Numerical Study on Fundamental Characteristics of Electro-Hydrodynamic Thruster for Mobility in Planetary Atmosphere

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The EHD (electro-hydrodynamic) thruster seems suitable for a device of mobility at planetary exploration because of its propellant-less nature. In this paper, the fundamental characteristics of the EHD thruster are investigated from a viewpoint of the applicability to the planetary exploration. The numerical analysis of the drift-diffusion equations shows that it works irrespective to the polarity of the electrodes because the polarity of the produced ions is also changed. This fact implies that it will work in different atmospheric composition with different polarity of the produced ions. It is also numerically found that the thrust tends to increase with the decrease in the ambient pressure. For simplicity of discussion, a simple analytical model to describe the energy conversion efficiency is derived, and it indicates that higher efficiency is expected in higher Knudsen number regime. Both the above results suggest that the EHD thruster is promising especially in planetary atmosphere with low density.

Key Words: EHD Thruster, Asymmetric Capacitor, Drift-diffusion Model, Planetary Exploration

Nomenclature

λ	: mean free path of molecules
d	: diameter of molecule
L	: distance between electrodes
Kn	: Knudsen number $\equiv \lambda/L$
n	: number density
r	: radius
e	: fundamental charge
ϵ_0	: permeability of vacuum
ϕ	: electro-static potential
γ	: second emission coefficient of electron
Ω	: computational domain
μ	: mobility
D	: diffusion coefficient
α	: ionization coefficient
η	: attachment coefficient
r_{ep}	: recombination coefficient between positive ion and electron
r_{pn}	: recombination coefficient between positive ion and negative ion
v_d	: drift velocity
S	: Surface
$\langle t \rangle$	: mean collision time of molecules
$\langle v \rangle$	: mean velocity of molecules
$\langle v^2 \rangle$	: mean power of velocity of molecules
m_{eff}	: effective mass

Subscripts

w	: wire
c	: collector

i	: represent all ions
n	: negative ion
p	: positive ion
e	: electron

1. Introduction

Recently, electro-kinetic devices called EHD (Electro Hydrodynamic) thruster are widely studied.¹⁻⁸⁾ This device can propel a vehicle without any on-board propellants in planetary atmosphere. EHD thruster is also called “CDPA (Corona Discharge Plasma Actuator)”¹³⁾. And it is distinguished with DBDPA (Dielectric Barrier Discharge Plasma Actuator) which is studied for flow control. The big structural difference between these devices is whether dielectric is inserted between electrodes. Because the induced flow speed of EHD thruster is faster than that of DBDPA, so EHD thruster is more suitable for propulsion than DBDPA.²⁾

Although there are a good number of researches on EHD thrusters, their main focus is on its behavior under 1-atm pressure condition. Kento et.al¹⁾ studied on EHD thruster experimentally. They varied the voltage and gap length between electrodes and measured the thrust and current. They also proposed a simple theory, but the effect of pressure is not described there. As a recent tendency, the attempts to increase thrust efficiency are intense. The study of Colas²⁾ and Martins³⁾ is typical. They used 3-pole type EHD thruster and succeeded in increasing efficiency. But their study is only on the optimization of its geometry. As for theoretical research, the study of AFMC⁵⁾ is representative. They assumed one dimensional EHD thruster, and by using a simple model of EHD thruster, they constructed an equation to estimate its

idealized efficiency and maximum thrust. In the study, the case where neutral fluid has velocity is also considered.

If this device still works under low pressure environment, the following applications are made to be possible: 1) The drag compensation system for satellites in LEO (Low Earth Orbit) to avoid orbital decay, 2) Arbitrary orbital maneuvering which could not be achieved without vast fuel consumption in case of conventional rocket propulsion.

In this study, we will reveal the following mechanisms which have not been revealed yet: 1) the reason why polarity is not dominant factor for thrust norm and direction. 2) Scale dependency of EHD thruster such as pressure or characteristics length. This investigation is especially important if we use this device in other planetary environment.

In section 2, structures of EHD thruster and its mechanism will be explained. In section 3, we introduce some formulation for numerical simulation and we will discuss the results of simulation and reconsider how this device works. In section 4, the feasibility of this device under low pressure environment is evaluated.

