times \overline{\iint r^2 \frac{(3+\nu)(1-\nu)\rho R^2 \nu_z k}{8E} \left(1 - \frac{1+\nu}{3+\nu} \frac{r^2}{R^2}\right) \frac{\partial w}{\partial \theta} d\theta dr} \right] \Omega^2\end{aligned}\quad (12)$$

$$\begin{aligned}\tilde{B}\Omega &= \frac{AU^2}{I_s} \cdot \frac{1}{2} \left\{ (2p_{n1} \cos \beta + p_{n2} + p_s \cos \beta) \overline{\iint r^2 \frac{\partial w}{\partial r} d\theta dr} \right. \\ &\quad \left. + 2p_s \cos \beta \overline{\iint r w d\theta dr} \right\} \\ &+ \frac{AU^2}{I_s} \left[-\frac{1}{2} (2p_{n1} \cos \beta + p_{n2} + p_s \cos \beta) \right. \\ &\quad \times \overline{\iint r^2 \frac{(3+\nu)(1-\nu)\rho R^2 \nu_z k}{8E} \left\{1 - \frac{3(1+\nu)}{3+\nu} \frac{r^2}{R^2}\right\} \frac{\partial w}{\partial r} d\theta dr} \\ &\quad \left. - p_s \cos \beta \overline{\iint r \frac{(3+\nu)(1-\nu)\rho R^2 \nu_z k}{8E} \left\{1 - \frac{3(1+\nu)}{3+\nu} \frac{r^2}{R^2}\right\} w d\theta dr} \right] \Omega^2\end{aligned}\quad (13)$$

$$\begin{aligned}\tilde{C} &= \frac{AU^2}{I_s} \cdot \frac{1}{2} (2p_{n1} \cos^2 \beta + 2p_{n2} \cos \beta + p_s \sin^2 \beta) \\ &\quad \times \overline{\iint r \frac{\partial w}{\partial \theta} d\theta dr} \\ &+ \frac{AU^2}{I_s} \left[-\frac{1}{2} (2p_{n1} \cos^2 \beta + 2p_{n2} \cos \beta + p_s \sin^2 \beta) \right. \\ &\quad \times \overline{\iint r^2 \frac{(3+\nu)(1-\nu)\rho R^2 \nu_z k}{8E} \left(1 - \frac{1+\nu}{3+\nu} \frac{r^2}{R^2}\right) \frac{\partial w}{\partial \theta} d\theta dr} \Big] \Omega^2\end{aligned}\quad (14)$$

Finally, the following equations are obtained.

$$\begin{bmatrix} \tilde{A}\Omega \\ \tilde{B}\Omega \\ \tilde{C} \end{bmatrix} = \begin{bmatrix} \tilde{A}_0 + \tilde{A}_2 \Omega^2 \\ \tilde{B}_0 + \tilde{B}_2 \Omega^2 \\ \tilde{C}_0 + \tilde{C}_2 \Omega^2 \end{bmatrix}\quad (15)$$

where $\tilde{A}_2$, $\tilde{B}_2$, and $\tilde{C}_2$ are the parameters determined by the natural sail shape without spin motion.

6. IKAROS Parameters

Here the above theory is applied to the IKAROS flight data to evaluate its parameter seen in Eq. (15). The data plotted in Fig. 5 is used, the curve fitting along with Eq. (15) is done, and the most likely parameters are calculated.

Fig. 7 is the result of the curve fitting. Note that the parameters plotted in the graph are expressed by the unit of rpm change per day. The obtained parameters are as follows.

$$\tilde{A}_0 = -4.3290 \times 10^{-8}, \quad \tilde{A}_2 = 2.2617 \times 10^{-7} \quad (16)$$

$$\tilde{B}_0 = 1.4514 \times 10^{-7}, \quad \tilde{B}_2 = -7.6183 \times 10^{-7} \quad (17)$$

$$\tilde{C}_0 = -4.8352 \times 10^{-8}, \quad \tilde{C}_2 = 5.1560 \times 10^{-7} \quad (18)$$

Fig. 7. The result of curve fitting of Eq. (15) applied to IKAROS flight data.

