Fuel-Optimal Control of Formation Flying in Near-Circular Orbits

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In this study, the formation flying of two spacecraft in near-circular orbits under the influence of a J2 perturbation is considered to obtain a control strategy for maintaining formation flying while minimizing fuel usage. This study utilizes a state transition matrix that describes the relative motion of a deputy spacecraft with respect to a chief spacecraft in terms of the argument of latitude of the chief spacecraft. Owing to the J2 perturbing force, the actual position of the deputy with respect to the chief deviates from the nominal position. The abovementioned state transition matrix enables the analytical calculation of the secular terms of the deviation, providing the strategy for optimized fuel position maintenance. In this study, a two impulsive control is considered to compensate for the secular terms of the position error. Additionally, the conditions for the chief spacecraft’s argument of latitude that minimize the velocity increments for the two impulsive controls are derived. The velocity increments are effectively reduced by adjusting the chief spacecraft’s argument of latitude while taking the influence of the J2 perturbation into account.

Key Words: Formation Flying, J2 Perturbation, Impulsive Control

Nomenclature

- a : semi-major axis
- e : eccentricity
- i : inclination of orbital plane
- J2 : second-order zonal harmonic coefficient of Earth gravitational potential
  \( ( = 1.0826 \times 10^{-3}) \)
- n : mean angular velocity
- r : distance from Earth to spacecraft \( ( = | r |) \)
- r : position vector of spacecraft with respect to Earth
- Re : equatorial radius of Earth
- Tc : orbital period
- \( \theta \) : argument of latitude of the chief
- \( \mu \) : gravitational constant of Earth
- \( \Omega \) : right ascension of ascending node

Subscripts

- 0 : initial state
- c : chief variables
- d : deputy variables
- n : nominal state without effects of J2

1. Introduction

Spacecraft formation flying is a concept in which multiple spacecraft work together in a group to gain benefits over a spacecraft flying alone. A spacecraft in a reference orbit is referred to as a chief and a target spacecraft is referred to as a deputy. The most famous equations describing the relative motion of spacecraft are the Hill-Clohessy-Wiltshire (HCW) equations.1) This is a set of linearized equations based on the assumptions that the chief spacecraft is in a circular reference orbit and that the Earth is spherically symmetric body. In spite of these assumptions, the chief and deputy orbits may deviate from each other owing to a variety of perturbing forces. This means that the HCW equations are not adequate for long-term predictions of the relative motion of the two spacecraft. The perturbing force with the largest impact on the relative motion is the J2 perturbation, which is due to the Earth’s oblateness. The effects of the J2 perturbation were incorporated into the HCW equations by many researchers. Schweighart and Sedwick extended the HCW equations and derived linear equations with constant coefficients that included the effects of the J2 perturbation.2) Roberts and Roberts calculated the gradient of the J2 perturbing force and derived the equations of relative motion with time-varying coefficients.3) If the reference orbit is elliptic, the linearized equations for the relative motion are called Tshauner-Hempel (TH) equations.4) Gim and Alfriend derived a state transition matrix (STM) of the TH equations that included the effects of the J2 perturbation.5) Their STM incorporated the difference of the mean orbital elements between the chief and the deputy and is suitable for the long-term prediction of the relative motion.5) Yamada et al. derived a general form of the STM of the TH equations in the presence of the J2 perturbation using another method.6) This STM employs osculating orbital elements, enabling the analytical treatment of the STM.7)

In the presence of the J2 perturbation, the relative position and velocity of the deputy with respect to the chief deviates from the nominal state. Therefore, a control for the orbit maintenance is needed. A simple and well-known maintenance method is an impulsive control. D’Amico and Montenbruck described the relative motion by means of relative eccentricity and inclination vectors, presenting an
impulsive control law.\(^7\) In this paper, the STM derived by Yamada et al. is utilized to obtain a control strategy for maintaining the relative state while minimizing fuel usage with respect to the argument of latitude of the chief spacecraft.

