Near-Earth Asteroid Deflection Mission Using Coulomb Force Attractor

By Kouhei YAMAGUCHI1) and Hiroshi YAMAKAWA1)

1)Research Institute for Sustainable Humanosphere, Kyoto University, Gokasho Uji City, Kyoto, Japan

(Received July 31st, 2015)

A method for controlling a Coulomb force attractor spacecraft in the vicinity of an asteroid is presented. A Coulomb force attractor tows and deflects an asteroid through a combination of mutual gravitational and Coulomb forces. We show asteroid deflection distances with time before impact and the required fuel consumption for efficient mission design with limited resources. By considering the asteroid and the spacecraft as a single body, motion is represented with the separation distance between the spacecraft and the asteroid and two Eulerian angles. We also investigate linearized dynamics and identify the stability requirements using the Routh–Hurwiz stability criterion. Numerical simulations are also performed and the feedback law to stabilize the position of the spacecraft is investigated. By investigating the interaction between the separation distance and Eulerian angles, we propose and evaluate a method for independently controlling each motion.

Key Words: Coulomb Force Attractor, Spacecraft Charging, Near-Earth Asteroid

1. Introduction

Astronomers and physicists have long warned of the potential threat posed by near-Earth asteroids (NEAs). As historical impact events like the Cretaceous mass extinction,1) the 1908 Tunguska event,2–4) and the 2003 Chelyabinsk event5) illustrate, an NEA collision with the Earth could be catastrophic. Though the probability of an impact resulting in severe ecological damage is low, we have to quantify the risk and prepare to deal with NEAs. To mitigate the hazard posed by NEAs, many techniques for deflecting asteroids away from an Earth-collision route have been proposed and investigated in some detail. Proposed methods include the gravity tractor (GT),6) which tows an asteroid using the small mutual gravitational force between an asteroid and the spacecraft, the kinetic impactor, low-thrust tugboats,7) mass drivers,8) solar collectors,9) and ion-beam shepherds.10) In this paper, we discuss the Coulomb force attractor (CFA), which tows the asteroid by means of the Coulomb force produced by artificially charging a spacecraft and the asteroid.

Figure 1 shows a conceptual diagram of a deflection mission with CFA. Introducing a Coulomb force to tow the asteroid is expected to more efficiently deflect asteroids using a spacecraft with smaller mass than is necessary for a GT. The Coulomb force acting on the asteroid and the spacecraft was estimated in a previous work.11) To investigate the deflection distance achieved with a CFA, we formulated the dynamics, proposed multiple CFA concepts, and performed numerical simulations.12) We concluded that a CFA can yield a greater deflection distance than can a simple GT. This paper proposes a method for controlling the position of the spacecraft in the vicinity of the asteroid. Natarajan and Schaub discussed the two-body Coulomb force tether problem by assuming two spacecraft as a slender near-rigid body.13) By introducing this concept, we analyze the linearized dynamics of a CFA–asteroid system. Through the analysis performed here, we introduce control laws to improve the efficiency of an asteroid deflection mission.

This paper is organized as follows. In section 2, we briefly explain our evaluation of the deflection distance. A new asteroid deflection chart based on fuel efficiency shows the importance of the deflection mission design with asteroid phasing consideration. Section 3 formulates the dynamics of a CFA spacecraft–asteroid system so that a deflection mission considering the phasing effect of the asteroid can be performed. Equations of motion formulated in the rotating reference frame are transformed into another form that contains the separation distance L as its variable. In addition, focusing on the dynamics in the vicinity of the asteroid’s centroid, the differential equations are linearized. Numerical simulations are performed and the results are provided in section 4. The required times, thrust forces, and interactions of each equation are investigated. Finally, section 5 gives a conclusion with a brief discussions of the results.

2. Asteroid Deflection Strategy

Asteroid deflection is sometimes characterized by the deflection distance on the b-plane, Δζ. The b-plane is oriented normal to the geocentric velocity of the target asteroid and gives a good estimation of the change in the asteroid orbit. The asteroid deflection formula, which allows us to calculate the Δζ that can be achieved with slow-push methods, such as GT and CFA, is formulated as

$$Δζ = \frac{3a}{m} v_{\perp} \sin \theta \int_{t_s}^{t_f} (t_s - \tau) v_{\ast} \cdot \frac{T(\tau)}{m_{\ast} + M_{\ast}} d\tau ,$$  

where \(a\) is the semi-major axis, \(m\) is the heliocentric gravitational constant, \(v_{\perp}\) is the magnitude of the heliocentric velocity.
1.4 0.4 1 0.6 1.8

