Empirical Data Driven Model for Sail Membrane Dynamics Estimation

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In order to predict the sail membrane dynamics precisely, it is necessary to use the on-orbit measurement data. However, the on-orbit sail membrane measurement data are incomplete (missing spatio-temporal data) because of sensor placement limitations and sunlight reflection. This paper proposes an empirical data driven model in which past measurement data and current incomplete measurement data are used to estimate sail membrane dynamics and the confidence intervals of the estimated dynamics are obtained via a bootstrap method. Further, the applicability of the proposed empirical data driven model is evaluated via numerical and vacuum chamber experiments. The experimental model used comprises a spin deployable space membrane structure consisting of a square membrane, a center hub, and tethers.

Key Words: Data Driven Model, Data Fusion, Sail Membrane Dynamics

1. Introduction

In recent times, ultralight membrane structures have been garnering increased attention, particularly in space engineering. These structures are being viewed as next-generation innovative space structures because they may provide the wrinkle-free design and active de-wrinkling of membranes required to successfully execute future missions. Thus, the ability to predict wrinkling and slack is one of the key capabilities necessary to successfully carry out future advanced missions. However, conventional on-orbit membrane structure measurement data are incomplete (missing spatio-temporal data) because of sensor placement limitations and sunlight reflection, as illustrated in Fig. 1.

In addition, because the structural characteristics vary as a result of thermal deformation and aging degradation, quantitative shape prediction of membrane space structures has not been possible. However, JAXA has recently managed to briefly detect the rough outline of the membrane space structure IKAROS using an on-board camera and also via numerical simulation.

Given this background, this paper proposes a new empirical data driven model for membrane space structure dynamics estimation in which partial on-orbit measurement data are fused with past measurement data as follows:

- The empirical mode is first derived from past experimental measurement data or numerical simulation data. Membrane dynamics are then predicted by fusing the derived empirical mode and spatio-temporal missing measurement data.
- A bootstrap method is then used to evaluate the uncertainties in the estimated dynamics arising from use of the empirical data driven model by obtaining confidence intervals.

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• The applicability of the empirical data driven model is evaluated by carrying out numerical and vacuum chamber experiments. The experimental model used comprises a spin deployable space membrane structure consisting of a square membrane, a center hub, and tethers.

The remainder of this paper is organized as follows. Section 2 explains the method used to construct the empirical data drive model, which is based on proper orthogonal decomposition. Section 3 discusses confidence interval calculations conducted using the boot strap method. Section 4 presents the experimental model used to evaluate the efficacy of the empirical data driven model. Section 5 presents and analyzes the results obtained.

2. The Empirical Data Driven Model

Calculation using the empirical data driven model is carried out in two steps. In the first step, the empirical mode is derived from past experimental measurement data and numerical simulation data. This method uses proper orthogonal decomposition (POD) to construct the empirical mode. The POD analysis yields a set of empirical mode that describes the dominant behavior of the given dataset. This mode is optimal in the sense that, for any given basis size, the error between the original and reconstructed data is minimized.

In the second step, the empirical mode and the measurement error between the original and reconstructed data is minimized. The POD analysis yields a set of empirical mode that describes the dominant behavior of the given dataset. This mode is optimal in the sense that, for any given basis size, the error between the original and reconstructed data is minimized.

Consider the time-series data \( x(t) \in \mathbb{R}^n, t_1 < t < t_n \) and the optimization problem defined by Eq. (1):

\[
\{ \phi_t \}_{t=1}^{t_n} = \arg\min_{\{ \phi_t \}_{t=1}} \int_{t_1}^{t_n} \| x(t) - \Pi x(t) \|^2 dt
\]

\[
\Pi = \sum_{t=1}^{t_n} \phi_t \phi_t^T \quad (\Pi^2 = \Pi, \sum_{t=1}^{t_n} \phi_t^T \phi_t = I_r)
\]

