Launched and Captures of the Space “Frisbees”:
Dynamics Modelling and Simulation

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The current paper presents a concept of the non-conventional method of handling/transfer of the space objects/payloads, using single and multiple cooperative robotic manipulators with extremely flexible arms. Practical cases may involve launch only (as in the case of the ejection of the spacecraft), capture only (as in the case of the capture of the dysfunctional spacecraft or debris) or launch and capture, combined together (as in the case of a payload transfer from one platform to another). This method is seen as having numerous advantages over the traditional methods, employing propulsive systems on the payloads. These include: simplicity, reliability, re-usability, ability to handle passive payloads, and small power requirements. In this study, various operational scenarios of the launches and captures of the payloads are considered and designed in detail. In particular, we demonstrate the feasibility of the launch of the outgoing rotating payloads (called “frisbees”) using a highly elastic robotic arm, capable of transferring its potential energy of the pre-deformed shape into the kinetic energy of the payload. Using the co-rotational FEM, we firstly simulate pre-launch phases, coiling the elastic elements in different shapes, for example, “U”, “S” or even more complex shapes, and then propose and simulate the scenarios of ejecting the payloads utilising the kinetic energy of the elastic members (playing role of catapults). We also demonstrate the feasibility of the ejection of the payloads with required dynamic parameters, using cooperative robotic manipulator arms, performing coordinated throwing manoeuvres. This is done in compliance with the kinematics of the system and dynamics laws. The cases of the ejection of the payload are simulated using fully non-linear formulation, employing so-called co-rotational FEM, which enables to deal with large deformations, large rotations and large translations of the simulated elastic robotic arms. Numerical simulation allows to observe, to analyse and to suppress the transient strains and stresses in the flexible arms. In the cases of the payload capture, the co-rotational FEM method is used to simulate the process of de-spin, slowing and stopping the spinning object. This is also supplemented with the analysis of the strains and stresses on the members of the robotic arm manipulators. Various study cases are illustrated with the animations of the representative cases in Virtual Reality.

Key Words: “Frisbee”; Spinning Payload; Launch/Capture; Co-Rotational Finite Element Method; Robotic Arm Manipulator

1. Introduction

Designers of aerospace systems and structures are forced by various constraints to reduce overall weight and improve performance. One of the biggest challenges they are faced is to deal with inherent flexibility of this equipment and non-linear nature of the created systems. Dynamic modelling of these systems is quite complex, as their motion is highly nonlinear and conventional linear techniques do not work. A co-rotational finite element method has been seen as a powerful engineering tool to efficiently simulate non-linear systems.

The co-rotational finite element method utilises multiple co-ordinate systems (a co-rotational co-ordinate system which rotates and translates with each element, and a global co-ordinate system) to cope with the geometric non-linearity present in either very flexible, or rotating systems. It was first proposed by Belytschko,1) and has since been the subject of many refinements, improvements and extensions.2–11) Initially the method was applied to the 2D systems, but then it was extended to 3D cases and specific applications.12–19) However, there is little evidence of the application of the co-rotational FEM to problems involving reconfigurable robotic systems, and the current pa-
per aims to contribute to this important area of research. It is based on the developed by the authors capability of modelling of flexible reconfigurable robotics, where it is possible to simulate manoeuvres involving connection to and disconnection from other manipulators or fixed points within the manipulators environment, as well as failure of the system due to high stresses; and aircraft structures. In all cases, strains and bending moments can be monitored throughout the simulation.

2. Overview of the Co-Rotational Finite Element Method

A co-rotational finite element method is used to model the structures discussed here. The technique is suited to modeling geometrically nonlinear structures which experience large deformations (static or dynamic cases) and/or large rotations and translations (dynamic cases). In most cases presented here, deformations are relatively small, and the nonlinearity present is largely due to rotation of the models.