2. Derivation of Impulsive Control

2.1. Equation of relative motion

To describe the relative motion, a coordinate system is defined. This coordinate system is centered at the chief spacecraft. The \(x\)-axis is directed from the Earth to the chief, the \(z\)-axis is normal to the orbital plane of the chief, and the \(y\)-axis completes the right-hand system with the \(x\) and \(z\)-axes. The TH equations that include the \(J_2\) perturbation can be written in the chief coordinate system as follows:\(^6\)

\[
\begin{align*}
\dot{x} - 2\omega_x y - (2k + \omega_x^2) x + \omega_x \omega_y z & = f_x(r) - f_x(r_0) \\
\dot{y} + 2\omega_y x - 2\omega_x z + (k - \omega_y^2) y + \omega_x \omega_y x & = f_y(r) - f_y(r_0) \\
\dot{z} + 2\omega_x y - \omega_y x + \omega_x + \omega_y y + k z & = f_z(r) - f_z(r_0)
\end{align*}
\]

(1)

where \(k\) is defined as \(k = \mu / r_0^3\), \(r\) and \(r_0\) are the position vectors of the chief and the deputy from the Earth, respectively, and \(\omega_x\), \(\omega_y\), and \(\omega_z\) are the angular velocities of the chief and the deputy with respect to the inertial coordinates expressed in the chief coordinate system. The force \(f_j(r)\) is the \(J_2\) perturbing force and is expressed as follows:

\[
f_j(r) = 3\mu J_2 R_1^3 \left[ 5r_j^2 - r_j^2 (1 - 2\gamma r_j^2) \right]
\]

(2)

where \(\vec{r}_j\) is a unit vector directed from the center of the Earth to the North pole, and \(n_j\) is defined as \(n_j = r \cdot \vec{r}_j\).

2.2. Deviation of formation flying

The orbits of the chief and the deputy may deviate from each other because of the \(J_2\) perturbation. The state vector \(x(t)\) is defined as follows:

\[
x(t) = [x(t) \ y(t) \ z(t) \ \dot{x}(t) \ \dot{y}(t) \ \dot{z}(t)]^T
\]

(3)

By means of the STM derived by Yamada et al., the state vector at time \(t\) is related to the state vector at time \(t_0 = 0\), as follows:

\[
x(t) = \Phi(t,t_0)x(t_0)
\]

(4)

The STM without the \(J_2\) perturbation is represented as \(\Phi_j(t,t_0)\). Therefore, the nominal relative state at time \(t\) is written as follows:

\[
x_n(t) = \Phi_j(t,t_0)x(t_0)
\]

(5)

The deviation of the relative state can be evaluated with respect to the time or the argument of latitude of the chief. The formation errors expressed in terms of time can be written as follows:

\[
\Delta x(t) = x(t; \theta + \Delta \theta) - x_n(t; \theta) = \Phi_j(t,t_0) - \Phi_j(t,t_0) x(t_0)
\]

(6)

\(x_n(t; \theta)\) is the nominal relative state without the effects of \(J_2\), whereas \(x(t; \theta + \Delta \theta)\) is the relative state subject to the effects of \(J_2\). In Eq. (6), \(\theta\) represents the argument of latitude of the chief spacecraft and \(\Delta \theta\) represents the deviation of \(\theta\) due to the \(J_2\) perturbing force. Because the argument of latitude of the chief deviates from the nominal argument of latitude owing to the effects of \(J_2\), the abovementioned \(\Delta x(t)\) is not an appropriate description of the formation errors. To rectify this, the effects of \(\Delta \theta\) in \(\Delta x(t)\) must be incorporated. Then, the formation errors in terms of the argument of latitude of the chief can be written as follows:

\[
\Delta x(t) = x(t; \theta + \Delta \theta) - x_n(t; \Delta \theta) = \left[ \Phi_j(t,t_0) - \Phi_j(t,t_0) \frac{\partial x}{\partial \theta} \Delta \theta \right] x(t_0)
\]

(7)

Next, the secular terms of \(\Delta x(t)\) in Eq. (7) are expressed as follows:

\[
\Delta x(t) = \frac{1}{2} \mu t (1 - 5 \cos^2 \theta) [-nx_x \sin(\theta - \theta_0) + \dot{x}_0 \cos(\theta - \theta_0)]
\]

(8)

\[
\Delta \dot{x}(t) = -\mu t (nx_x \sin(\theta_0 + 1 - 5 \cos^2 \theta) \sin(\theta - \theta_0) + 6ny_0 \cos \theta_0 \cos \theta \sin^2 \theta + 16nz_0 \sin \theta_0 \sin \theta \cos \theta + 4z_0 \cos \theta \sin i \cos i]
\]

