Numerical Analysis of Potential Structure around Electric Solar Wind Sail Tethers

By Kento HOSHI,1) Hirotsugu KOJIMA2) and Hiroshi YAMAKAWA2)

1) Graduate School of Engineering, Kyoto University, Kyoto, Japan
2) Research Institute for Sustainable Humanosphere, Kyoto University, Kyoto, Japan

(Received July 31st, 2015)

We perform the first three-dimensional particle-in-cell simulations of the electric solar wind sail, which is a recently proposed new propulsion system. We investigate the potential structure around the tethers of the electric solar wind sail. As a result, the potential distribution is greatly lower than the potential expression proposed by the previous studies. We proposed the new method to estimate the potential around the tether numerically without performing the time-consuming PIC calculation. The proposed numerical solution has a very good agreement with PIC results under two difference plasma environments, and its agreement shows the validity of the method. The proposed method is useful to estimate the thrust of E-sail when a precise thrust operation is required.

Key Words: Electric Solar Wind Sail, Tether, Particle-in-cell

Nomenclature

\( b \) : approximated parameter
\( F \) : force
\( h \) : grid spacing
\( I_b \) : beam current
\( j_{ph0} \) : current density of photoelectrons
\( K \) : proportional factor
\( k_B \) : Boltzmann constant
\( k_D \) : debye wave number
\( m_p \) : proton’s mass
\( n_0 \) : background plasma density
\( n_e \) : background electron density
\( n_p \) : background proton density
\( r_0 \) : distance from the tether where the vacuum potential becomes zero
\( r_s \) : sheath radius
\( r_w \) : tether’s radius
\( T_b \) : beam temperature
\( T_e \) : electron temperature
\( T_p \) : proton temperature
\( v_d \) : drift velocity
\( V_{b0} \) : tether’s electrical potential
\( V_a \) : beam accelerating potential
\( W \) : exponential factor for Fourier Transform
\( \lambda_d \) : debye length
\( \epsilon_0 \) : the permittivity of vacuum
\( \varepsilon \) : the perpendicularity of vacuum

\[ V(r) = V_0 \frac{\ln \left( 1 + \left( \frac{r_0}{r} \right)^2 \right)}{\ln \left( 1 + \left( \frac{r_0}{r_w} \right)^2 \right)} \]

where \( r_0 = 2\lambda_d = 2\sqrt{\frac{e_0 k_B T_e}{e n_e}} \) (\( \lambda_d \) is the debye radius), \( V_0 \) is the tether’s electrical potential, and \( r_w \) is the tether radius.

The force per unit length acting on the sail is given by

\[ \frac{dF}{dz} = \frac{K m_p n_0 v^2 r_0}{\sqrt{\exp \left( \frac{n_p v^2}{\epsilon_0} \ln \left( \frac{r_0}{r_w} \right) - 1 \right)}} \]

where \( z \) is the axis along the tether, \( K \) is a proportional factor (~3.09 obtained from their Monte Carlo simulation), \( m_p \) is the proton’s mass, \( n_0 \) is the electron density, \( v_d \) is the drift velocity of solar wind. Successful and efficient mission trajectory in the solar system was calculated using the analytic expression. 3) In addition, Janhunen proposed the thrust may become at most 5 times larger than that estimated from Eq. (2) due to trapping and reducing electrons around the tether by using 1-D test particle simulations. 4)

Sanchez-Torres investigated more precise thrust modeling considering ion-scattering. 5) He used the potential expressed as

1. Introduction

An electric solar wind sail is a new propulsion system recently proposed by Janhunen, 1) which consists of several kilometre-long, thin, conducting tethers. Tethers are kept at a high positive electrical potential on the order of several kilovolts with an electron gun. The positively charged tethers deflect the momentum of solar wind protons so that the tethers obtain a propulsive force. The system requires only electron sources and electrical power to generate thrust. Hence, it is expected to be a new propellantless space propulsion.