The characteristic of the technique, which allows modelling of this type of motion, is the fact that each element within the system is assigned a unique set of co-ordinates, which is allowed to rotate and translate with the element. All deformations and therefore strains within the element are described relative to the co-rotational co-ordinates, and providing element sizes are chosen appropriately, can be said to be linear relative to this co-ordinate system. Nodal forces due to deformation are therefore evaluated relative to these co-ordinates. System velocities and accelerations, and thus nodal forces proportional to these, are described relative to a global co-ordinate system. This leads to a non-standard form of the system’s equation of motion,

\[ M \ddot{\mathbf{x}} + C \dot{\mathbf{x}} + \mathbf{f} = \mathbf{F}; \quad \mathbf{f} = Q^T \mathbf{K} \ddot{\mathbf{x}}, \]

where \( M, C \) and \( K \) are the mass, damping and stiffness matrices respectively, \( \mathbf{f} \) is the nodal force vector due to deformation, \( Q \) is a transformation matrix, the superscript \( i \) represents elemental quantities, and an overbar (’) represents quantities expressed relative to the co-rotational co-ordinates. The quantity \( \mathbf{f} \) is evaluated relative to the co-rotational co-ordinates for each element, then transformed to the global co-ordinates for inclusion in the global deformation force vector. Rotation of the co-rotational co-ordinates, however, also requires that mass and stiffness matrices be transformed from co-rotational to global co-ordinates at each time step. Proportional damping is used to evaluate \( C \), where \( C = \alpha M + \beta K \), and \( \alpha \) and \( \beta \) are chosen to be 0.01 and 0.05 respectively.

Strains are evaluated using an expression for bending strain,

\[ \varepsilon_{xy} = -y \frac{\partial^2 v}{\partial x^2}, \]

where \( \varepsilon_{xy} \) is the strain, \( y \) is the distance from the neutral axis of the beam element at which the strain is to be evaluated, \( v \) is the deformation of the element in the direction of the \( y \) axis of the co-rotational co-ordinates and \( x \) represents distance along the \( x \) axis of the co-rotational co-ordinates.

The equations of motion are solved at each time step using the Newton-Raphson iterative procedure and Newmark integration. All models consist of two-dimensional, two-noded, six degree-of-freedom, linear beam elements, which simplifies the modelling procedure and significantly reduces the computational cost of performing analyses.

2.1. Implementation of the Co-rotational FEM

The co-rotational finite element method implemented here follows closely that outlined by Elkaranshawy & Dokainish. Traditional finite element methods are capable of modelling structures in which it can be assumed that deformations are relatively small (i.e., the system can be modelled linearly). A co-rotational formulation however, allows modelling of a geometrically non-linear structure, such as a beam experiencing large deformations, or undergoing rotation.

This is achieved by using multiple coordinate systems to describe the position/motion of the beam. Figure 1 shows the different coordinate systems, where \((X, Y)\) represents the global coordinates, and \((\bar{x}, \bar{y})\) are the co-rotational coordinates which can rotate and translate with each element. The co-rotational coordinates are utilised to describe the deformations of each element, such that relative to these axes, small deformations can be assumed and traditional mass and stiffness matrices for an element can be used. The global coordinates are used to describe overall positions, velocities and accelerations of the system. The non-linearity present in the system is introduced via rotation of the system matrices from the co-rotational to global coordinates.

The equations of motion for the system are developed from Lagrange’s equations, which can be represented as follows,

\[ \frac{d}{dt} \left( \frac{\partial T^i}{\partial \dot{q}^j} \right) - \frac{\partial T^i}{\partial q^j} + \frac{\partial U^j}{\partial \dot{q}^i} = \mathbf{F}^i, \]

where \( T \) is the system’s kinetic energy, \( U \) is the strain or potential energy and \( \mathbf{d} \) and its derivatives with respect to time \( t \) represent the nodal displacements, velocities and accelerations of the system. \( \mathbf{F} \) is the nodal force vector applied to the system, and the superscript \( i \) represents the \( i^{th} \) element of the system.

First, the relevant terms for strain energy must be evaluated,

\[ U^i = \frac{1}{2} \int_{V^i} \sigma^j \epsilon^j \, dV^i, \]

where \( V^i \), \( \sigma^j \) and \( \epsilon^j \) represent the elemental volume, stress and strain, respectively. Similarly, the kinetic energy can be found from,

\[ T^i = \frac{1}{2} \int_{V^i} \rho^i \bar{v}^j \bar{v}^j \, dV^i, \]
where \( \rho \) is the elemental density, and \( \mathbf{f} \) is the velocity of a point in the element.