(9)

\[
\Delta x(t) = \frac{1}{2} \mu t (1 - 5 \cos^2 \theta) [-nx_x \sin(\theta - \theta_0) + \dot{x}_0 \cos(\theta - \theta_0)]
\]

(10)

\[
\Delta \dot{y}(t) = -\mu t (nx_y \sin(\theta_0 + 1 - 5 \cos^2 \theta) \cos \theta \sin^2 \theta + 6ny_0 \cos \theta_0 \cos \theta \sin \theta \cos \theta + 4z_0 \cos \theta \sin i \cos i]
\]

(11)

\[
\Delta x(t) = \mu t (nx_z \sin(\theta_0 + 1 - 5 \cos^2 \theta) \sin \theta \sin^2 \theta + 6nz_0 \sin \theta_0 \sin \theta \cos \theta + 4z_0 \cos \theta \sin i \cos i]
\]

(12)


Equations (8)–(13) show only the secular terms of \(\Delta x(t)\) and the time derivatives of Eqs. (8)–(10) do not correspond to Eqs. (11)–(13).

Let us assume that the nominal orbit of the chief is circular, i.e., \(e = 0\), for simplicity and that the initial state \(x(t_0)\) is given as follows:

\[
x_0 = r_{c0} \sin \alpha, \quad y_0 = 0, \quad z_0 = r_{c0} \cos \alpha
\]

\[
\dot{x}_0 = nz_0, \quad \dot{y}_0 = -2nx_0, \quad \dot{z}_0 = -nx_0
\]

(16)
Under these conditions, the analytical solution of the HCW equations shows that the deputy moves in a circle in the x-z plane of the chief coordinate system with a radius \( r_{cz} \) and an initial phase \( \theta_1 \). This formation is named a helix formation. Using this, Eqs. (8)–(13) are expressed as follows:

\[
\Delta \theta(t) = \frac{1}{2} r_{cz} m \left[ (1 - 5 \cos^2 i) \cos(\alpha + \theta_1, \theta_1) \right] \cdot \delta(t)
\]

\[
\delta(t) = -r_{cz} m \left[ (1 - 5 \cos^2 i) \sin(\alpha + \theta_1, \theta_1) \right]
\]

\[
+ 3 \sin \alpha (3 \cos^2 i + 3 \cos^2 \theta_1, \sin^2 i - 2)
\]

\[
+ 4 \sin i \cos i (4 \cos \alpha \sin \theta_1, - \sin \alpha \cos \theta_1)
\]

\[
\Delta \epsilon(t) = r_{cz} m \sin(\alpha - \theta_2) \cos \theta_2 \sin^2 i
\]

\[
\delta(t) = -\frac{1}{2} r_{cz} m \left[ (1 - 5 \cos^2 i) \sin(\alpha + \theta_2) \right]
\]

\[
\delta(t) = -r_{cz} m \left[ (1 - 5 \cos^2 i) \sin(\alpha + \theta_2) \right]
\]

\[
\Delta \epsilon(t) = -r_{cz} m \left[ \sin(\alpha - \theta_2) \right] \sin \theta_2 \sin^2 i
\]

2.3. Formation maintenance by two impulsive controls

In this section, two impulsive controls are considered to compensate for the formation deviation. The first and the second impulses control the deputy when the argument of latitude of the chief is \( \theta_1 \) and \( \theta_1 + \phi \), respectively, and the formation deviation is assumed to be perfectly compensated at the end of the second impulse. The relative position and velocity vectors are given by \( p \) and \( v \), respectively, and the nominal relative position and velocity vectors are given by \( p^* \) and \( v^* \), respectively. The position and velocity deviations are defined as \( p_\epsilon = p - p^* \) and \( v_\epsilon = v - v^* \), respectively. By ignoring the effects of \( J_2 \) from the first impulse to the second one, the following equations are obtained:

\[
\begin{bmatrix}
    p^*(\theta_1 + \phi) \\
    v^*(\theta_1 + \phi)
\end{bmatrix} = \begin{bmatrix}
    0 \\
    \Delta v(\theta_1 + \phi) + \Phi_s(\theta_1 + \phi) \Phi_s(\theta_1 + \phi)
\end{bmatrix}
\]

\[
\begin{bmatrix}
    p(\theta_1) \\
    v(\theta_1)
\end{bmatrix} = \begin{bmatrix}
    \Phi_s(\theta_1 + \phi) \\
    \Phi_s(\theta_1 + \phi)
\end{bmatrix}
\]

\[
\begin{bmatrix}
    p^*(\theta_1 + \phi) \\
    v^*(\theta_1 + \phi)
\end{bmatrix} = \begin{bmatrix}
    \Phi_s(\theta_1 + \phi, \theta_2) \\
    \Phi_s(\theta_1 + \phi, \theta_2)
\end{bmatrix}
\]