However, its thrust characteristics are not clear at present. Janhunen and Sandroos performed the one-dimensional test particle simulation to estimate the potential structure around the tether and performed the two-dimensional particle-in-cell (PIC) simulations to evaluate the force acting on the tether in 2007. 2) The potential around the tether they proposed was expressed as

\[ V(r) = V_0 \frac{\ln \left( 1 + \left( \frac{r_0}{r} \right)^2 \right)}{\ln \left( 1 + \left( \frac{r_0}{r_w} \right)^2 \right)} \]

1. Introduction

An electric solar wind sail is a new propulsion system recently proposed by Janhunen, 1) which consists of several kilometre-long, thin, conducting tethers. Tethers are kept at a high positive electrical potential on the order of several kilovolts with an electron gun. The positively charged tethers deflect the momentum of solar wind protons so that the
Fig. 1. Definition of the tether model and plasma environment. Background plasmas have solar wind velocity along the $x$-axis. Photoelectrons are emitted towards solar wind. Electron beam is emitted along the $y$-axis from the edge of the tether.

$$V(r) = V_0 \begin{cases} \ln \left( \frac{r e^{-b}}{r_s} \right) & (r \leq r_s e^{-b}) \\ \ln \left( \frac{r e^{-b}}{r_w} \right) & (r > r_s e^{-b}) \end{cases}$$

where the approximated parameter $b$ is 0.65 for the 10-40 kV potential bias, and $r_s$ is the sheath radius for a high positive bias tether. 6)

However, all of these previous studies were one-dimensional or two-dimensional studies under ideal conditions. They assumed the surface potential was fixed to tens of kilovolts, and do not vary in their calculations. The electron beam emission and photoelectron emission were not included. Background electrons and ions impacting were also ignored.

To estimate the realistic performance of the E-sail, it is necessary to reveal the potential distribution under the realistic conditions, and it is the purpose of the paper.

We perform the first three-dimensional particle-in-cell simulations of the long tethers in solar wind environment. We include electron beam emission from the edge of the tether and the impinging of particles on the surface so that there is no assumption of biased the tether’s potential. We show the electric potential structure and plasma density distributions around the tether, and compare our results with the potential distributions proposed by previous studies. Additionally, we proposed a new potential estimation method using Fast Fourier Transform (FFT) and validate its correctness.

2. PIC Simulation

2.1. Simulation settings

In interplanetary space, Magnetohydrodynamics (MHD) codes and Hybrid codes are often adopted for analysis (e.g. Magneto Sail). However, we wanted to simulate the electron beam emission from the surface and its returning to the surface. The Particle-in-cell method can treat this situation easily, so we adopted the PIC method in this paper. We used the simulation code called HiPIC, which was developed by the Japan Aerospace Exploration Agency’s Engineering Digital Innovation Center (JEDI). 7) HiPIC has 3-D rectangular grids and uses the full static PIC method for calculating collisionless kinetic plasma. HiPIC solves Newton’s equations of motion for each particle by using Buneman-Boris method, and solves Poisson’s equations for obtaining an electric potential distribution in the computational domain by using discrete sine transformation. PIC methods use “super particles” as representatives of real particles to reduce the calculation times. $n_{\text{super}}$ denotes the super particle’s representative number and is calculated in each simulation as $n_{\text{super}} = n_e \times \Delta x^3 / p_{\text{cell}}$.

Table 1. Simulation Parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Background plasma species</td>
<td>Electron, Proton</td>
</tr>
<tr>
<td>Background plasma density $n_0$</td>
<td>$10^7$ (10/cc)</td>
</tr>
<tr>
<td>Background electron temperature $T_e$</td>
<td>100 eV (case a), 12 eV (case b), Maxwellian</td>
</tr>
<tr>
<td>Background proton temperature $T_i$</td>
<td>12 eV, Maxwellian</td>
</tr>
<tr>
<td>Drift velocity of background plasma $v_d$</td>
<td>400 km/s</td>
</tr>
<tr>
<td>Debye length $\lambda_d$</td>
<td>23.5 m (case a), 8.14 m (case b)</td>
</tr>
<tr>
<td>Magnetic field</td>
<td>None</td>
</tr>
<tr>
<td>Photoelectron current density</td>
<td>$144 \mu A/m^2$</td>
</tr>
<tr>
<td>Photoelectron temperature</td>
<td>2.0 eV</td>
</tr>
<tr>
<td>Electron beam temperature $T_b$</td>
<td>0.1 eV, (Drift–Maxwellian)</td>
</tr>
<tr>
<td>Time step width</td>
<td>10 ns</td>
</tr>
<tr>
<td>Grid spacing</td>
<td>0.3 m</td>
</tr>
<tr>
<td>System length</td>
<td>$76.8 \times 153.6 \times 76.8 m (256 \times 512 \times 256 grids)$</td>
</tr>
<tr>
<td>Super particle per cell, $p_{cell}$</td>
<td>30/cell</td>
</tr>
<tr>
<td>Boundary conditions</td>
<td>$V = 0$ V at edges of the system.</td>
</tr>
</tbody>
</table>