viding this by $x = 0.5$ N. Note that to maintain generality, the $u$ where $f$ is the true anomaly of the asteroid. With Eq. (2), we can roughly investigate the efficiency of the concept. Figure 3 shows a new deflection chart for 99942 Apophis, indicating the required fuel to achieve a 100 km change in the distance between the asteroid and Earth in the $b$-plane. We used $T_{\text{max}} = 1$ N, $f_1 = 90$ deg., and $T_{\text{min}} = 0.1$, 0.3, and 0.5 N. Note that to maintain generality, the $x$-axis is modified to indicate fuel consumption $\times I_{\text{sp}}$ (specific impulse). Dividing this by $I_{\text{sp}}$ gives the amount of fuel consumption. The orbital parameters were semi-major axis $a = 0.922$ AU, eccentricity $e = 0.191$, inclination $i = 3.331$ deg., ascending node $\Omega_{\text{asc}} = 204.46$ deg., and node of perihelion $\omega_{\text{per}} = 126.39$ deg. We used $4.6 \times 10^{10}$ kg as the mass of Apophis. Note that in our simulation, $t_S \geq t_P$ is always achieved. The dashed black line in Fig. 3 indicates continuous 1 N deflection, corresponding to the

Fig. 2. Asteroid deflection considering phasing effect.

3. Formation Dynamics for One Spacecraft

This section formulates the dynamics of a CFA spacecraft in the vicinity of the centroid of its target asteroid. As described in the previous section, more efficient slow-push deflection can be performed by changing the towing force considering the phasing effect of the asteroid. To this end, an efficient way to control the separation distance $L$ is required. By introducing the Natarajan and Schaub method, the separation distance $L$, which accounts for the value of the towing force, becomes the variable of the equations of motion. Derived equations are linearized to allow analysis of the dynamics. We also give a brief explanation of the linearized asteroid–spacecraft system.

3.1. Differential equation of the separation distance $L$

To calculate the motion of the charged spacecraft around the asteroid, we introduce the reference frame $O(x, y, z)$, which rotates relative to the inertial frame $N$ shown in Fig. 4. In Fig. 4, the $x$-axis is along the direction opposite to the transverse direction, the $y$-axis is along the radial direction, and the $z$-axis is chosen to make the reference system right-handed. Note that the $O$ frame is fixed to the initial motion of the asteroid, and

$$T(\tau) = \begin{cases} T_{\text{max}} & (\cos f \geq \cos f_1) \\ T_{\text{min}} & (\cos f < \cos f_1) \end{cases},$$

Figure 3 indicates continuous 1 N deflection, corresponding to the example shown in Ref. 15). Here, we assume that the time to change the towing force $T$ is small compared with the mission time. Figure 3 shows that the fuel efficiency of the slow-push method can be improved by changing the towing force, which can be achieved by changing the separation distance between the asteroid and the spacecraft. The towing force achieved by a CFA is expressed by the following equation:

$$T(\tau) = G \frac{M_{\text{ast}} m_{\text{sc}}}{L^2} - q_{\text{sc}} \varphi_{\text{ast}} r_{\text{ast}} \frac{1}{L} \exp \left( - \frac{L - r_{\text{ast}}}{\lambda} \right) \left( \frac{1}{L} + \frac{1}{\lambda} \right)$$

(3)

Here, $G$ is the gravitational constant, $L$ is the distance between the asteroid and the spacecraft, $q_{\text{sc}}$ is the charge of the spacecraft, $\varphi_{\text{ast}}$ is the surface voltage of the asteroid, $r_{\text{ast}}$ is the diameter of the asteroid, and $\lambda$ is the effective shielding length of the asteroid. It is clear that we can change the magnitude of towing force by controlling the separation distance $L$. A more fuel-efficient deflection mission can thus be performed by choosing an appropriate $L(\tau)$.
the position and velocity of the asteroid are indicated with the subscript \( \text{ast} \). Three-body dynamics is used to investigate the CFA dynamics. The equations of motion, which are called the Clohessy–Wiltshire–Hill equations, are expressed as