where \( r \) and \( \Pi \) are low-dimensional number and projection map, respectively. The projection map subsequently becomes a least square map that minimizes the re-projection error. The above optimization problem can be solved using its eigen-equation, Eq. (2):

\[
G \phi = \lambda \phi, \quad \lambda_1 \geq \cdots \geq \lambda_r \geq 0
\]

where, \( G = \int_{t_1}^{t_n} x(t) x(t)^T dt \in \mathbb{R}^{r \times n} \)

where eigenvalue \( \lambda_t \) expresses the contribution ratio for the given dataset. Thus, \( \mu_t \) expresses the relative contribution ratio for the dataset that can be used to select the number of modes, as defined in Eq. (3), where \( n \) is the number of dimensions in the full-order dimension of the given dataset.

\[
\mu_r = \sum_{t=1}^{r} \lambda_t \sum_{j=1}^{n} \lambda_j
\]

The problem of estimating missing data using empirical modes was first considered by Everson and Sirovich for an image reconstruction static problem in the context of reconstructing human face images. It was subsequently extended and tested on flow problems by Tan and Wilcox. In this paper, we propose to extend empirical modes to the membrane dynamics estimation problem—an extension of the structural dynamics estimation problem. The expression for the structural dynamics estimation is as follows.

To represent the spatio-temporal missing node measurements, a mask vector, \( m(x, t) \in \mathbb{R}^n \), is defined. Spatio-temporal missing data \( x_{\bullet}(t) \in \mathbb{R}^n \) can be written as

\[
x_{\bullet}(t) = m(x, t) \cdot \hat{x}(t)
\]

where \( \hat{x}(t) \in \mathbb{R}^n \) is the complete dataset, \( n \) is the degrees of freedom possessed by the model, and \( m(x, t) \) tracks the spatio-temporal missing data, \( \cdot \) is point-wise multiplication.

\[
m(x, t) = \begin{cases} 1 & \text{if the component is known in } (x, t) \\ 0 & \text{if the component is missing in } (x, t) \end{cases}
\]

Denoting \( \hat{x}(t) \) by a complete dataset based on some initial guess, such as numerical analysis dataset and experimental measurement dataset, we can perform POD on \( \hat{x}(t) \) to obtain the guessed spatial and temporal empirical modes. This decomposition has the form,

\[
\hat{x}(t) = \sum_{k=1}^{M} \tilde{\phi}_k(t) \phi_k(x)
\]

where \( \tilde{\phi}_k(t) \) is the \( k \)-th guessed empirical temporal mode and \( \phi_k(x) \) is the \( k \)-th guessed empirical spatial mode. Once the spatio-temporal missing measurement dataset is acquired, the difference between the incomplete measurement dataset and the empirical mode superposition is minimized as shown in Eq. (7):

\[
F_j(\tilde{\phi}_k) = \left\| x(t) - \sum_{k=1}^{M} \tilde{\phi}_k(t) \phi_k(x) \right\|_m^2
\]

where the gappy inner product, \( (u, v)_m = (m \cdot u, (m \cdot v)) \), and the corresponding gappy norm, \( ||v||_m = (v, v)_m \), can be defined. \( M \) is the number of modes used in the reconstruction process. Minimization of this functional leads to the linear system of algebraic equations

\[
\sum_{j=1}^{M} (\phi_j(x), \tilde{\phi}_k(x)) m \tilde{\phi}_k(t) = (x(t), \tilde{\phi}_k(x))_m
\]

Solving Eq. (8) for unknown expansion coefficient \( \tilde{\phi}_k(t) \), the estimate for the missing values, \( \hat{x}_{\bullet} \), can be calculated using the following equation:

\[
\hat{x}_{\bullet}(t) \approx \sum_{k=1}^{M} \tilde{\phi}_k(t) \phi_k(x)
\]

Finally, the spatio-temporal complete \( x_{\bullet} \) representing the empirical data driven model estimation value is reconstructed by replacing the missing elements in \( x_{\bullet} \) by the
corresponding estimated elements in $\hat{x}_j$; that is, $x_j = \hat{x}_j$, if $m(x, t) = 0$. As mentioned above, the empirical data driven model estimates the spatio-temporal missing data by superposition of the empirical mode $\phi_k(x)$. Therefore, the estimation isn’t accurate if the missing data is orthogonal to the empirical mode $\phi_k(x)$.