Fig. 2. Potential distribution around the tether. a) The slice along the $x$-axis, the potential in the sunward side of the tether becomes slightly higher than that in the sunshade side. b) The slices along the $y$-axis. The potential nearby the beam-electron emission point are lower than the potential at the center.
Fig. 3. Density distribution of background plasmas. a) Background electron density. Electrons are accumulated around the tether attracted by the tether’s potential. b) Background proton density. Protons are deflected by the potential and decreased behind the tether.

where \( p_{\text{eff}} \) is a number of super particle in one cell. HiPiC calculates an interaction between plasmas and the spacecraft model, which is modeled as rectangular internal boundaries. If super particles impact the model’s surface, their charges accumulate on the surface and contribute to the electric potential of the model.

To show a potential structure around the E-sail’s tether, we modeled the tether as shown in Fig. 1. The tether is located on the center of the computational domain, and the surface potential of the tether is initially set to 0 V. The tether’s length is 60 meters and the tether’s radius is 15 centimeters. The background plasma in solar wind environment (at about 0.5 AU) is assumed. Electron’s and proton’s density are set to \( n_0 = 10 \text{ cc} = 1.0 \times 10^7 \text{ m}^{-3} \), their temperature are set to \( k_B T_e = 100 \text{ eV} \), \( k_B T_i = 12 \text{ eV} \), and they have the drift velocity \( v = 400 \text{ km/s} \) towards the \( x \)-axis. The background plasma particles are injected from all the domain boundaries in each time step. The magnetic field is assumed as zero, not considering the interplanetary magnetic field (IMF) because it is very small and ignorable for the particle’s motion. Photoelectrons are emitted from the tether’s surface towards the sun (\( x = 0 \)) as shown Fig. 1. Photoelectron current density \( j_{\text{ph0}} \) is estimated from the current density at 1 AU. \( j_{\text{ph0}} \) of conductive materials is in \( 30 - 50 \mu \text{A/m}^2 \) at 1 AU, \( \delta \) so we extrapolated \( j_{\text{ph0}} \) to 0.5 AU and set \( j_{\text{ph0}} \) to 144 \( \mu \text{A/m}^2 \). Secondary electrons are neglected in these simulations because they are returned to tether’s surface if the tether’s potential is higher than +10 V. In order to simulate active charging, an electron beam is emitted from the edge of the surface, \( (x, y, z) = (127.5, 356.0, 127.5) \), as shown in Fig. 1. The accelerating potential of an electron beam is set to \( V_b = 200 \text{ V} \) and the beam current is set to \( I_b = 5 \text{ mA} \). The electric potential of the surface rises up after the beam emission starts and converges at a beam accelerating potential. We summarize the parameters that are used in this paper in Table 1.

We performed simulations on these conditions and evaluated the potential structure around the tether surface. Note that the tether’s radius \( r_e \) in our simulations (= 15 cm) are much larger than the actual (ideal) E-sail radius (~ 0.1 \( \mu \text{m} \)) due to the technical difficulty of simulations. But both radii are smaller than the debye length \( (d_d = 23.5 \text{ m}) \), so this condition will not cause a significant difference. The simulated radius in the previous paper \( \delta \) was \( r_e = 1.0 \text{ m} \), hence our settings are more close to the real conditions.

2.2. Simulation results

Figure 2a shows the potential distribution around the tether along the \( x \)-axis, and Fig. 2b shows that of along the \( y \)-axis. The tether’s electrical potential becomes \( V_0 = 240 \text{ V} \), which is higher than the beam accelerating potential \( V_b \). This is due to the thermal temperature of the beam electrons. If background electron current at \( V_b \) is much smaller than the beam current \( I_b \), the current balance is satisfied at higher potential. The actual potential can be calculated an active charging model which considers the distribution function of the beam electrons. \( \delta \) The difference between \( V_0 \) and \( V_b \) is not the point of this paper, so we use \( V_0 = 240 \text{ V} \) which is obtained from the simulation results as the known value.

It is noteworthy that the potential nearby the electron emission point is lower than the potential at the center, as shown in Fig. 2b. This is the effect of the emitted electrons. Even though the potential is converged to \( V_0 \), the beam electrons still exist in space, and they are composing current-loop around the emission point. They contribute the potential as the negative space charge. Its effect on the potential may not cause a significant decline of the thrust. However, if much electron current (and so many electron guns) is required to maintain the surface potential in practice, the potential decreasing would affect the thrust characteristics.