From these three equations, the two impulsive controls are given as follows:

\[
\Delta v(\theta_1) = -P^{-1} \Phi_s(\theta_1 + \phi, \theta_1) \left[ p_s(\theta_1) + \Phi_s(\theta_1 + \phi, \theta_1) \Phi_s(\theta_1 + \phi, \theta_1) \right]
\]

\[
P = \begin{bmatrix}
    \Phi_{s12}(\theta_1 + \phi, \theta_1) \\
    \Phi_{s22}(\theta_1 + \phi, \theta_1)
\end{bmatrix}
\]

where \( \Phi_{s12}(\theta_1 + \phi, \theta_1) \) and \( \Phi_{s22}(\theta_1 + \phi, \theta_1) \) are block matrices of \( \Phi_s(\theta_1 + \phi, \theta_1) \), as follows:

\[
\Phi_s(\theta_1 + \phi, \theta_1) = \begin{bmatrix}
    \Phi_{s11} & \Phi_{s12} \\
    \Phi_{s21} & \Phi_{s22}
\end{bmatrix}
\]

\[
P^{-1} \Phi_s \] is a function of \( \phi \) and the components are expressed as follows:

\[
\begin{bmatrix}
    n(3\phi_s - 4s_p) \\
    3\phi_s + 8c_p - 8 \\
    2n(3\phi_s + 7c_p - 7)
\end{bmatrix} = \begin{bmatrix}
    0 \\
    0 \\
    0
\end{bmatrix}
\]

\[
\begin{bmatrix}
    3\phi_s + 8c_p - 8
\end{bmatrix} = \begin{bmatrix}
    0
\end{bmatrix}
\]

\[
\begin{bmatrix}
    3\phi_s + 8c_p - 8 \\
    3\phi_s + 8c_p - 8 \\
    3\phi_s + 8c_p - 8
\end{bmatrix} \times \begin{bmatrix}
    n \\
    s_p \\
    0
\end{bmatrix} = \begin{bmatrix}
    0 \\
    0 \\
    0
\end{bmatrix}
\]

where \( s_p \) and \( c_p \) are set as \( s_p = \sin \phi \) and \( c_p = \cos \phi \), respectively. Eq. (29) shows that the in-plane and out-of-plane components are mutually decoupled and, therefore, that the in-plane and out-of-plane controls can be conducted at different phases.

3. Comparison of Analytical Solution and Simulation

In this section, analytical solutions of the two impulsive controls given by Eq. (26) are compared with the results of numerical simulations. The initial phase of the helix formation \( \alpha \) is given for the following three cases: \( \alpha = 0 \) [°] for Case A, \( \alpha = 90 \) [°] for Case B, and \( \alpha = \theta_1 \) for Case C. The initial orbital elements of the chief are set as follows: \( a = R_e + 628 \text{ [km]} \), \( e = 0 \), \( i = 97.9 \) [°], and \( \Omega = 0 \) [°]. The radius of the helix formation \( r_{cz} \) is set to \( r_{cz} = 3 \) [km].

3.1. Case A (\( \alpha = 0 \) [°])

The following assumptions are made for this case: the initial argument of latitude of the chief is \( \theta_1 \), the first impulsive control is conducted after \( N \) orbital periods, and the second phases of the out-of-plane and in-plane impulses are defined by \( \phi_2 = \pi / 2 \) and \( \phi_2 = \pi \), respectively. Under these conditions, the two impulsive controls are analytically described by the following equations:

\[
\Delta v_{i1} = -\frac{1}{32} r_{cz} m^2 N^2 \left[ 3\pi (1 - 5 \cos^2 i) + 128 \sin i \cos i \sin i \theta_1 \right]
\]

\[
\Delta v_{i2} = \frac{1}{8} r_{cz} m^2 N^2 \left( 1 - 5 \cos^2 i \right)
\]

\[
\Delta v_{i1} = -\frac{1}{32} r_{cz} m^2 N^2 \sin \theta_1 \sin^2 i
\]

\[
\Delta v_{i2} = -\frac{1}{32} r_{cz} m^2 N^2 \left[ 3\pi (1 - 5 \cos^2 i) + 128 \sin i \cos i \sin i \theta_1 \right]
\]

\[
\Delta v_{i1} = -\frac{1}{8} r_{cz} m^2 N^2 \theta_1 \sin \theta_1 \cos \theta_1 \sin^2 i
\]