3. Confidence Interval Calculation

The confidence interval of the estimated data is calculated using the bootstrap method\(^5\) which provides the upper and lower limits of the estimated value. The confidence interval is derived from a probabilistic distribution of the estimated error. In this paper, the estimated error data, $e(t) \in \mathbb{R}^n$ (where $w$ is the number of test data), is defined as the difference between the test data and the estimated data. The test dataset comprises data taken from the known dataset that are treated as missing data.

**Step 1:** Randomly extract $w$ samples, $E_1(t)$, $E_2(t)$, ..., $E_w(t)$, from the error dataset, $e_1(t)$, $e_2(t)$, ..., $e_w(t)$, while allowing repetition.

**Step 2:** Calculate the root mean square error, $E_{\text{RMSE}}$, from the randomly extracted data, $E_1(t)$, $E_2(t)$, ..., $E_w(t)$.

**Step 3:** Repeat Steps 1 and 2 $B$ times, where $B$ is a large number, in order to create $B$ samples, such as $E_{\text{RMSE}}^{(1)}(t)$, $E_{\text{RMSE}}^{(2)}(t)$, ..., $E_{\text{RMSE}}^{(B)}(t)$, and sort in ascending order.

**Step 4:** When the required confidence level is $1-\alpha$, the confidence interval is given as follows (percentile bootstrap confidence interval):

$$[C_{\text{upper}}(t), C_{\text{lower}}(t)]_{\text{RMSE}} = \left[ E_{\text{RMSE}}^{(\alpha/2)}(t), E_{\text{RMSE}}^{(1-\alpha/2)}(t) \right]$$

4. Experimental Spin-deployment Membrane Model

Figure 2 shows the experimental setup used in our evaluation.

The experiment was conducted in a vacuum chamber in order to eliminate the effect of air drag. The chamber used had an inner diameter of φ1800 mm and a height of 1000 mm. The pressure was reduced to $10^{-3}$ Pa. A motor was connected to a jig inside the vacuum chamber. The spin membrane model comprised a center hub, square membrane, and tethers, with the center hub set to rotate the motor at an arbitrary rotational frequency. The membrane was a 7.5 μm thick and 420 mm long square. Feature points were attached to the membrane, as shown in Fig. 3.

The deformation of the spinning membrane was measured using two sets of cameras, with the pixel coordinates of each feature point detected by two cameras. Consequently, the three-dimensional deformation could be reconstructed by using these pixel coordinate datasets and the internal/external camera parameter of the two sets of cameras. In this paper, the internal/external camera parameter is calculated using Zhang’s method,\(^6\) which can calculate the parameter from multiple sets (different angles) of a two-dimensional checkerboard pattern placed at approximately the same distance from the camera as the measurement objects. It does not require a large high-accuracy three-dimensional calibration structure. Thus, it is suitable for large space structure camera calibration.

The spinning rate of the experimental model was 3 Hz. The three-dimensional membrane shape was reconstructed for a second by the high speed camera. Fig. 4 and 5 show the images of the spin membrane acquired and the three-dimensional reconstruction results at 0.0 s and 1.0 s.
5. Results and Discussion

5.1. Numerical example

In this section, we present the numerical example used to evaluate the applicability of the empirical data driven model. First, we prepared two sets of numerical calculation datasets, dataset A and dataset B. The structural parameter of these calculation models was different. Second, the empirical model was derived from dataset B. Third, a spatio-temporal incomplete dataset was artificially created from dataset A. Finally, the spatio-temporal missing part of the incomplete data in dataset A was estimated by fusing the empirical model and the known part of dataset A.