Figure 3 shows the background plasma’s density distribution. Electrons accumulated around the tether and they do not show the asymmetry in the sunward side and in the sunshade side. In contrast, proton’s density behind the tether becomes smaller than the nominal value \( (n_0 = 10/\text{cc}) \). This proton deflection of the
Before comparing the simulation results with the proposed Eq. (1) and Eq. (3), we performed another calculation with 512 × 512 × 512 grids. Figure 4 shows the results with two different system lengths. The center of Fig. 4’s horizontal axis corresponds to \( X = 38.585 \) m of Figs. 2 and 3. The black line shows the potential distribution with 256 grids. The red line shows the potential distribution with 512 grids. The potential around the tether is almost the same, so the simulation results mostly are not affected by the Dirichlet boundary condition. With 512 grids, there is a potential asymmetricity between in the sunward side and that in the sunshade side. This potential asymmetricity may be caused by lack of background proton density behind the tether. There is also a potential asymmetricity in the result with 256 grids, but it is smaller than 512 grids results. For this reason, we use the potential distribution of the 512 grids simulation in following sections to compare with Eq. (1) and Eq. (3).

Figure 5a, which is the slice of the simulation results (with 512 grids) at \( z = 38.585 \), shows the potential dependence on the distance from the tether. Black lines show our PIC simulation results. Red lines and blue lines show the potential proposed by previous studies. Red lines are the Eq. (1) and blue lines are Eq. (3). \( \nu = 80.1 \) was applied for Eq. (3).

We also performed the simulation under a similar plasma condition in the previous study, that is, the plasma conditions near 1 AU. We set the background electron temperature \( T_e = 12 \) eV. Other settings are the same as shown in Table 1. The debye length \( \lambda_d = 8.14 \) m in this environment, and Fig. 6a shows the potential distribution under these conditions.

Clearly, the simulation results are greatly lower than proposed expressions. This is because Eq. (1) does not consider the debye shielding and Eq. (3) can only be applied for very high positive potential as Sanchez-Torres mentioned. Figure 3a shows that the potential shielding will occur in a realistic environment, so Eq. (1) should be modified to include the shielding effects. The model may cause the significant difference between the actual potential and the calculated potential if a precise thrust operation is required. Hence, it is important to construct the method that can calculate the potential around the tether more accurately.

3. Potential Calculation using FFT

In this section, we propose a numerical method to calculate the potential without performing time-consuming PIC simulations.

First, let us consider the Poisson’s equation:

\[
\nabla^2 V(x) = -\frac{\rho(x)}{\varepsilon_0} \tag{4}
\]

where \( \rho \) is a space charge density, and \( \varepsilon_0 \) is the permittivity of vacuum. When we consider a two-dimensional case, we can convert the partial differential equation Eq. (4) into the difference equation as:

\[
\frac{1}{h^2} \left( V_{j-1,k} + V_{j+1,k} + V_{j,k-1} + V_{j,k+1} - 4V_{j,k} \right) = -\frac{\rho_{j,k}}{\varepsilon_0} \tag{5}
\]

where \( h \) is the grid length, \( (j, k) \) shows the grid number. Using Fourier Transform, Eq. (5) can be solved for \( \varphi \) in \( k \)-space.

\[
V_{mn} = \frac{h^2}{\varepsilon_0} \frac{n_0}{4 - n^2} \left[ W + W^* \right], \quad W = e^{2iz/N} \tag{6}
\]

where \( (m, n) \) shows the grid number in \( k \)-space, \( N \) is the total number of lattices, and \( i \) is the imaginary number. The inverse Fourier Transform for Eq. (6) then gives the potential infinite cylinder. This is a well-known procedure to solve the Poisson’s equation numerically using (Fast) Fourier Transform.

The Poisson equation in plasma is modified with considering electron’s density \( n_e \) and the proton’s density \( n_p = n_0 \).

\[
\nabla^2 V(x) = -\frac{\rho(x)}{\varepsilon_0} - \frac{e}{\varepsilon_0} (n_p - n_e) \tag{7}
\]

From the velocity distribution function of electrons, the electron’s density distribution becomes the Boltzmann-distribution:

\[
n_e(x) = n_0 \exp \frac{eV(x)}{k_BT_e} \tag{8}
\]

Assuming the normalized potential \( eV(x)/k_BT_e \) becomes small in the distance, Eq. (7) becomes

\[
\nabla^2 V(x) = -\frac{\rho(x)}{\varepsilon_0} + \frac{e^2 n_0}{\varepsilon_0 k_BT_e} V(x) \tag{9}
\]

with the first-order approximation. Defining \( k_D = e^2 n_0/\varepsilon_0 k_BT_e \) as the debye wave number, we obtain
To solve this equation in three-dimensional space with a unit charge, the well-known Yukawa potential be obtained. Instead, to solve this in two-dimensional space, the shielded potential around the infinite cylinder will be obtained. However, it is difficult to solve it analytically.