It is then possible through lengthy manipulation involving evaluation of Lagrange’s equations to establish the elemental equations of motion for the system in global coordinates as,

\[
M' \ddot{\mathbf{d}}' + \mathbf{f}' = \mathbf{F}'.
\]  

(6)

In this case, \( M' \) represents the elemental mass matrix, and can be found from,

\[
M' = Q^T M Q',
\]

(7)

where \( Q' \) is a transformation matrix,

\[
\bar{M}' = \int_V \rho N^T \bar{N}^T \mathbf{d} V'
\]

and \( \mathbf{f}' \) is the internal force vector and is equal to

\[
\mathbf{f}' = Q^T \mathbf{K}' \bar{\mathbf{d}}',
\]

(9)

where \( \mathbf{K}' = Q^T K Q' \) and \( \bar{\mathbf{d}}' \) is the elemental deformation and external nodal force vector.

The elemental equations of motion are then expanded with the inclusion of proportional damping

\[
C = \alpha M + \delta \mathbf{K},
\]

(11)

giving the final form of the equations of motion as,

\[
M \ddot{\mathbf{d}} + C \dot{\mathbf{d}} + \mathbf{f} = \mathbf{F}.
\]

(12)

In numerical methods, this equation can be further rewritten and interpreted in terms of the inertial nodal force vector \( \mathbf{F}' \), nodal force vector \( \mathbf{F}' \) due to damping, nodal force vector \( \mathbf{F}' \) due to deformation and external nodal force vector \( \mathbf{P} \), requiring the numerical procedures during the solution process to force the residual nodal force vector \( \mathbf{F}' \) to be less than a specified tolerance:

\[
\mathbf{F}' = \mathbf{F}' + \mathbf{F}' - \mathbf{P} = \mathbf{F}' = 0.
\]

(13)

2.2. Strain analysis

For the planar beam elements chosen to model the reconfigurable manipulator, the axial strain relationship utilised in Eq. (4) is as follows,

\[
\varepsilon_\alpha = \frac{\partial u}{\partial x} + \frac{1}{2} \left( \frac{\partial u}{\partial x} \right)^2 - y \frac{\partial^2 v}{\partial x^2},
\]

(14)

where \( u \) and \( v \) are the deformations of a point along the beam. This expression for strain incorporates terms for pure axial, membrane and bending strains respectively. This is also, therefore, the expression used to calculate the strain within a manipulator at any point in the simulation process.

In order to evaluate this expression, it is first necessary to calculate the derivatives of the deformations with respect to the \( x \) axis (in this case, \( x \) represents the position along the co-rotational axis \( \bar{x} \). This is done through the multiplication of the derivatives of the relevant shape functions by the current nodal coordinates relative to the co-rotational axes. The displacement and slope of any point along an element can be expressed as,

\[
\begin{bmatrix}
\frac{\partial u}{\partial x} \\
\frac{\partial v}{\partial x} \\
\frac{\partial^2 v}{\partial x^2}
\end{bmatrix} = N'(x) \cdot \bar{d}'.
\]

(15)

where \( N'(x) \) represents the shape functions for the chosen element (in this case a planar beam element), and \( \bar{d}' \) are the nodal displacements of the current element.

The shape functions \( N'(x) \) can be seen below.

\[
N' = \begin{bmatrix}
N_1 & 0 & 0 & N_4 & 0 & 0 \\
0 & N_2 & N_3 & 0 & N_5 & N_6
\end{bmatrix}
\]

(16)

where \( \zeta = x' / l' \) and \( l' \) is the length of the element.