Let us consider the spin-type sail membrane model modeled using the geometrically nonlinear finite element model (FEM) depicted in Fig. 6. The incomplete dataset is artificially created from the complete FEM numerical calculation by omitting some data. The black FEM nodes represent the missing FEM nodes that lack x, y, and z position data (Fig. 6). The calculation parameters of the models (datasets A and B) are listed in Table 1.

![Fig. 6. Square type spin-deployment membrane model.](image)

Table 1. Material and geometrical parameters.

<table>
<thead>
<tr>
<th>Item</th>
<th>Value (dataset A)</th>
<th>Value (dataset B)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Center hub</td>
<td>180.0 rad/sec</td>
<td>180.0 rad/sec</td>
</tr>
<tr>
<td>Initial Angular velocity</td>
<td>5.0 deg</td>
<td>5.0 deg</td>
</tr>
<tr>
<td>Initial nutation angle</td>
<td>(48.3, 50.7, 66.0) kgm²</td>
<td>(48.3, 50.7, 66.0) kgm²</td>
</tr>
<tr>
<td>Moment of inertia (I_{xx},I_{yy},I_{zz})</td>
<td>(48.3, 50.7, 66.0) kgm²</td>
<td>(48.3, 50.7, 66.0) kgm²</td>
</tr>
<tr>
<td>Young's modulus</td>
<td>3.0 GPa</td>
<td>3.0 GPa</td>
</tr>
<tr>
<td>Cross section</td>
<td>1.0x10^{-6} m</td>
<td>1.0x10^{-6} m</td>
</tr>
<tr>
<td>Mass density</td>
<td>1.42x10^{3} kg/m³</td>
<td>1.42x10^{3} kg/m³</td>
</tr>
<tr>
<td>Compressive stiffness coefficient</td>
<td>1.0x10^{-6} m</td>
<td>1.0x10^{-6} m</td>
</tr>
<tr>
<td>Membrane</td>
<td>0.5 kg</td>
<td>0.5 kg</td>
</tr>
<tr>
<td>Tip mass</td>
<td>0.5 kg</td>
<td>0.5 kg</td>
</tr>
</tbody>
</table>

![Fig. 7. Sequence of deformations during membrane deployment.](image)

Figure 7 illustrates the deformed shapes obtained for the conditions listed in Table 1. The empirical eigenvalues and corresponding top three empirical modes are shown in Fig. 8. The 1st and 2nd mode express the z-axis rotation and x, y-axis stretch of membrane. The 3rd mode mainly expresses the z-axis deflection. The x and y axis deflection are very small. In this calculation, the selected empirical mode number is 99.9% of the cumulative contribution ratio $\mu$, as expressed in Eq. (3). Thus, the empirical mode number $M$ is 38.

![Fig. 8. Empirical eigenspectrum.](image)

Figure 9 shows the x, y, and z axis positions at Node C (see Fig. 6). The solid red line represents the estimated solutions for the x, y, and z axis positions using the empirical model. The black dashed line represents the exact solutions for the x, y, and z axis positions. The gray area represents the confidence interval of the estimated data calculated using the test dataset.

The difference between dataset A and dataset B is the compressive stiffness coefficient $\alpha$. This model consists of a square membrane and a center hub. The membrane comprises four trapezium membranes connected by tethers. The square membrane is connected to the probe vehicle by the center tether. In addition, the tip of the square membrane is connected to the tip mass via the tip tether.
In this numerical example, 10% of the FEM node data is extracted to form the test dataset from the known FEM node data and treated as missing data in order to calculate the confidence interval. The bootstrap resampling number, $B$, is 2000, and the confidence level is set at $1-\alpha = 95\%$. The empirical model can approximate the exact solution by only superposition of 38 models. Further, the confidence interval covers the error between the estimated data and the exact data with sufficient accuracy using only a small test dataset.