\[
\begin{align*}
\ddot{x} &= \omega y + 2\omega \dot{y} + \omega^2 x - \frac{\mu}{R^3} x + E_{\text{grav}}(L)(x_{\text{ast}} - x) + T_x/m_{\text{sc}}, \\
\ddot{y} &= -\omega x + 2\omega \dot{x} + \omega^2 y - \frac{\mu}{R^3} y + E_{\text{grav}}(L)(y_{\text{ast}} - y), \\
\ddot{z} &= -\frac{\mu}{R^3} z + E_{\text{grav}}(L)(z_{\text{ast}} - z) + T_z/m_{\text{sc}}.
\end{align*}
\] (4)

where \( \omega \) is the orbital rate of the asteroid, \( R_1 \) is the distance between the sun and the asteroid, \( E_{\text{grav}}(L) \) indicates the mutual gravity and Coulomb acceleration, and \( T(T_x, T_y, T_z) \) is the control thrust force. As studied in Ref. 11, \( E_{\text{grav}}(L) \) can be expressed as

\[
E_{\text{grav}}(L) = G M_{\text{ast}} \frac{M_{\text{sun}}}{L^3} - \frac{q \kappa \kappa R_1^3}{L^2 m_{\text{sc}}^2} \exp\left(-\frac{L - r_{\text{ast}}}{\lambda} \right) \left( \frac{1}{L} + \frac{1}{\lambda} \right)
\] (5)

and separation distance \( L \) is given by

\[
L^2 = (x_{\text{ast}} - x)^2 + (y_{\text{ast}} - y)^2 + (z_{\text{ast}} - z)^2.
\] (6)

To simplify the subsequent analysis, we neglect the change in the asteroid orbit. Since we assume that the orbital control of the spacecraft is completed in a short time as compared with \( t_p \), it is reasonable to assume that \((x_{\text{ast}}, y_{\text{ast}}, z_{\text{ast}}) = (0, 0, 0)\). To adopt the Natarajan and Schaub scheme, we introduce the body-fixed frame \( B \) \((b_1, b_2, b_3)\), where \( b_1 \) is along the asteroid–spacecraft line. Let \( B \) and \( O \) be identical when the spacecraft is located on the \( x \)-axis. Because the \( B \) frame is fixed to the asteroid–spacecraft body, the spacecraft position in the \( B \) frame is always \((b_1, b_2, b_3) = (L, 0, 0)\). We also introduce the Euler angles \((\theta, \varphi)\) to represent the \( B \) frame attitude relative to the \( O \) frame (See Fig. 5). Note that we assume only one spacecraft towing the asteroid, so we can neglect rotation about the \( b_1 \)-axis (yaw). The rotation matrix \([BO]\), which relates \( O \) and \( B \), is

\[
[BO] = \begin{bmatrix}
\cos \theta \cos \varphi & \cos \theta \sin \varphi & -\sin \theta \\
-\sin \varphi & \cos \varphi & 0 \\
\sin \theta \cos \varphi & \sin \theta \sin \varphi & \cos \theta
\end{bmatrix}
\]

\[
\approx \begin{bmatrix}
1 & \varphi & -\theta \\
-\varphi & 1 & 0 \\
\theta & 0 & 1
\end{bmatrix}
\] (7)

Note that small-angle approximations are used for trigonometric functions. By using Eq. (7), we can obtain the linearized position and velocity in the \( O \) frame as

\[
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix} = [BO]^T \begin{bmatrix}
L \\
\varphi L \\
-\theta L
\end{bmatrix},
\] (8)

\[
\begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{z}
\end{bmatrix} = \begin{bmatrix}
L + \frac{T_x}{m_{\text{sc}}} + \frac{\varphi T_y}{m_{\text{sc}}} - \theta T_z \\
\varphi L + \varphi L \\
-\theta L - \theta L
\end{bmatrix}.
\] (9)

By substituting Eqs. (8) and (9) into the second derivative of Eq. (6), we get the linearized differential equation accounting for the change in \( L \):

\[
\ddot{L} = \omega^2 + 2\omega \omega - \frac{\mu}{R_1^3} - E_{\text{grav}}(L_0) + T_x/m_{\text{sc}} + \varphi T_y/m_{\text{sc}} - \theta T_z/m_{\text{sc}}.
\] (10)

The higher-order Euler-angle terms are neglected. Since small-angle approximations are performed, the \( x \)-axis thrust force \( T_x \) becomes a dominant term for \( L \) motion.