The sum of squares of residual error of A , B , and C are

$$\begin{bmatrix} J_A \\ J_B \\ J_C \end{bmatrix} = \begin{bmatrix} 1.2218 \times 10^{-14} \\ 16.7179 \times 10^{-14} \\ 0.4727 \times 10^{-14} \end{bmatrix} \quad (19)$$

7. Consideration to Natural Deformation and Equivalent Young's Modulus

It is shown in Fig. 7 and Eqs. (16) – (18) that the parameters of IKAROS were estimated, but residual error of B is larger compared to those of A and C . This shows that the radial deformation is more sensitive than the circumferential deformation to the spin rate change or other disturbance, which is a proper consideration because the circumferential deformation is limited geometrically.

Now it is possible to estimate the natural deformation of IKAROS when it is in a state of rest by viewing $\tilde{A}_0$, $\tilde{B}_0$, and $\tilde{C}_0$. For simplification, Eq. (C-3) is used to see a tendency of deformation by replacing it with linear deformations. When the Sun angle is sufficiently small, Eq. (C-3) can be written as

$$\begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} \frac{\pi R^3}{3} (2p_{n1} + p_{n2} + p_s) \xi \\ \frac{\pi R^3}{3} \left\{ (2p_{n1} + p_{n2} + 3p_s) \eta - 3p_s \frac{h}{R} \right\} \\ \frac{\pi R^3}{3} (2p_{n1} + 2p_{n2}) \xi \end{bmatrix} \quad (20)$$

With Eq. (20) and IKAROS properties shown in Table 1, and by supposing

$$R = 7.65 \text{ m} \quad (21)$$

(this is the equivalent radius if IKAROS sail is modeled as an axisymmetric sail) η and ξ when it is not spinning are calculated as

$$\eta = 0.76 \text{ deg} \quad (22)$$

$$\xi = \begin{cases} -0.28 \text{ deg} & (\text{derived from } A) \\ -0.33 \text{ deg} & (\text{derived from } C) \end{cases} \quad (23)$$

Note that η can be derived from A (spin axis - based) and

C (spin rate - based).

Table 1. IKAROS properties.

I_s	779 kgm ²
C_a	0.163
C_s	0.719
C_d	0.118

Furthermore, it is possible to evaluate the order of magnitude of the equivalent Young's modulus. From Eqs. (12), (14) and (15), the following is obtained.

$$\begin{aligned} \left| \frac{\tilde{A}_2}{\tilde{A}_0} \right| &= \left| \frac{\tilde{C}_2}{\tilde{C}_0} \right| = \left| \frac{\iint r \frac{(3+\nu)(1-\nu)\rho R^2 \nu_z}{8E} \left(1 - \frac{1+\nu}{3+\nu} \frac{r^2}{R^2} \right) \frac{\partial w}{\partial \theta} d\theta dr}{\iint r \frac{\partial w}{\partial \theta} d\theta dr} \right| \\ &\approx \iint \frac{(3+\nu)(1-\nu)\rho R^2 \nu_z k}{8E} \left(1 - \frac{1+\nu}{3+\nu} \frac{r^2}{R^2} \right) d\theta dr \\ &\approx \frac{\rho R^3 k}{E} \end{aligned} \quad (24)$$

Here, the equivalent Young's modulus is defined as

$$E' = \frac{E}{k} \quad (25)$$

Assuming that the sail is made of polyimide, the physical property is

$$\rho = 1.4 \times 10^3 \text{ kg/m}^3, \quad E = 3.6 \times 10^9 \text{ Pa} \quad (26)$$

and the order of the equivalent Young's modulus can be calculated from Eqs. (16), (18), (21) and (26) as

$$E' \approx 10^{-5} \sim 10^{-4} \text{ GPa} \quad (27)$$

Compared to the material Young's modulus of polyimide

$$E = 3.6 \text{ GPa} \quad (28)$$

equivalent Young's modulus is much smaller, which is intuitively appropriate because there are many wrinkles on the sail. This would be an index of how much the sail with wrinkles stretches when the force is applied to it.

8. Conclusions

In this research, a method to evaluate the parameters of the attitude motion and the deformation of the solar sail is established from the view point of the structure. By considering the natural sail shape in a state of rest and stretched shape caused by the centrifugal force, the parameters were formulized. As a result, it was revealed that the variations of the parameters are proportional to the squares of the spin rate. Furthermore, with the theory derived, the flight data of IKAROS was evaluated through the parameters, and the natural deformation when it is not spinning and its equivalent Young's modulus were estimated.