It is, therefore, possible to differentiate Eq. (15) giving,

\[
\begin{bmatrix}
\frac{\partial u}{\partial x} \\
\frac{\partial v}{\partial x} \\
\frac{\partial^2 v}{\partial x^2}
\end{bmatrix} = \frac{\partial N'(x)}{\partial \bar{d}} \cdot \bar{d}'.
\]

(17)

It can be seen that the relevant terms for evaluating Eq. (14) are available in Eq. (17), and therefore, by substituting in appropriate values of \( x \), the strain at any location along the current element, and therefore at any location along the length of the manipulator can be evaluated. A value of \( -1 \) is chosen for \( y \) in Eq. (14), meaning that the strain calculated is indicative of that which may be seen on the lower surface of the manipulator.

2.3. Reconfigurable finite element technique

Reconfiguration of the manipulator was achieved by making changes to the degrees of freedom within the finite element model over time. For instance, in the case of a separation manoeuvre or failure of the system, the beam is effectively broken at the prescribed location by the addition of extra degrees of freedom at that location, in order to make independent the motion of the previously joined ends of the now separated beams. An example of this alteration of the degrees of freedom can be seen for a simple two element system in Fig. 2.

Connection to another manipulator was achieved by eliminating the required degrees of freedom once the end of one manipulator came into close proximity with the end of another. This process is effectively the opposite of the separation procedure outlined above.

In cases where it was necessary to simulate connection to or disconnection from a particular location, this was achieved by altering the boundary conditions of the system at the required
time. For instance, fixing the degrees of freedom at the end of a beam to a particular location enables connection to its current location, whilst eliminating these boundary conditions provides the ability to disconnect that end of the manipulator.

### 2.4. Multiple bodies

Multiple bodies within the system are joined by a ball-joint type hinge. This hinge is modelled in the same manner as proposed for a two-dimensional or planar system in Refs. 20,26,27. This involves the two bodies sharing a node at their connection point, meaning the position of the connected ends of the two bodies must be the same. However, additional rotational degrees-of-freedom are added to the node, such that the orientation of the two bodies can be different, allowing them to rotate independently. Figure 3 shows a depiction of this procedure for a two-dimensional system.

![Method of implementing hinges](image)

**Fig. 3.** Method of implementing hinges.

### 2.5. Solution of equations of motion

To solve the equations of motion the Newton-Raphson iterative method and Newmark direct integration are used.4, 17, 28) This combination of methods is commonly used for the solution of the equations of motion for a system having non-linear characteristics, and therefore requiring the use of an iterative procedure in conjunction with an integration method. To perform a simulation, the solution procedure is performed at each timestep.

The procedure described here is used for solving the equations of motion developed using either two- or three-dimensional co-rotational formulations.

The first step in the solution process, is to perform an initial estimate of the system vectors representing change in position, \( \Delta D \), velocity, \( \dot{D} \), and acceleration, \( \ddot{D} \).

When calculating the position, velocity and acceleration of the system at each timestep, it is assumed that the system is in dynamic equilibrium at end of the previous timestep (at time \( t \)), and that the nodal velocities and angular velocities, \( \dot{D} \), and accelerations and angular accelerations, \( \ddot{D} \), at that time are known.

An initial estimate for \( \Delta D \) (the change in positions and rotations) is also made, which generally contains zeros, apart from at those degrees of freedom where a displacement is prescribed by the imposed boundary conditions. These boundary conditions can be time variant or constant.

At the first iteration for the current timestep, Newmark integration is used to make a prediction of the nodal velocities and accelerations at the end of the current timestep (at time \( t + \Delta t \)), based on the initial estimate for \( \Delta D \), along with \( \dot{D} \) and \( \ddot{D} \) at the end of the previous timestep. Dynamic equilibrium is deemed to have been achieved if the residual force vector in Eq. (13) is less than a specified tolerance.