3.2. Attitude dynamics of asteroid–spacecraft system

Here, we derive the differential equations for \( \theta \) and \( \varphi \) to calculate the attitude of the \( B \) frame relative to the \( O \) frame. To this end, we start from Euler’s equation of motion, given as

\[
N = \frac{dH}{dt} + \omega \times H = [I] \omega + [I] \omega + \omega \times [I] \omega,
\] (11)

where \( N \) is the external torque vector, \( H \) is the angular momentum vector, \([I]\) is the inertia matrix, and \( \omega \) is the angular velocity vector. The inertia matrix is usually expressed as \([I] = -m_L[\dot{p}_1][\dot{p}_1] - m_L[\dot{p}_2][\dot{p}_2]\), where \([\dot{\cdot}]\) is equivalent to the cross-product operator and \([\dot{p}_1]\) is a position vector of the \( p_1 \) body relative to the centroid of the system. Because the mass of the asteroid is much greater than that of the spacecraft, the

![Asteroid and spacecraft diagram](image)
inertia matrix of the $B$ frame can be expressed in a relatively simple form as

$$^B[I] = -m_c \begin{bmatrix} 0 & 0 & 0 \\ 0 & -L^2 & 0 \\ 0 & 0 & -L^2 \end{bmatrix}. \quad (12)$$

The external torque vector $^B\mathbf{N}$ can be written as the sum of the gravity gradient torque $^B\mathbf{L}_G$ and the thruster torque $^B\mathbf{L}_T$. These can be written as

$$^B\mathbf{L}_G \approx \frac{3GM_{sun}}{R_c^2} \begin{bmatrix} 0 \\ 0 \\ \varphi I \end{bmatrix}, \quad (13)$$

and

$$^B\mathbf{L}_T \approx \begin{bmatrix} 0 \\ -L(\delta T_x + T_x) \\ L(-\varphi T_x + T_y) \end{bmatrix}. \quad (14)$$

where $R_c$ is the distance between the sun and the spacecraft and $I = m_c L^2$. Because $R_c$ is on the order of $10^{11}$ m, the magnitude of $^B\mathbf{L}_G$ is typically very small in comparison with that of $^B\mathbf{L}_T$. Using the preceding equations, Eq. (11) can be written as

$$^B\mathbf{L}_G + ^B\mathbf{L}_T = ^B[I] \beta \omega_{BN} + ^B[I] \phi \omega_{BN} + ^B[I] \beta \omega_{BN} + ^B[I] \phi \omega_{BN}. \quad (15)$$

The subscript $B/N$ denotes a parameter of the $B$ frame relative to the inertia frame $N$. The orbital rate transformed from inertial frame $N$ to $B$ frame $\omega_{BN}$ in Eq. (14) is given in Ref. 13).

Finally, we can obtain the differential equations as

$$\ddot{\vartheta} + \frac{\dot{\vartheta}}{I} \vartheta + \omega^2 \vartheta = -L \frac{\delta T_x + T_x}{I}, \quad (16)$$

and

$$\ddot{\varphi} + \frac{\dot{\varphi}}{I} \varphi + \frac{\dot{\varphi}}{I} \omega = \frac{3GM_{sun}}{R_c} \varphi + L \frac{\delta T_x}{I} (-\varphi T_x + T_y). \quad (17)$$

A linearized form of these differential equations of the attitude will be developed later as the differential equation of the separation distance $L$.

### 3.3. Static formation analysis

Here, we provide an example of linear analysis, noting the following special case. Since we assume that the position of the spacecraft is controlled over a very short time, the change in the asteroid orbit is neglected. We assume that the spacecraft tows the asteroid suspended at the artificial equilibria of $L = 0$, $\vartheta = 0$, and $\varphi = 0$, and $L = L_0$. These are given as

$$L = L_0 + \delta L,$$

$$T = (T_x, T_y, T_z) + (\delta T_x, \delta T_y, \delta T_z). \quad (18)$$

Reference thrust force $T_0$ is determined by solving $\mathbf{x} = 0$, $\vartheta = 0$, and $L = L_0$. These are given as

$$T_x/\mu_{sc} = \frac{\mu_{sc}}{r_{sc}} L_0 - \omega^2 L_0 + E_{p0}(L_0) L_0 \quad (19)$$