Acknowledgments

This work was supported by Japan Society for the Promotion of Science Grant-in-Aid for Scientific Research, 23560964.

Appendix

Here Generalized Spinning Sail Model (GSSM) is introduced. Although this method is already established by Tsuda,³⁾ the process to derive the theory is important for this research, so it is introduced here again.

A. Equation of Motion of Spinning Spacecraft

The spin-free coordinates is used, in which z axis corresponds to the spin axis of the s/c, and x axis and y axis do not rotate with the spin of the body.

Here a specific spin-free coordinates is defined in which x axis is on the orbital plane, and y axis is decided by the right-handed system. In this coordinates (denoted by Σ_{SF} hereinafter), the following is also defined.

$$\mathbf{I} = \begin{bmatrix} I_T & & \\ & I_T & \\ & & I_S \end{bmatrix}, \quad \boldsymbol{\omega} = \begin{bmatrix} \omega_x \\ \omega_y \\ \Omega \end{bmatrix}, \quad \tilde{\boldsymbol{\omega}} = \begin{bmatrix} \omega_x \\ \omega_y \\ 0 \end{bmatrix} \quad (\text{A-1})$$

Then the equation of motion can be expressed as

$$\mathbf{I}\dot{\boldsymbol{\omega}} + \tilde{\boldsymbol{\omega}} \times \mathbf{I}\boldsymbol{\omega} = \mathbf{T} \quad (\text{A-2})$$

With α and δ , the following equation is obtained.

$$\frac{d}{dt} \begin{bmatrix} \omega_x \\ \omega_y \\ \Omega \\ \alpha \\ \delta \end{bmatrix} = \begin{bmatrix} -\tilde{\Omega}\omega_y + \tilde{T}_x \\ \tilde{\Omega}\omega_x + \tilde{T}_y \\ \tilde{T}_z \\ \frac{\omega_y}{\cos \delta} \\ -\omega_x \end{bmatrix} \quad (\text{A-3})$$

where

$$\tilde{T}_x = \frac{T_x}{I_T}, \quad \tilde{T}_y = \frac{T_y}{I_T}, \quad \tilde{T}_z = \frac{T_z}{I_T}, \quad \tilde{\Omega} = \frac{I_S}{I_T} \Omega \quad (\text{A-4})$$

Supposing that time constant of $\mathbf{T}$ is much slower than those of ω_x and ω_y , Eqs. (A-3-1) and (A-3-2) can be solved as follows.

$$\omega_x = -\frac{\tilde{T}_y}{\tilde{\Omega}} + a \cos(\tilde{\Omega}t + \phi), \quad \omega_y = \frac{\tilde{T}_x}{\tilde{\Omega}} + a \sin(\tilde{\Omega}t + \phi) \quad (\text{A-5})$$

Extracting the long period motion, Eqs. (A-5) become

$$\omega_x = -\frac{\tilde{T}_y}{\tilde{\Omega}}, \quad \omega_y = \frac{\tilde{T}_x}{\tilde{\Omega}} \quad (\text{A-6})$$

With Eqs. (A-3) and (A-6), eventually the following equations are obtained.

$$\dot{\alpha} = \frac{1}{I_S \Omega} \frac{T_x}{\cos \delta}, \quad \dot{\delta} = \frac{1}{I_S \Omega} T_y, \quad \dot{\Omega} = \frac{T_z}{I_S} \quad (\text{A-7})$$

B. Relation between Shape of Spacecraft and SRP

The force from the SRP on a small surface dA can be expressed as

$$d\mathbf{f} = P[\mathbf{s} \cdot \mathbf{n}](C_a + C_d)\mathbf{s} + (\mathbf{s} \cdot \mathbf{n})\{2[\mathbf{s} \cdot \mathbf{n}]C_s + B_f C_d\}\mathbf{n}dA$$

$$\left(P = \frac{S_0}{c} \left(\frac{1}{AU}\right)^2\right) \quad (\text{B-1})$$