Our idea is to modify Eq. (6) corresponding with Eq. (10). The modified difference equation becomes

$$\left( V^{2} - k_{D}^{2} \right) V(x) = -\frac{\rho(x)}{\epsilon_{0}}$$ \hspace{1cm} (10)$$

Thereby, we can solve the shielded potential in $k$-space as:

$$V_{m,n} = \frac{h^{2}}{\epsilon_{0}} \sum_{i}^{n} \left( V_{i-1,k} + V_{i+1,k} + V_{i,k-1} + V_{i,k+1} - 4V_{i,k} \right) - k_{D}^{2}V_{i,k}$$ \hspace{1cm} (11)$$

Where

$$\rho_{j,k} = \frac{\rho_{j,k}}{\epsilon_{0}}$$

We consider a tether’s charge per tether’s length $q_{\text{tether}} = 2\pi a_{0}V_{0}/\log(r_{0}/r_{w})$ at the center of two-dimensional space and solve for the potential using the proposed method. The cyan lines in Fig. 5b and Fig. 6b show the solution of Eq. (12) using FFT, with $V_{0} = 240V$, $r_{w} = 15 cm$, $h = 0.02 m$, $N = 8192$. $q_{\text{tether}}$ is uniformly distributed to center grids which are within $r_{w}$.

The solutions have a very good agreement with the 3D PIC simulation results in both cases. The method does not assume any non-physical constraint ($r_{0} = 2\lambda_{D}$, assumed in Eq. (1)) and do not require an advance calculation of the sheath equation for $r_{s}$, which is required in Eq. (3). Instead, the tether’s charge $q_{\text{tether}}$ is given as a weak assumption. As mentioned above, the
tether’s potential $V_0$ can be determined by using semi-analytical active charging model,\(^{10}\) so $q_{\text{tether}}$ can be adjusted to make the tether’s potential the same as $V_0$ obtained from the semi-analytical model. The background plasma environment, the tether’s radius $r_w$ are only required to calculate. Thus, the method is useful to estimate the accurate potential of the tether without time-consuming full PIC simulation.

Equation (9) is under the first order approximation. We assume a normalized potential $eV(x)/k_BT_e$ becomes small in the distance from the tether, so the validity of the proposed method with higher $V_0$ is still unknown. The normalized potential $eV_0/k_BT_e$ in our test cases is relatively low. $eV_0/k_BT_e = 2$ in case a, and $eV_0/k_BT_e = 20$ in case b. Actually, The expected potential range of E-sail is $V_0 = 2kV \sim 20$ kV. In this range, $eV_0/k_BT_e$ becomes $20 \sim 200$ at $T_e = 100$ eV, and $166 \sim 1666$ at $T_e = 12$ eV. Hence, validations in the range of $eV_0/k_BT_e = 100 \sim 2000$ should be performed to test the proposed method. Though the validity of the method on the more high potential is not confirmed in this paper, it is clear that the proposed method is useful to estimate the thrust of E-sail when the small thrust operation is required.

4. Conclusion

In this paper, we have performed the first 3-D Particle-in-cell simulations of the electric solar wind sail. Our simulations calculated electron beam emission from the edge of the tether and charged particles impinging on the surface, both have not been considered in previous studies. We have shown the electric potential distribution and density distribution around the tether. The potential is $\sim 50\%$ lower than that of proposed by the previous studies, so the thrust will be lower than the conventionally estimated value. We have proposed the numerical method using Fast Fourier Transform to calculate the potential around the tether with no assumption. The proposed solution has a very good agreement with the 3D PIC simulation results. Though the tested range of the method in this paper have been limited, the proposed method definitely useful to estimate the thrust of E-sail.

Our future tasks are to validate the proposed method in a higher potential region. The 3D PIC simulation with $V_0 = 2\sim 20$ kV should be performed to validate the proposed method with the potential. We then investigate the correct thrust characteristics using the proposed method and the PIC simulations.

Acknowledgments

The computations in the present study were performed using the KDK system in the Research Institute for Sustainable Humanosphere (RISH) at Kyoto University. The present study was supported by JSPS KAKENHI Grant Number 15K06600, and Grant-in-Aid for JSPS Fellow Number 15J08941.

References