By substituting Eq. (19) into the derived Eqs. (10), (16), and (17), neglecting the higher-order terms of $\delta L$, $\vartheta$, $\varphi$, and $\delta T$, the linearized equations of motion around the equilibria are given by

$$\delta L = \left\{ \begin{array}{l} \omega^2 - \frac{\mu}{R_1^3} + \frac{GM_{sun}}{L_0^2} \frac{q_g \varphi_{ast}}{\mu_{sc} \varphi_{ast}} \exp \left( -\frac{L_0 - r_{ast}}{\lambda} \right) \\
+ \frac{1}{L_0} \left( \frac{2}{L_0 + \frac{1}{\lambda}} + \frac{1}{\lambda} \left( \frac{1}{\lambda} + \frac{1}{L_0} \right) \right) \delta \varphi \\
+ \frac{\omega L_0 \varphi}{2} + \omega L_0 \varphi + \delta T_x \frac{m_{sc}}{L_0} \end{array} \right\} \varphi \quad (20)$$

and

$$\ddot{\vartheta} = -\frac{\mu}{R_1^3} + E_{p0}(L_0) \vartheta - \frac{\delta T_x}{m_{sc} L_0} \quad (21)$$

and

$$\ddot{\varphi} = -\frac{\omega}{L_0} \omega \delta L - \frac{\omega}{L_0} \omega \delta L \quad (22)$$

Note that we assume that $R_c \approx R_1$. It is clear that the $\theta$ accounting for out-of-plane motion of the $B$ frame is decoupled from $\delta L$ and $\varphi$ motion. Since Eq. (21) is a simple oscillator, in practice, $z$-axis control with $T_z$ is required. However, $\delta L$ and $\varphi$ motion interact with each other, and thus we can control both by introducing only $\delta T_x$ or $\delta T_y$. However, considering heliocentric orbital motion, $\omega$ and $\varphi$ are very small relative to geocentric motion of the spacecraft. Such differences will be investigated in the numerical simulation section.

### 4. Numerical Simulations

The linearized dynamics of the system is investigated through numerical simulations. A feedback control law is constructed, and its applicability is demonstrated with linearized and full nonlinear dynamics.

#### 4.1. Position control with $T_x$

Table 1 summarizes parameters of the target asteroid and the CFA spacecraft. These correspond to the values assumed in Ref. 12). For all simulations, differential equations are integrated at double precision by the fourth-order Runge–Kutta scheme.

For simplicity, we consider a circular heliocentric orbital motion of the asteroid. In this case, the asteroid has a constant orbital rate $\omega = \sqrt{\mu/R_1^3}$ ( fixed), and its derivative becomes zero. In addition, $R_1 = 1$ AU is used. Under such conditions, station-keeping with $T_0$ is performed first. Figure 6 shows the motion of the spacecraft generated with linearized and full nonlinear dynamics. Here, full nonlinear dynamics means the results produced by the original nonlinear equations in Ref. 12) with the control force sum $T = T_0 + \delta T$ inserted. Of course,
in this station-keeping mission, \( \delta T = 0 \) is always achieved. It is clear from Fig. 6 that linearized dynamics provides a very good approximation to the full nonlinear dynamics. By introducing \( T_0 \), the desired equilibrium is achieved and thus the transformed equations of motion can be used for the analysis. As described in the previous section, \( \delta L \) motion is coupled to \( \varphi \) motion. Hence, we introduce only the \( x \)-axis control force \( \delta T_x \) and we investigate the requirements to meet the stability conditions of the Routh–Hurwitz stability criterion. The small thrust \( \delta T_x \) is defined as the partial derivative control form

\[
\frac{\delta T_x}{m_{sc}} = \left[ -\frac{\mu}{R'_0^2} + \frac{2GM_{\text{sat}}}{L_0^3} - \frac{q_{\text{sat}}r_{\text{sat}}}{L_0^2m_{sc}} \exp\left(-\frac{L_0 - r_{\text{sat}}}{\lambda}\right) \right] \delta L + C_2 \delta \dot{L}.
\]

(23)

Then, Eqs. (20) and (22) are simplified as

\[
\delta \dot{L} = (\Omega^2 - C_1) \delta L + C_2 \delta \dot{L} + 2\Omega L_0 \varphi,
\]

(24)

\[
\dot{\varphi} = -\frac{2}{L_0} \Omega \delta \dot{L} + [3\Omega^2 - E_{\text{sat}}(L_0)] \varphi.
\]

(25)

Constant coefficients \( C_1 \) and \( C_2 \) are feedback gains introduced to relate the control thrust force to the changes in \( \delta L \) and \( \delta \dot{L} \). Coefficient \( C_2 \), which relates the derivative of the separation distance \( \delta L \) to \( \delta T_x \), is necessary to achieve stable closed-loop dynamics. Equations of motion independent from orbital rate \( \Omega \) are given by introducing the transformation

\[
\begin{align*}
\tau &= \Omega t, \\
\dot{\varphi} &= \Omega \frac{\delta \dot{L}}{\delta L} (\equiv \dot{\tau}'), \\
\ddot{\varphi} &= \Omega^2 \frac{\delta \ddot{L}}{\delta L} (\equiv \ddot{\tau}'').
\end{align*}
\]