Now to make it easy to calculate the SRP torque, another specific spin-free coordinates is defined in which the Sun is on the first quadrant of xz plane (denoted by Σ_* hereinafter, Fig. 8). In this coordinates, $\mathbf{s}$ and $\mathbf{r}$ are expressed as follows.

$$\mathbf{s} = \begin{bmatrix} -\sin \beta \\ 0 \\ -\cos \beta \end{bmatrix}, \quad \mathbf{r} = \begin{bmatrix} r \cos(\theta + \Omega t) \\ r \sin(\theta + \Omega t) \\ z(r, \theta) \end{bmatrix} \quad (\text{B-2})$$

The sail normal vector is then derived as

$$\mathbf{n} = \frac{\partial \mathbf{r}}{\partial r} \times \frac{\partial \mathbf{r}}{\partial \theta} = \begin{bmatrix} -\frac{\partial z}{\partial r} \cos(\theta + \Omega t) + \frac{\partial z}{r \partial \theta} \sin(\theta + \Omega t) \\ -\frac{\partial z}{\partial r} \sin(\theta + \Omega t) - \frac{\partial z}{r \partial \theta} \cos(\theta + \Omega t) \\ 1 \end{bmatrix} \quad (\text{B-3})$$

Fig. 8. Relation between three coordinate systems.

The SRP torque exerted on the sail is then expressed using Eqs. (A-8) - (A-10) as

$$\mathbf{T}^* = \mathbf{T}_{n1}^* + \mathbf{T}_{n2}^* + \mathbf{T}_s^* \quad (\text{B-4})$$

where

$$\mathbf{T}_{n1}^* = \overline{\int \int \mathbf{r} \times p_{n1} |\mathbf{n} \cdot \mathbf{s}| (\mathbf{n} \cdot \mathbf{s}) \mathbf{n} r d\theta dr} \quad (\text{B-5})$$

$$\mathbf{T}_{n2}^* = \overline{\int \int \mathbf{r} \times p_{n2} (\mathbf{n} \cdot \mathbf{s}) \mathbf{n} r d\theta dr} \quad (\text{B-6})$$

$$\mathbf{T}_s^* = \overline{\int \int \mathbf{r} \times p_s |\mathbf{n} \cdot \mathbf{s}| \mathbf{s} r d\theta dr} \quad (\text{B-7})$$

where $\bar{x}$ indicates the spin-average of x and

$$p_{n1} = P(2C_s), \quad p_{n2} = P(B_f C_d), \quad p_s = P(C_a + C_d) \quad (\text{B-8})$$

To transform the torque expressed in Σ_* into Σ_{SF} , the following rotation about z axis is required.

$$\begin{bmatrix} T_x^* \\ T_y^* \\ T_z^* \end{bmatrix} = \begin{bmatrix} \cos \Theta & -\sin \Theta \\ \sin \Theta & \cos \Theta \\ 1 \end{bmatrix} \begin{bmatrix} T_x^* \\ T_y^* \\ T_z^* \end{bmatrix} \quad (\text{B-9})$$

where Θ is decided by noting the spherical trigonometry as

$$\sin \Theta = m \frac{|\sin(\delta_s - \delta) + \sin \delta \cos \delta_s (1 - \cos(\alpha_s - \alpha))|}{\sin \beta} \quad (\text{B-10})$$

$$\cos \Theta = \frac{\cos \delta_s \sin(\alpha_s - \alpha)}{\sin \beta} \quad (\text{B-11})$$

$$\cos \beta = \sin \delta \sin \delta_s + \cos \delta \cos \delta_s \cos(\alpha_s - \alpha) \quad (\text{B-12})$$

where

$$m = \frac{\cos^2 \delta \sin \delta_s - \sin \delta \cos \delta \cos \delta_s \cos(\alpha_s - \alpha)}{|\cos^2 \delta \sin \delta_s - \sin \delta \cos \delta \cos \delta_s \cos(\alpha_s - \alpha)|} \quad (\text{B-13})$$

Now by introducing the parameters

$$\begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} -\frac{T_x^*}{\sin \beta} \\ -\frac{T_y^*}{\sin \beta} \\ T_z^* \end{bmatrix}$$

$$= \frac{1}{2} \iint \left[\begin{aligned} &(2p_{n1} \cos \beta + p_{n2} + p_s \cos \beta) r \frac{\partial z}{\partial \theta} \\ &(2p_{n1} \cos^2 \beta + 2p_{n2} \cos \beta + p_s \sin^2 \beta) r \frac{\partial z}{\partial \theta} \end{aligned} \right] d\theta dr \quad (\text{B-14})$$

the following equations are obtained from Eqs. (A-7), (B-4) - (B-14).