2. Description of EHD Thruster

The overview of typical EHD thruster is shown in Fig.1. EHD thruster is composed of two electrodes; wire and collector. These electrodes are distinguished by its curvature of the surface. The curvature of wire is larger than that of collector in general. If we put current between these electrodes, then corona discharge will occur in the vicinity of the wire, and the generated charged particles (as of this point, the sign of charge is not specified) are accelerated toward the collector. According to Newton's second law, the momentum would be exchanged from scalar potential (spatial voltage) energy to the kinetic energy of the ambient gas. We call "Nominal direction" if the wire is at higher potential and "Reverse direction" if the collector is at the higher potential. Although there is a number of simulations in the case of Nominal direction, the mechanism how the Reverse direction also works has not been sufficiently investigated so far.

Fig. 1. Typical (2-pole) EHD thruster.

3. Numerical Simulation

3.1. Hydrodynamic model

Before constructing governing equations and setting transport parameter, some assumptions are made.

A) Single-moment description.

This means that charged particles are considered to be a continuum. This assumption also means that the relaxation time of electron is relatively short against the characteristic time scale, and all parameters such as electron impact ionization coefficient and mobility are represented as a function of E/N (local reduced field).

B) Treatment of neutral fluid

In this study, neutral fluid is not solved. Only the result of short timespan simulation is reliable because we consider the moment where neutral particles stay.

3.2. Governing equations

To describe plasma behavior, we use drift-diffusion equations, which is widely accepted in the field of DBDPA.^{10, 11, 13, 14)} These equations are well known as the three-fluid model.

$$\frac{\partial n_e}{\partial t} + \nabla \cdot \Gamma_e = (\alpha - \eta) | \Gamma_e | - r_{ep} n_e n_p. \quad (1)$$

$$\frac{\partial n_p}{\partial t} + \nabla \cdot \Gamma_p = \alpha | \Gamma_e | - r_{ep} n_e n_p - r_{pn} n_p n_n. \quad (2)$$

$$\frac{\partial n_n}{\partial t} + \nabla \cdot \Gamma_n = \eta | \Gamma_e | - r_{pn} n_p n_n. \quad (3)$$

The Nikonov's parameter¹²⁾ is used as these coefficient except recombination coefficients and diffusion coefficient of ions. As for these coefficients, we use $r_{ep} = r_{pn} = 2.0 \times 10^{-13} [\text{m}^3 \text{s}^{-1}]$ and $D_i = 0.180 [\text{m}^2 \text{s}^{-1}]$. These parameters assume the plasma reaction under the "air-like" gas¹²⁾. They are valid as long as we consider the atmospheric gas near ground. Γ is the flux and given by the equations as follows.

$$\Gamma_e = n_e \mu_e E - D_e \nabla n_e. \quad (4)$$

$$\Gamma_p = n_p \mu_p E - D_p \nabla n_p. \quad (5)$$

$$\Gamma_n = n_n \mu_n E - D_n \nabla n_n. \quad (6)$$

Note that mobility can take positive and negative values depending on the species. And then, to close the equation system, we need to calculate electric field by solving Poisson equation, as follows.

$$\nabla \nabla \phi = -\frac{e}{\epsilon_0} (n_p - n_e - n_n). \quad (7)$$

From Eqn.(7), we can calculate electric field directly.

$$\mathbf{E} = -\nabla \phi \quad (8)$$

Thrust (body force) of EHD thruster is estimated by the equation.

$$\mathbf{F} = \iint_{\Omega} e (n_p - n_e - n_n) \mathbf{E} ds. \quad (9)$$

The current which flows in EHD thruster is calculated by a line integral as follows.

$$I = \oint_{\text{Electrode}} e(n_e \mu_e + n_p \mu_p + n_n \mu_n) \mathbf{E} \cdot d\mathbf{l}. \quad (10)$$

FEM (Finite Element Method) is employed to solve drift-diffusion equation and Poisson equation. SUPG (Stream Upwind Petrov Galerkin method)¹⁵⁾ is applied as spatial discretization and implicit Crank-Nicolson method¹⁶⁾ is applied as time discretization. These equations are solved by weak coupling.

3.3. Initial conditions and boundary conditions

As initial conditions, we set $n_e = 1.0 \times 10^9 \text{ [m}^{-3}\text{]}$, $n_p = 1.0 \times 10^9 \text{ [m}^{-3}\text{]}$, $n_n = 1.0 \times 10^6 \text{ [m}^{-3}\text{]}$ in all computational domains. The reason why negative ion density has finite value which is not zero, is to avoid instability of matrix inversion when we solve finite element equations.

The boundary conditions are imposed as follows.