(26)

This is proposed in Ref. 13 to avoid numerical issues caused by the small \( \Omega \) of low-Earth orbit (LEO), and is more important when considering the type of heliocentric motion assumed here. We also introduce non-dimensional feedback gains \( \tilde{C}_1 = C_1/\Omega^2 \) and \( \tilde{C}_2 = C_2/\Omega \). Finally, linearized equations of motion in terms of \( \delta L \) and \( \varphi \) are derived as

\[
\delta \dot{L}'' = (1 - \tilde{C}_1) \delta L + \tilde{C}_2 \delta \dot{L}' + 2L_0 \varphi',
\]

(27)

\[
\varphi'' = -\frac{2}{L_0} \delta L' + (3 - A) \varphi,
\]

(28)

where \( A = E_{\text{sat}}(L_0) \) normalized by \( \Omega^2 \). The natural frequencies of Eqs. (27) and (28) are \( \sqrt{\tilde{C}_1 - 1} \) and \( \sqrt{A - 3} \), respectively. Since the orbital rate \( \Omega \) is usually much smaller than that of the satellite at geosynchronous orbit (GEO) or LEO, \( A \) could be on the order of \( 10^5 \). This is the most significant difference with a Coulomb-tether formation flight around Earth, \( 13 \) and this will be seen in the analysis results.

Stability analysis of the asteroid–spacecraft system is performed using linearized differential equations. The characteristic equation can be derived by Laplace transformation of Eqs. (27) and (28), as

\[
s^4 - \tilde{C}_2 s^3 + (A + \tilde{C}_1)s^2 - \tilde{C}_2(A - 3)s + (A - 3)(\tilde{C}_1 - 1) = 0.
\]

(29)

Table 2 is the Routh table, which gives conditions ensuring stability of the system as \( C_2 < 0 \) and \( \tilde{C}_1 > 1 \). To relate these non-dimensional feedback gains \( \tilde{C}_1 \) and \( \tilde{C}_2 \), we introduce the following relations:

\[
\begin{align*}
\tilde{C}_1 &= A + n, \\
\tilde{C}_2 &= -\alpha \sqrt{\tilde{C}_1 - 1}.
\end{align*}
\]

(30)

When the gain \( \alpha = 2 \), Eq. (27) has only real negative solutions if the \( \varphi \) motion is neglected. This can occur under conditions where \( \delta L \) is quickly damped. If \( n = -2 \), the two natural frequencies become the same value. Since the gravity-gradient torque and orbital rate \( \Omega \) are much smaller than the condition assumed in Ref. 13, several values of \( n \) were tested to investigate the behavior of the motion.

When \( n = -2 \), the time to damp \( \delta L \) and \( \varphi \) goes up to several years. Since linearized solutions do not provide good approximation of the full nonlinear dynamics over such a long simulation time, the control law cannot be applied. Hence, \( n \) is determined with focusing on the time required for \( \delta L \) to be damped. The parameter \( n \) in Eq. (30) was determined on the basis of the
The value of $n$ was determined using this relation. Figure 7 summarizes the results for several values of $n$. Figure 7a) shows the time histories of $\delta L$, 7b) shows $\theta$, 7c) shows $\varphi$, and 7d) is a 2D-orbit diagram of the spacecraft. In that figure, dashed lines show linearized dynamics and continuous lines show full nonlinear dynamics. The initial conditions are $L_0 = 150$ m, $\delta L = 10$ m, $\theta = 0$, and $\varphi = 5.730$ deg. The completion time is 60 min. The damping constants of cases 1 to 4 are 60, 120, 300, and 600 s, respectively. As Fig. 7a) shows, $\delta L$ can be damped with $\delta T_x$. In the first two cases, linear solutions give good approximations of the full nonlinear dynamics calculated in the rotating frame $O(x, y, z)$. Differences between linearized and nonlinear dynamics increase with the damping constant, as can be seen in cases 3 and 4, but the approximations are still reasonable. Since the $\theta$ motion is decoupled from the other two, as Fig. 7b) shows, the initial value is preserved during the simulation for all cases. Figure 7a) shows the capability to control the motion of $\delta L$ with $\delta T_x$. However, $\varphi$ is still about 6.1 deg at the end of the simulation, as indicated in Fig. 7c). Hence, the position of the spacecraft cannot converge to the artificial equilibrium. This can be seen from the 2D-orbit diagram in Fig. 7d).