$$\frac{d}{dt} \begin{bmatrix} \alpha \\ \delta \end{bmatrix} = \frac{1}{I_s \Omega} \begin{bmatrix} \frac{A}{\cos \delta} & -\frac{B}{\cos \delta} \\ \cos \delta_s \sin(\alpha - \alpha_s) \\ -m |\sin(\delta - \delta_s) - \sin \delta \cos \delta_s (1 - \cos(\alpha - \alpha_s))| \end{bmatrix} \quad (\text{B-15})$$

$$\frac{d}{dt} \Omega = \frac{C}{I_s} \quad (\text{B-16})$$

A , B , and C are the parameters and the functions of the shape of the s/c, its optical parameter, the Sun angle, and the Sun distance to determine the attitude motion.

If it is assumed that $\alpha, \delta, \alpha_s, \delta_s \ll 1$, Eq. (B-15) can be

$$\frac{d}{dt} \begin{bmatrix} \alpha \\ \delta \end{bmatrix} = \frac{1}{I_s \Omega} \begin{bmatrix} A & -B \\ B & A \end{bmatrix} \begin{bmatrix} \alpha - \alpha_s \\ \delta - \delta_s \end{bmatrix} \quad (\text{B-17})$$

With another assumptions that A , B , and C are time-invariant (i.e. s/c is rigid), the equilibrium point of Eq. (B-17) can be calculated as follows.

$$\begin{bmatrix} \alpha_{eq} \\ \delta_{eq} \end{bmatrix} = \begin{bmatrix} \alpha_s \\ \delta_s \end{bmatrix} + \frac{I_s \Omega}{A^2 + B^2} \begin{bmatrix} A & B \\ -B & A \end{bmatrix} \begin{bmatrix} \dot{\alpha}_s \\ \dot{\delta}_s \end{bmatrix} \quad (\text{B-18})$$

Using this and by assuming that the spin rate variation is

sufficiently slow, the solution for Eqs. (B-16) and (B-17) is given by

$$\alpha = \alpha_{eq} + D \exp\left(\frac{A}{I_s \Omega} t\right) \cos\left(\frac{B}{I_s \Omega} t + \varphi\right) \quad (\text{B-19})$$

$$\delta = \delta_{eq} + D \exp\left(\frac{A}{I_s \Omega} t\right) \sin\left(\frac{B}{I_s \Omega} t + \varphi\right) \quad (\text{B-20})$$

$$\Omega = \Omega_0 + \frac{C}{I_s} t \quad (\text{B-21})$$

This indicates that the spin axis performs circles about the equilibrium point and the circle radius varies exponentially (spiral behavior), and that the spin rate varies linearly (windmill behavior). These behaviors were actually observed for IKAROS (Fig. 2).

C. Example of Parameters of a Specific Deformation

Considering an approximation that the sail shape is axisymmetric, the optical property is uniform, and the deformation is expressed with the deflection and the torsion parameters. These assumptions are expressed as

$$\eta = \frac{\partial z}{\partial r} = \text{const.}, \quad \xi = \frac{\partial z}{r \partial \theta} = \text{const.}, \quad z = r\eta \quad (\text{C-1})$$

$$p_{n1} = \text{const.}, \quad p_{n2} = \text{const.}, \quad p_s = \text{const.} \quad (\text{C-2})$$

Under this approximation, the parameters are calculated as follows.

$$\begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} \frac{\pi R^3}{3} (2p_{n1} \cos \beta + p_{n2} + p_s \cos \beta) \xi \\ \frac{\pi R^3}{3} \left\{ (2p_{n1} \cos \beta + p_{n2} + 3p_s \cos \beta) \eta - 3p_s \cos \beta \frac{h}{R} \right\} \\ \frac{\pi R^3}{3} (2p_{n1} \cos^2 \beta + 2p_{n2} \cos \beta + p_s \sin^2 \beta) \xi \end{bmatrix} \quad (\text{C-3})$$

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