If \( t \) represents the end of the previous timestep, and \( t + \Delta t \) represents the end of the current timestep, we can state that,

\[
\Delta D^0_{t+\Delta t} = \frac{\Delta D^0_{t+\Delta t}}{\beta \Delta t^2} - \frac{\Delta D_t}{\beta \Delta t} - \left[ \frac{1}{2} \right] \ddot{D}_t;
\]

\[
\Delta D^0_{t+\Delta t} = \dot{D}_t + \Delta t \left( 1 - \gamma \right) \ddot{D}_t + \gamma \dot{D}^0_{t+\Delta t},
\]

where the superscript 0 represents the iteration number, and \( \beta \) and \( \gamma \) are parameters which provide control over the stability and accuracy of the Newmark integration procedure. In all cases presented here, values of \( \beta = 0.25 \) and \( \gamma = 0.5 \) are used, as proposed for this type of application by Hsiao & Jang,5) Elkaranshawy & Dokainish6) and Hsiao et al.7) amongst others. According to Cook,25) this combination of \( \beta \) and \( \gamma \) represent an implicit form of the Newmark method. These values were proven to be quite robust in almost all situations encountered, and have also been utilised when using a co-rotational finite element method to model other elastic systems, such as the flexible wing of a manoeuvring aircraft.25)

Using \( \Delta D \), whose only non-zero components at the first iteration are the prescribed displacements, it is then possible to update the co-rotational co-ordinates in order to perform evaluation of the current nodal forces, and to check for equilibrium.

### 2.6. Residual force vector

After the co-rotational co-ordinates have been updated, it is necessary to calculate the residual force vector based on the equations of motion shown in Eq. (13).

This involves firstly generating mass and stiffness matrices for each element and transforming them to the global coordinate system such that they can be assembled into the system matrices. The damping matrix can then be calculated based on the system mass and stiffness matrices as per Eq. (11).

It is then possible to determine the required system nodal force vectors, which allows determination of the residual nodal force vector, \( \mathbf{F}^R \) using Eq. (13) - the residual or unbalanced force vector, the magnitude of which will be less than a specified tolerance, or approximately equal to zero, when the system is in dynamic equilibrium. If the system is not in equilibrium, the residual force vector is then used to determine an update to the change in position vector \( \Delta D \).

Using the Newton-Raphson iterative method, an updated change in position vector \( \Delta D \) can be determined from,

\[
\Delta D_{t+\Delta t}^{n+1} = \Delta D_{t+\Delta t}^n + \mathbf{R}; \quad \mathbf{R} = -\mathbf{K}_c^{-1} \mathbf{F}^R \tag{18}
\]
where \( K_T \) is the tangent stiffness matrix and is equal to,

\[
K_T = K + \frac{\gamma}{\beta\Delta} C + \frac{1}{\beta\Delta^2} M. \tag{19}
\]

The current estimates for \( D \) and \( \dot{D} \) can then be updated, again using Newmark integration as below,

\[
\dot{D}^{n+1} = \dot{D}^n + \frac{\gamma}{\beta\Delta} \Delta D^{n+1}, \tag{20}
\]

\[
\ddot{D}^{n+1} = \ddot{D}^n + \frac{\gamma}{\beta\Delta} \Delta \ddot{D}^{n+1}. \tag{21}
\]

At this point, the co-rotational co-ordinates can be updated based on the new estimate of \( \Delta D \), and the procedure repeated until convergence is deemed to have been achieved. In this case, following Hsiao & co-workers,\(^{17}\) the equation,

\[
\left| F^p \right| / \sqrt{N} \leq e_{tol} \tag{22}
\]

is deemed to be a sufficiently close approach to convergence, where \( N \) is the number of degrees in the freedom of the system, and \( e_{tol} \) is a specified tolerance. A value of \( e_{tol} = 1 \times 10^{-4} \) is used in cases presented here.

3. Study Cases: Launch & Capture of "Frisbee" Payloads

3.1. Technique-1: super-elastic single arm catapult

We propose to utilise high flexibility elastic elements for payload ejection. To test the capabilities of the co-rotational method and to ensure that it could be the right numerical tool for the simulation of the space ejection systems, we consider two supplementary testing cases of deforming the elastic beams, which are not deformed at the beginning of the numerical experiment, but for which the slowly increased commanded clamped angles at the ends are applied. One end of the beam is prevented from \( x \) and \( y \) displacements, but the second is allowed to move in the \( x \)-direction only (the direction of the initially undeformed and undistorted beam). The co-rotational FEM performed well not only for the stage of deformation of the elastic elements, but worked extremely well for more complex extension of the experiment with the self-ejection of the elastic beams. The characteristic snap-shots of the simulations in virtual reality are presented in Fig. 4.