Figure 10 shows the relation between the x axis position and the y axis position at the plane of section (Fig. 6). Fig. 11 shows the relation between the x axis position and the z axis position at the plane of section (Fig. 6). The solid red line represents the empirical data driven model, whereas the black dashed line represents the exact solution. It is clear that the empirical data driven model can express the exact shape.

Figure 12 shows the time variation of the root mean square error. The black line with the black circular marker represents the empirical data driven model. The root mean square error does not increase significantly as the time elapses. The blue line with the inverted triangular marker and the green line with the upright triangular marker represent the conventional FEM calculation. The root mean square errors increase gradually as the time elapses. In the case of the conventional numerical analysis, the root mean square error increases significantly when the parameter is marginally different. Conversely, it does not increase significantly when the parameter is different. The empirical data driven model is a more robust model for estimating membrane dynamics.

5.2. Vacuum chamber experiment

In this section, we examine the results of a vacuum chamber experiment conducted to evaluate the applicability of the empirical data driven model. First, we derived the empirical model from the spinning membrane FEM analysis, as shown in Fig. 13 and Table 2.

Second, we artificially created an incomplete dataset from the vacuum chamber experiment data by omitting some of the data. The black feature points represent the missing points that lack x, y, and z position data (Fig. 14).
Table 2. Calculation parameters of the square membrane model.

<table>
<thead>
<tr>
<th>Item</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Center hub</strong></td>
<td></td>
</tr>
<tr>
<td>Initial angular velocity</td>
<td>18.85 rad/s</td>
</tr>
<tr>
<td>Radius</td>
<td>0.0225 m</td>
</tr>
<tr>
<td>Initial nutation angle</td>
<td>1.803 deg</td>
</tr>
<tr>
<td>Moment of inertia ((Ixx, Iyy, Izz))</td>
<td>(3.02, 3.02, 4.28)x10^{-5} kgm²</td>
</tr>
<tr>
<td><strong>Cable</strong></td>
<td></td>
</tr>
<tr>
<td>Young’s modulus</td>
<td>3.0 GPa</td>
</tr>
<tr>
<td>Cross section</td>
<td>1.0x10^{-6} m</td>
</tr>
<tr>
<td>Mass density</td>
<td>1.42x10^{3} kg/m³</td>
</tr>
<tr>
<td>Compressive stiffness coefficient</td>
<td>1.0x10^{-4}</td>
</tr>
</tbody>
</table>

Figure 16 shows the z axis direction displacement at Nodes A, B, C, and D. The solid red line represents the displacement estimated using the empirical data driven model, and the dotted black line represents the exact displacement. Figure 17 compares the overall shape of the membrane. Minor differences exist, but the general movements and shape are very similar. This confirms that the proposed empirical data driven model is applicable for membrane space structure dynamics and shape estimation.

Finally, the complete dynamics were estimated using the empirical data driven model. In addition, the applicability of the empirical data driven model was evaluated. The eigenvalues of the empirical mode are shown in Fig. 15. In this calculation, the selected empirical model number was 99.999% of the cumulative contribution ratio $\mu$, as expressed in Eq. (3). Thus, the empirical mode number $M$ was 8.
6. Conclusion

This paper proposed an empirical data driven model as a new dynamics estimation model for sail membranes. We estimated the membrane deformation with a confidence interval using spatio-temporal missing measurement data and the empirical model derived from past experimental measurement and calculated numerical data. The proposed method makes it possible to estimate the membrane motion more precisely than the conventional method using simple numerical prediction.

The results of numerical and ground experiments confirm that the proposed method is applicable to membrane space structure dynamics estimation problems. In addition, we believe that the confidence interval can be used for appropriate empirical model selection. Further investigations will clarify this issue.

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References