The same amounts of velocity increments are required for the first and second impulsive controls and the \( y \)-component of the in-plane impulsive control is independent of the initial argument of latitude \( \theta_1 \). The condition for the \( x \)-component of the in-plane impulse to be zero is given as follows:

\[
\sin \theta_1 = -\frac{3\pi (1 - 5 \cos^2 i)}{128 \sin i \cos i}
\]

Because the inclination is set to \( i = 97.9 \) [°], \( \theta_1 \) is calculated to be \( \theta_1 = 29.3 \) or 150.3 [°].
To evaluate the analytical solutions given by Eqs. (30)–(35), the dynamics of the chief and the deputy are simulated and the two impulsive controls are numerically conducted. In the simulation, the control cycle $N$ is set to $N = 2$. Fig. 2 shows the changes of the velocity increments with respect to the argument of latitude of the chief. In this figure, the solid lines and the dots show the analytical and simulation results, respectively. The dotted line shows the total velocity increment of the analytical results. As shown in this figure, the results of the analysis and the simulation agree. This figure also shows that the total velocity increment is minimized if the initial argument of latitude $\theta_i$ is set to $3.3^\circ$ or $176.7^\circ$ and the minimum total delta-v is 41.3 [mm/s].

3.2. Case B ($\alpha = 90^\circ$)

In this case, the initial phase angle of the helix formation $\alpha$ is assumed to be $90^\circ$. The second phases $\phi_2$ and $\phi_3$ are set to $90^\circ$ and $180^\circ$, respectively, as in Case A. Then, the analytical solutions of the impulsive controls are expressed as follows:

$$\Delta v_{1i} = -\frac{1}{4} r_{xy} m^2 N \tau \{2 + 5 \cos^2 i - 9 \sin^2 \theta_1 \sin^2 i \}$$
$$- 4 \cos \theta_1 \sin i \cos i$$
$$\Delta v_{1i} = 0$$
$$\Delta v_{12} = r_{xy} m^2 N \tau \sin \theta_1 \cos \theta_1 \sin^2 i$$
$$\Delta v_{13} = -\frac{1}{4} r_{xy} m^2 N \tau \{4 - 5 \cos^2 i - 9 \sin^2 \theta_1 \sin^2 i \}$$
$$- 4 \cos \theta_1 \sin i \cos i$$
$$\Delta v_{13} = 0$$

In this case, the $y$-components of the in-plane impulsive control become zero. Fig. 3 shows the comparison of the analytical solutions and the numerical simulation; the figure shows that they agree with each other. This figure also shows that the total velocity increment is minimized if the initial argument of latitude $\theta_i$ is set to $44.2^\circ$ or $315.8^\circ$ and that the minimum total delta-v is 79.0 [mm/s], respectively. Fig. 4 shows the simulation results of the out-of-plane velocity increments, and can be calculated by solving $\Delta v_{1i} = 0$ or $\Delta v_{12} = 0$. The solutions of $\Delta v_{1i} = 0$ are given by $\theta_1 = 31.7^\circ$ and $224.5^\circ$ and those of $\Delta v_{12} = 0$ are given by $\theta_1 = 41.6^\circ$, $138.1^\circ$, $168.3^\circ$, and $233.3^\circ$. Among these six nondifferentiable points, $\theta_1 = 168.3^\circ$ corresponds to the point of the minimum velocity increments in total. The minimum total delta-v is calculated as 17.1 [mm/s].