At this stage of the conceptual design process, being not bound with the particular specification parameters, for the illustrative example, we select the length of the elastic robotic arm (called also her as "catapult") equal to 4.8 m, with its bending stiffness \( EJ = 154,688 \text{ Nm}^2 \). We also consider the case of the ejection of the "frisbee" payload having 525 kg mass and 1.2 m in diameter.

The super-flexible robotic arm has been deformed for 7 seconds to form an arc. As an example, the kinematic command angle was applied to the root of the robotic arm, with its gradual increase from 0 to 180 degrees. During this phase of the process, the end of the robotic arm, attached to the payload, was constrained from the motion in \( y \) direction. The payload was allowed to slide in the horizontal direction, as shown in Fig. 5, gradually reaching its commanded horizontal displacement of 4 m at \( t = 7 \text{ s} \). Then the system was left undisturbed for 1 second for any excited oscillations to settle, and at the instant \( t_{e1} = 8 \text{ s} \), the payload was unconstrained, still being connected via a hinge to the tip of the catapult. This was sufficient to trigger the process of transfer of the part of the potential energy of the deformed catapult into the kinetic energy of the payload to be ejected. At the time \( t_{e2} = 8.25 \text{ s} \), while in motion, the payload was disconnected from the hinge, connecting it to the catapult, and from this moment it was completely free from any constraints. Note that before ejection, the payload was initially constrained at two points (attaching it to the base and to the robotic arm). Therefore, there were two consecutive "releases" of the payload at these two points: firstly the payload was disconnected from the base at \( t_{e1} \) (this triggered transfer of the potential energy of the elastic arm into the kinetic energy of the arm and the payload); then the payload was disconnected from the "catapult" robotic arm at \( t_{e2} \) (this enabled the payload to fly as frisbee). In the presented example, the simulation has been completed for 10 seconds (Fig. 6).

A 3D plot of the time history of the strains in the systems is presented in Fig. 7. It shows that the maximum level of strains in the beam at the pre-launch deformation stage was at the order of 1.5%, with a rapid increase up to 2.7%, maintained for a very short period of time.

The analysis of the results of the simulation case enables to conclude, that it was possible to eject the payload with the outgoing velocity of \( t_{e1} = 21.3 \text{ m/s} \) and rotational angular velocity \( \omega = 18.3 \text{ rad/s} \), which corresponds to almost 3 angular rotations of the "frisbee" per second.

For comparison, a series of additional experiments were run, in which the time of ejection of the "frisbee" was set to other values, including \( t_{e2} = 8.1 \text{ s} \), \( t_{e2} = 8.2 \text{ s} \), and \( t_{e2} = 8.3 \text{ s} \). As it can be seen from Fig. 8, the outgoing direction, velocity of the payload flight, and the parameters of the rotational motion are strongly depending upon the time of the ejection of the payload. For example, in this series of tests, case \( t_{e2} = 8.3 \text{ s} \) is characterised with the lowest angular velocities of the "frisbee". Obviously, the disconnection time dictates the way of interaction between the catapult and the payload. In the cases of very late disconnection the spin and speed of the "frisbee" are dropping and in the exaggerated case the payload may not be allowed to fly.

3.2. Technique-2: ejection and capture of the "Frisbee" 
payload, using two cooperative two-link robotic arms

The author\(^{24}\) has reported about the use of two cooperative robotic arm manipulators for launching payloads. The modelling process involved the following critical stages, illustrated with Fig. 9: (1) The system has been initially modelled as a straight beam with the following segments joined together: end of the Robot-1 link-1 was joined to the Robot-1 link-2 (denoted as point B); then the end of the Robot-1 link-2 was joined to
"frisbee" (point C); then the opposite side of "frisbee" (point D) was joined to the end of the Robot-2 link-2; at last, the start-

ing end of the Robot-2 link-2 was joined to the Robot-2 link-1. Note, that all segments were provided with their individ-

Fig. 5. Pre-launch stage: the super-elastic robotic arm is deformed ($t = 0 - 8 \text{ s}$).