Figure 8 shows the time histories of the thrust force. As that figure shows, shorter damping times require greater thrust force.

![Graphs showing orbital control](image)

The completion time is 60 min. The damping constants of cases 1 to 4 are 60, 120, 300, and 600 s, respectively. As Fig. 7a) shows, $\delta L$ can be damped with $\delta T_x$. In the first two cases, linear solutions give good approximations of the full nonlinear dynamics calculated in the rotating frame $O(x, y, z)$. Differences between linearized and nonlinear dynamics increase with the damping constant, as can be seen in cases 3 and 4, but the approximations are still reasonable. Since the $\theta$ motion is decoupled from the other two, as Fig. 7b) shows, the initial value is preserved during the simulation for all cases. Figure 7a) shows the capability to control the motion of $\delta L$ with $\delta T_x$. However, $\varphi$ is still about 6.1 deg at the end of the simulation, as indicated in Fig. 7c). Hence, the position of the spacecraft cannot converge to the artificial equilibrium. This can be seen from the 2D-orbit diagram in Fig. 7d).

![Graphs showing thrust force](image)

The value of $n$ was determined using this relation. Figure 7 summarizes the results for several values of $n$. Figure 7a) shows the time histories of $\delta L$, 7b) shows $\theta$, 7c) shows $\varphi$, and 7d) is a 2D-orbit diagram of the spacecraft. In that figure, dashed lines show linearized dynamics and continuous lines show full nonlinear dynamics. The initial conditions are $L_0 = 150$ m, $\delta L = 10$ m, $\theta = 0$, and $\varphi = 5.730$ deg. The completion time is 60 min. The damping constants of cases 1 to 4 are 60, 120, 300, and 600 s, respectively. As Fig. 7a) shows, $\delta L$ can be damped with $\delta T_x$. In the first two cases, linear solutions give good approximations of the full nonlinear dynamics calculated in the rotating frame $O(x, y, z)$. Differences between linearized and nonlinear dynamics increase with the damping constant, as can be seen in cases 3 and 4, but the approximations are still reasonable. Since the $\theta$ motion is decoupled from the other two, as Fig. 7b) shows, the initial value is preserved during the simulation for all cases. Figure 7a) shows the capability to control the motion of $\delta L$ with $\delta T_x$. However, $\varphi$ is still about 6.1 deg at the end of the simulation, as indicated in Fig. 7c). Hence, the position of the spacecraft cannot converge to the artificial equilibrium. This can be seen from the 2D-orbit diagram in Fig. 7d).

Figure 8 shows the time histories of the thrust force. As that figure shows, shorter damping times require greater thrust force.

![Graphs showing thrust force](image)
the angle deviation \( \varphi \) and write it as

\[
\frac{\delta T_y}{L_{0\text{msc}}} = \left(-D_1 + E_{y\varphi}(L_0)\right) \varphi + D_2 \dot{\varphi},
\]

(32)

where \( D_1 \) and \( D_2 \) again denote the feedback gains. As described above, \( \delta T_y \) is related to only \( \varphi \) terms. We also assume the transformation of the variables shown as Eq. (26) and introduce the dimensionless gains \( \tilde{D}_1 = D_1/\Omega^2 \) and \( \tilde{D}_2 = D_2/\Omega \). We then have the following differential equation for \( \varphi \) motion:

\[
\varphi'' = (3 - \tilde{D}_1) \varphi + \tilde{D}_2 \dot{\varphi}.
\]

(33)

Note that by assuming that \( \delta L \) and \( \varphi \) are decoupled, the \( \delta L \) and \( \delta \varphi \) terms are negligible.

The conditions for feedback gains to stabilize the system, which can be calculated with the same scheme for \( \delta L \) motion, are \( \tilde{D}_1 < 0 \) and \( \tilde{D}_2 < 0 \). Eq (33) is critically damped when \( \tilde{D}_1 = \tilde{D}_2^2/4 + 3 \), because solutions of the characteristic equation are only negative real numbers. The value of the gain \( \tilde{D}_2 \) is also determined from the damping constant of \( \varphi \), as performed for \( \delta L \). The damping constant \( \tau_{\varphi\text{d}} \) is

\[
\tau_{\varphi\text{d}} = \frac{1}{\sqrt{\tilde{D}_1 - 3}}.
\]

(34)