The simulation results when $\alpha = \theta_1 = 168.3^\circ$ are presented in Fig. 5. This figure shows the time histories of the position deviations $\delta x(t)$, $\delta y(t)$ and $\delta z(t)$. Fig. 6 shows the velocity deviations $\delta \dot{x}(t)$, $\delta \dot{y}(t)$ and $\delta \dot{z}(t)$. The position and velocity deviations are maintained within $100$ [m] and $50$ [mm/s], respectively. Fig. 7 shows the simulation results of the velocity increments of each impulsive control. As expected from Fig. 5, the out-of-plane components of delta-v are approximately zero, and the total amount of delta-v is approximately 17 [mm/s]. Finally, Figs. 8 and 9 show the comparison of the absolute formation deviation with and without the maintenance control. Without the maintenance control, the position and velocity deviations increase linearly as time progresses (solid line). Alternatively, the deviation can be effectively suppressed by applying the formation maintenance control (dotted line).

$$\Delta v_{1i} = 0$$

Fig. 4 shows the results of the analytical solutions and the numerical simulation. In this case, it is possible to reduce the total velocity increments by setting $\theta_1$ to approximately $170^\circ$. This angle corresponds to the nondifferentiable point of the in-plane velocity increments, and can be calculated by solving $\Delta v_{1i} = 0$ or $\Delta v_{12} = 0$. The solutions of $\Delta v_{1i} = 0$ are given by $\theta_1 = 31.7^\circ$ and $224.5^\circ$ and those of $\Delta v_{12} = 0$ are given by $\theta_1 = 41.6^\circ$, $138.1^\circ$, $168.3^\circ$, and $233.3^\circ$. Among these six nondifferentiable points, $\theta_1 = 168.3^\circ$ corresponds to the point of the minimum velocity increments in total. The minimum total delta-v is calculated as 17.1 [mm/s].

The simulation results when $\alpha = \theta_1 = 168.3^\circ$ are presented in Fig. 5. This figure shows the time histories of the position deviations $\delta x(t)$, $\delta y(t)$ and $\delta z(t)$. Fig. 6 shows the velocity deviations $\delta \dot{x}(t)$, $\delta \dot{y}(t)$ and $\delta \dot{z}(t)$. The position and velocity deviations are maintained within $100$ [m] and $50$ [mm/s], respectively. Fig. 7 shows the simulation results of the velocity increments of each impulsive control. As expected from Fig. 5, the out-of-plane components of delta-v are approximately zero, and the total amount of delta-v is approximately 17 [mm/s]. Finally, Figs. 8 and 9 show the comparison of the absolute formation deviation with and without the maintenance control. Without the maintenance control, the position and velocity deviations increase linearly as time progresses (solid line). Alternatively, the deviation can be effectively suppressed by applying the formation maintenance control (dotted line).

$$\Delta v_{1i} = 0$$

Fig. 2. Velocity increments of the two impulsive controls for Case A (The solid lines and dots show the analytical and simulation results, respectively. The dotted line shows the total velocity increments of the analytical results.)
Fig. 3. Velocity increments of the two impulsive controls for Case B (The solid lines and dots show the analytical and simulation results, respectively. The dotted line shows the total velocity increments of the analytical results.)

Fig. 4. Velocity increments of the two impulsive controls for Case C (The solid lines and dots show the analytical and simulation results, respectively. The dotted line shows the total velocity increments of the analytical results.)

Fig. 5. Deviations of the relative position under the orbital maintenance control for Case C ($\alpha = \theta_1 = 168.3^\circ$).

Fig. 6. Deviations of the relative velocity under the orbital maintenance control for Case C ($\alpha = \theta_1 = 168.3^\circ$).

Fig. 7. Simulation results of the velocity increments of each impulsive control for Case C ($\alpha = \theta_1 = 168.3^\circ$).

Fig. 8. Deviation of position (norm of error vector) for case C ($\alpha = \theta_1 = 168.3^\circ$). (The dotted line and solid line show the results with and without the orbit maintenance control, respectively.)
4. Conclusions

In this paper, a two impulsive control law of a helix formation flying subject to $J_2$ perturbations is considered and the velocity increments of the two impulsive controls are analytically derived. The formation error is evaluated with respect to the argument of latitude of the chief spacecraft, and the results are applicable to a flying formation whose mission sequence is determined by the chief's argument of latitude. The analytically derived two impulsive control shows that the in-plane control is decoupled from the out-of-plane control. This means that the total velocity increments can be effectively reduced by adjusting the argument of latitude of the chief. The analytical solutions are compared with the numerical simulation results, and they agree well with each other.