Fig. 6. Launch stage: "frisbee" payload is unclamped from the base, and from $t = 8.25 \text{ s}$ is completely unconstrained.
with the desired outgoing speed and rotational angular velocity, in the cited paper,\textsuperscript{24} this was achieved by firstly specifying a particular scenario for the distance $d_e$ traveled by the mass center of the payload and its rotational angle $\theta$. As an illustration example, a scenario described with Eqs. (23-24) has been considered.

\begin{equation}
\theta_e = \theta_0 + (t/t_e)^2 (\theta_1 - \theta_0); \quad (23)
\end{equation}

\begin{equation}
d_e = d_{e0} + (t/t_e)^2 (d_{e1} - d_{e0}). \quad (24)
\end{equation}

For $d_{e0} = \theta_0 = 0$, this scenario ensures launch of the resting payload, as it features its zero initial angular velocity, in addition to zero initial velocity of the mass center of the payload.

Careful selection of the ejection values of $d_e$ and $\theta_e$ enabled us to achieve the desired outgoing linear and rotational velocities of the payload. In particular, for the study case we selected $t_f = 1.1$ s, $d_{e0} = 2.1$ m and $\theta_e = \pi$.

Using the selected ejection history for $d_e(t)$ and $\theta(t)$, presented with Eqs. (23-24), two inverse kinematic tasks for two cooperative robotic arms were solved, resulting in the required time histories for the angles of the robotic arms, their angular velocities and accelerations, required by the co-rotational method. In order to implement the intended ejection scenario, these angles must be kinematically controlled by the motors of the robotic arms. Fifty intermediate positions of both cooperative robotic arms are shown in Fig. 9.

The time history of the robotic link’s position angles, their angular velocities and angular accelerations are presented in the original publication\textsuperscript{24} and results of the simulation of a representative case of the controlled ejection of the payload is shown in virtual reality in Fig. 10. In the study case, the diameter of the $m = 300$ kg payload and the length of each link are all equal to 2 m ($L_1 = L_2 = L_{1R} = L_{2R} = 2$ m). Also, $EI_1 = EI_{1R} = 469,330$ Nm$^2$; $EI_2 = EI_{2R} = 91,670$ Nm$^2$.

Developing this concept further, it is proposed in this paper to revert the process of ejection for simulation of the capture of the payloads. With this approach, we take the linear and angular velocities of the "frisbee" in Fig. 10 and assign them as the parameters of the ingoing "frisbee", to be captured by the cooperative robotic arms. We also assume the same control angles to the robotic arms, but applied in retrospective, i.e. backwards in time. This scenario is then simulated using the co-rotational FEM package, enabling calculation of the dynamic response of the robotic arm after capture.

Reverted characteristics of the cooperative robotic arms, capturing the "frisbee" are presented in Figs. 11 and 12. The capture case was successfully simulated. The animated snap-shots for the capturing case looked very similar to the ejection case images in Fig. 10, providing that they were displayed in the reverse order. The strains in the system were also determined. Their analysis showed that despite of the local disturbances, the capture process was rather smooth, with the maximum strains raising up to 0.84%.

\subsection{3.3 Brief discussion of the ejection techniques}

The payload ejection Technique-1 has a few significant advantages, including the simplicity of the used ejection system, which has only one elastic link. However, the most appealing advantage (over many other ejection methods, including Technique-2), is its low power requirements. Indeed, the catapult system does not need to produce substantial power instantly.
during the ejection, instead it utilises the substantial potential energy of the deformed elastic robotic arm (catapult), accumulated at the pre-launch stage, which can be completed during long time. This consideration may be essential for space applications, where the solar batteries are often used as a power supply. Saying this, the Technique-1 has a few associated challenges in achieving the following required performance parameters of the ejected payload: (1) outgoing velocity $v_c$, (2) directional angle of flight $\theta_c$ and (3) spin angular velocity $\omega$. To achieve the desired values of these, in case of the single flexible arm catapult, the variation of following design parameters can be used: attitude of the base, stiffness of the arm, pre-launch initial deformed shape of the arm and attitude of the payload, time $t_{e1}$ of the release from the base, time $t_{e2}$ of the release from the robotic arm. Unfortunately, the change in most of these design parameters would lead to simultaneous change in the performance parameters $v_c$, $\theta_c$ and $\omega$. Their appropriate combination to achieve the mission objectives can be determined via computer simulations. This paper, in particular, demonstrated that the time $t_{e2}$ of the release of the payload has significant impact on the angular spin and directions of the frisbee. This can be used to select appropriate $t_{e2}$ to get the required spin, and then variation of the attitude of the base may be needed to also achieve the desired direction. This coupled influence of the design parameters should be specially considered and treated in a similar way, as coupling between yaw, pitch and roll in the aircraft dynamics.