For simplicity, we assume that the two damping constants are equal (i.e., that \( \tau_\phi = \tau_{\varphi\text{d}} \)). Figure 9 summarizes the results using four simulation cases. In that figure, continuous lines indicate the results of full nonlinear dynamics and dashed lines show linearized dynamics. The completion time is 60 min. Figure 9a) shows the time histories of \( \delta L \), 9b) shows \( \theta \), 9c) shows \( \varphi \), and 9d) is the 2D-orbit diagram of the spacecraft. As these figures show, the linearized dynamics provides a good approximation to the full nonlinear dynamics. Since the coupling of \( \delta L \) and \( \varphi \) motion is still weak, introducing a control force exerting directly on the \( \varphi \) direction is efficient. The initial value of \( \theta \) is preserved during the simulation, as in the results of the former simulation using \( \delta T_y \). The time histories of \( \varphi \) in Fig. 9c) show the advantage of introducing \( \delta T_y \); the angle offsets converge to zero, depending on the damping time \( \tau_{\varphi\text{d}} \). However, comparing Figs. 7a) and 9a), the correspondence with \( \delta L \) becomes poorer by introducing the control force \( \delta T_y \). This notable difference is caused by the nonlinear term, which is neglected through the linearization process. The neglected higher-order term of the control force \( \delta T_y \) in the differential equation of the separation distance is \( \delta T_y \phi \). This term is clearly smaller than the first-order term of \( \delta T_y \), but could be greater than the initial terms characterizing the angular rate \( \Omega \). For example, when the damping time is set to 60 s, the term \( \delta Y_{y\varphi} \) is about \( 10^5 \) times the first-order term \( 2\Omega L_0 \phi \dot{\phi} \). The angular offset \( \phi_0 \) can be converged to zero by introducing the control force \( \delta T_y \). However, higher-order terms of the control force \( \delta T_y \) could degrade the approximation of the change in the separation distance \( \delta L \). Differences between the linear and nonlinear dynamics for the angle \( \varphi \) are also caused by the higher-order term. The higher-order term \( \frac{1}{2} \delta T_y \phi \dot{\phi} \) also degrades the approximation of the angle \( \varphi \). As cases using only \( \delta T_y \), these errors are also validated with the orbit diagram. Figure 9d) shows the orbits of spacecraft as provided by full nonlinear and linearized dynamics. Compared with the case controlled with only \( \delta T_y \) force (Fig. 7d)), the distances between the artificial equilibrium and spacecraft at the end of the simulation are clearly improved. Figure 9d) also shows that linearized dynamics still provides good approximation of the full nonlinear.
dynamics. Though Fig. 9 suggests degradation of the approximation to nonlinear dynamics proposed with linearized dynamics, in practice, introducing $\delta T_y$ improves the position control of the spacecraft. The position errors shown in Fig. 7d) are around 1 m and the accuracy of the control is enough. To implement the control law for the mission, the linearized dynamics would be recomputed.

Figure 10 shows the required thrust force $\delta T_y$ for cases 1 to 4. In that figure, a control force pointing in the negative $y$-direction is required to damp $\varphi$. The magnitude of the required thrust increases as the damping constant $\tau_d$ becomes smaller. Because the final position of the spacecraft does not strongly depend on $\tau_{d1}$, as shown in Fig. 9d), we can design feedback gains while considering the amount of fuel that can be used to reposition the spacecraft.

![Time histories of the thrust along the y-axis.](image)

**5. Conclusion**

We investigated the dynamics of a Coulomb force attractor spacecraft in the vicinity of the centroid of an asteroid, and briefly explained the importance of designing an asteroid deflection mission with a slow-push method considering the phasing effect. Focusing on the distance between the asteroid and the spacecraft, the equations of motion were transformed to a form containing the separation distance between the spacecraft and the asteroid as the variable. By linearizing the derived differential equations around the artificial equilibrium, a feedback law was constructed. The law was demonstrated and its capability investigated through numerical simulations. The linearized solutions provided good approximations to the nonlinear dynamics, and the separation distance could be controlled with the law. However, since the origin of the reference frame rotates with the heliocentric orbital motion of the asteroid, two Euler angles representing the attitude of the asteroid–spacecraft system could not be controlled without introducing additional thrust forces. This fact could be considered as the decoupling of each motion, and was investigated by introducing additional control force. The results showed that though the correspondence of linearized and nonlinear dynamics is degraded, each motion could be controlled independently.

**Acknowledgements**

This study was supported by JSPS KAKENHI Grant Number 15J08268.

**References**