1) Electrode(Anode)

$$\frac{\partial n_e}{\partial \mathbf{n}} = 0. \quad (11)$$

This condition means that the gradient of electron density is equal to zero at the anode.

$$\Gamma_p = 0. \quad (12)$$

This means that positive ion flux to the anode is equal to zero.

$$\frac{\partial n_n}{\partial \mathbf{n}} = 0. \quad (13)$$

$$\phi = \phi_A. \quad (14)$$

This means that anode's potential takes certain value at any time.

2) Electrode(Cathode)

$$\Gamma_e = -\gamma \Gamma_i. \quad (15)$$

This means that if ions collide with cathode, then electron is emitted from the electrode (second electron emission).

$$\frac{\partial n_p}{\partial \mathbf{n}} = 0. \quad (16)$$

$$\Gamma_n = 0. \quad (17)$$

$$\phi = \phi_C. \quad (18)$$

As for the external boundary, the zero gradient condition is imposed for all quantities.

Here, ϕ_A and ϕ_C are time-dependent and given by following equations.

$$\phi = V_{\text{supply}} - R_b I - C_b R_b \frac{d\phi}{dt}. \quad (19)$$

This equation means that we use current restriction resistance and blocking capacitor so as to stabilize this system.

In order to put current gently, we use hyperbolic tangent function. This method is important because numerical instability is emerged by impulsive addition of high voltage.

$$V_{\text{supply}} = V_0 \tanh\left(\frac{t}{\tau}\right). \quad (20)$$

where, V_0 is the voltage of power supply. There is a degree of freedom to select the potential, ϕ_A and ϕ_C . We selected as shown.

$$\phi_A = -\phi_C = \frac{\phi}{2}. \quad (21)$$

Detailed simulation conditions are listed in table 1.

Table 1. Simulation Condition (common in this section)

Time Step	$1.0 \times 10^{-8} \text{ [s]}$
Time Integral Method	Crank-Nicolson (Implicit)
Numerical Integration	Gauss Legendre (2 nd order)
V_0	$1.0 \times 10^4 \text{ [V]}$
Pressure	760[Torr]
C_b	$1.0 \times 10^{-12} \text{ [F]}$
γ	0.02
τ	$1.0 \times 10^{-6} \text{ [s]}$
L	$2.0 \times 10^{-3} \text{ [m]}$
r_w	$1.0 \times 10^{-4} \text{ [m]}$
r_c	$3.2 \times 10^{-2} \text{ [m]}$

Fig. 2. Mesh near the wire

R_b is $1.0 \times 10^6 \text{ [ohm]}$ for nominal direction, and $1.0 \times 10^7 \text{ [ohm]}$ for reverse direction.

Fig. 3. Overview of EHD thruster in analysis (with electro-static potential).

The unstructured mesh near the wire is shown in Fig.2. EHD thruster's geometry which is to be analyzed is shown in Fig.3 with electro-static potential after 2.0[us] in nominal

direction. It is obvious that the gradient of electro-static potential near the wire is very strong.

3.4. Effect of the polarity

As pointed out in section.1, the polarity of EHD thruster rarely affects the thrust efficiency and the direction of the thrust at atmospheric environment. It is needed to investigate

Fig. 4. Time dependency of particle number (nominal direction).

Fig. 5. Time dependency of particle number (reverse direction).

this reason because this characteristic may cause fatal disadvantages when we use this device in other planets. To discuss the effect of polarity, two cases are simulated. One is nominal direction, and the other is reverse direction. Fig.4 and Fig.5 indicate the time dependency of the number of the charged particles in the whole computational domain shown in Fig.3. In Fig.4 (nominal direction), positive particle is dominant. On the contrary, negative particle is dominant in Fig.5 (reverse direction). In both case, electron number decreases at first. This is because recombination or attachment of electron occurs due to low local reduced field strength. It is obvious that if the polarity of the electrodes is changed, the dominant species of charged particle are also changed and the polarity of the ionized gas is changed. This is the reason why thrust direction is not changed even if the polarity of the

electrode is changed.

By applying this nature, it is suggested that even in the case of the planetary environment where negative or positive ion cannot be generated, we can use EHD thruster by only changing the polarity.

4. Effectiveness of Pressure

Atmospheric pressure is not necessarily the optimum environment for EHD thruster. It is because in the field of DBDPA, the trend that the more pressure decreases, the more thrust increase near the environment of atmospheric pressure is experimentally confirmed¹⁷⁾. As pointed out in section.1, EHD thruster is partially similar to DBDPA, there is a possibility that this device also perform well in other pressure rather than atmospheric pressure especially in low pressure. We studied it analytically, and validated the results by numerical simulation.

4.1. Assumptions toward constructing equations

Before constructing the equation, we put some assumptions.