In this regard, as demonstrated in the paper, the ejection of the payload with two cooperative robotic arms, despite of the complexity of the system (where two robots involved), this method possesses more flexibility in the independent control of the $v_c$, $\theta_c$ and $\omega$. It can be achieved via control of the angular positions of each of the two links of each of the two arm systems, requiring, however, substantial immediate power during the launch.

4. Main Results and Conclusions

This paper proposes, considers and substantiates the use super-flexible robotic manipulators for the deployment, capture and handling of payloads in space. It illustrates that their capability for large elastic deformations can be very useful and can be used, for example, to build-up the potential energy via application of bending loading to the elastic members of the robots, which then can be subsequently transferred into the kinetic energy of the ejected payloads, providing them with the required outgoing velocity and rotational spin. In a similar way, the high energy absorption capacity of the elastic members can be utilised for capturing of the on-going payloads. The complexity of these and other related tasks is not only in the large rotations and translations of the systems’ elements, but also in the reconfigurable nature of various practical scenarios of operation of the robotic systems. A procedure for non-linear modelling of single and multiple super-flexible flexible reconfigurable ma-
nipulators has been proposed and successfully implemented. It is based on the co-rotational finite element method, capable of modelling systems, undergoing large rotations and translations and experiencing large deformations as a result of their inherent flexibility.

We were able to formulate and solve these highly non-linear problems numerically, without the need to apply popular linearized models and multiple assumptions, which would inevitably lead to the loss of accuracy. Solving the tasks in their fully non-linear formulation widens the range of the application of these systems and adds on to the confidence and realism of the simulation methods.

The main illustration examples, which were successfully completed using the co-rotational FEM, include launch of the payloads with the desired high speed and rotation spin. This has been implemented using two different techniques: (1) using one single super-flexible robotic arm, which can be highly deformed and play a role of the launching catapult; (2) using two controlled cooperative robotic arms (each having two flexible links).

Fig. 11. Angular displacements $\theta_{1 R}$, angular velocities $\dot{\theta}_{1 R}$ and angular accelerations $\ddot{\theta}_{1 R}$ ($i = 1, 2$) for Robot-1 (of two cooperative robotic arms), ensuring controllable capture of “frisbee”.

Fig. 12. Angular displacements $\theta_{i}$, angular velocities $\dot{\theta}_{i}$ and angular accelerations $\ddot{\theta}_{i}$ ($i = 1, 2$) for Robot-2 (of two cooperative robotic arms), ensuring controllable capture of “frisbee”.
For the super-flexible catapult it has been quantitatively demonstrated that the launching parameters are significantly influenced by the instant of ejection (complete disconnection from the catapult).

For the cooperative robotic systems it has been quantitatively demonstrated that the system enables to regulate and achieve the desired parameters for the "frisbee" and possesses high accuracy.

The method is quite generic and enables to plan, simulate and execute various practical scenarios of the operation of the robotic manipulators with super-flexible arm members, making the application of these systems more feasible.

In addition to the numerical implementation of the co-rotational numerical implementation of the co-rotational FEM, capable of modelling of the reconfigurable systems with variable degrees-of-freedom and variable boundary conditions, a set of special interactive computer programs was developed to animate and represent results in Virtual Reality, full of impressive colorful graphics and navigation features.

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References