1. Electric field is uniform. This implies that sheath domain is not considered and wire curvature is negligible.
2. The particle which takes momentum transfer is only positive ion.
3. The electron's mobility is empirically decided. (Nikonov's parameter¹²⁾) This means that our equation may contain large errors where pressure is extremely high or low relatively to atmospheric pressure.

4.2. Derivation of thrust efficiency

Consider three-dimensional space. The total electric energy consumptions are given by the equation as follows.

$$W = IV_0 = en_p \langle v \rangle SV_0. \quad (22)$$

Also, the total kinetic energy transferred from electric energy is given by the equations as follows.

$$K \approx \frac{1}{2} m_p \langle v^2 \rangle \cdot n_p SL \cdot \frac{1}{\langle t \rangle}. \quad (23)$$

The average speed of the charged particle is given by

$$\begin{aligned} \langle v^2 \rangle &= \frac{3k_B T + m_g v_d^2}{m_{eff}} \approx 6k_B T + 2m_p \mu_p^2 \frac{V_0^2}{L^2} \\ &= 6k_B T + 2m_p \cdot \frac{V_0^2}{L^2} \cdot C_0^2 \frac{2\lambda^2 \pi^2 d^4}{k_B^2 T^2} \\ &= 6k_B T + 2m_p V_0^2 C_0^2 \frac{2\pi^2 d^4}{k_B^2 T^2} \cdot \text{Kn}^2. \end{aligned} \quad (24)$$

where, $C_0 = 2.43 \times 10^1 \left[\frac{\text{m}^2 \cdot \text{Pa}}{\text{V} \cdot \text{s}} \right]$

$$\begin{aligned} \eta_{eff} &\approx \frac{K}{W} = \frac{m_p \langle v \rangle L}{2eV_0 \langle t \rangle} = \frac{m_p \langle v^2 \rangle L}{2eV_0 \lambda} = \frac{m_p \langle v^2 \rangle}{2eV_0 \text{Kn}} \\ &= \frac{m_p}{2eV_0 \text{Kn}} \cdot \left(6k_B T + 2m_p V_0^2 C_0^2 \frac{2\pi^2 d^4}{k_B^2 T^2} \cdot \text{Kn}^2 \right). \end{aligned} \quad (25)$$

$$\frac{\partial \eta_{eff}}{\partial \text{Kn}} = 0. \quad (26)$$

If we assume $T = 300[\text{K}]$, $m_p = 1.7 \times 10^{-27}[\text{kg}]$, $V_0 = 10[\text{kV}]$, $d = 3.7 \times 10^{-10}[\text{m}]$, and calculate Eqn.(26), we yield

the value when thrust efficiency takes global minima.

$$Kn_{\minima} = \sqrt{\frac{3k_B^3 T^3}{2\pi^2 m_p V_0^2 C_0^2 d^4}} \approx 7 \times 10^{-5}. \quad (27)$$

The characteristic length L of a typical EHD thruster is around 10[mm]. Then the pressure to give above Knudsen number is 9.4×10^4 [Pa]. In other words, when the ambient pressure is lower than that value, the thrust efficiency increases with the decrease in the pressure.

Fig. 6. Pressure dependency of body force (nominal direction).

4.3. Results and discussion

In the previous section, it is analytically shown that higher thrust efficiency is expected at lower ambient pressure.

To directly show the possibility of EHD thruster under the different pressure, a numerical simulation of EHD thruster around atmospheric pressure is executed in a similar manner as in section. 2. The geometry of EHD thruster and simulation condition are the same except pressure. It is clearly understood by Fig.6 that the produced body force decreases with the increase in the ambient pressure. This is because ionization coefficient in calculation domain became larger as local reduced field increases.

Following these results, it seems that there is optimum condition where thrust efficiency or thrust takes global maxima. Hence there is a possibility EHD thruster can be used as a propulsion system in low pressure atmosphere at planetary exploration, for example, in the Martian atmosphere.

5. Conclusions

The numerical and theoretical analysis of EHD thruster is made by solving the drift-diffusion equations. It is revealed that if the polarity of electrodes is changed, the dominant particle's polarity is also changed. This fact indicates that EHD thruster can be used in planet where positive or negative ion is difficult to emerge in nature by alternating its polarity. The pressure dependency is also examined. In this procedure, thrust efficiency is the function of non-dimensional number

(Knudsen number). It is also expected that the EHD thruster may work well at higher efficiency in the condition rather than one-atmospheric pressure. It is suggested that EHD thruster seems promising as a fuel-less device for mobility at planetary explorations.

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