Escape Trajectories from Sun–Earth Distant Retrograde Orbits

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The purpose of this work was to develop escape trajectories from distant retrograde orbits in the Sun–Earth circular restricted three-body problem. Previous studies have suggested installing a space port outside the gravitational sphere of influence of Earth for future deep space exploration. This paper proposes the placement of a space port on a Sun–Earth distant retrograde orbit (SE-DRO), which is stable over a long period. The characteristics and fuel consumption efficiency of escape trajectories from the space port on the SE-DRO using one or two impulses $\Delta V$ were investigated and compared with those obtained in previous studies that considered a space port located in the vicinity of the Sun–Earth L2 Lagrangian point. The connection of the escape trajectories from an SE-DRO with Earth gravity-assist (EGA) trajectories was then investigated based on the results of the analysis with the impulse $\Delta V$. Finally, a series of trajectories from SE-DRO to EGA were shown to have a high efficiency from the perspective of both $\Delta V$ and the flight time.

Key words: Circular Restricted Three-Body Problem, Lagrange Point, Distant Retrograde Orbit

Nomenclature

- $m_E$: mass of Earth
- $m_S$: mass of the Sun
- $C_j$: Jacobi integral
- $\lambda$: eigenvalue
- $\alpha$: angle of the position on the DRO at which an impulse is applied (clockwise)
- $\phi$: angle of the escape velocity
- $\xi$: angle of the escape position
- $V_\infty$: velocity at infinity
- $\nu$: efficiency of escaping from the DRO
- $\Delta V$: impulse applied to a spacecraft in an orbit
- $\Delta V_{\text{esc1,2}}$: first and second impulses applied in the escape trajectory from the DRO
- $\Delta V_{\text{tot}} = \Delta V_{\text{esc1}} + \Delta V_{\text{esc2}}$: total impulse in two-impulse escape trajectory from the DRO
- $\Delta V_{\text{DVEGA1,2}}$: first and second impulses applied in $\Delta V$ Earth gravity-assist trajectory
- $\rho$: efficiency of Earth gravity-assist
- $u$: input of electric $\Delta V$ Earth gravity-assist trajectory
- $t_f$: flight time of electric $\Delta V$ Earth gravity-assist trajectory

1. Introduction

Currently, deep space exploration missions are enabled by electric propulsion systems that can achieve a high specific impulse. It is expected that deep space exploration missions, such as the exploration of planets and asteroids, will be conducted more frequently. Studies on such missions conducted by the Japan Aerospace Exploration Agency (JAXA) have used a “space port” located outside Earth’s gravitational sphere of influence.1,2 A space port acts as a fueling station for spacecraft (S/Cs). An S/C launched from Earth approaches a space port using chemical propulsion and changes the propulsion system from chemical to electric at the space port. Subsequently, the S/C enters deep space with an electric low-thrust propulsion system. Generally, a chemical propulsion system is useful within Earth’s gravitational sphere of influence because it can generate a high thrust with a low specific impulse. In contrast, an electric propulsion system is suitable for interplanetary flight because it can continuously generate low thrust with little fuel consumption. Therefore, the use of a space port is expected to significantly increase the efficiency of deep space exploration in terms of energy and cost.

Some previous studies have considered a space port located in the vicinity of the Sun–Earth L2 Lagrangian point (SEL2).3) SEL2 is one of five dynamical equilibrium points in the Sun–Earth gravitational field and is advantageous in that its relative position with respect to Earth is fixed. Because SEL2 is an unstable equilibrium point, active and highly accurate orbit control is necessary to maintain a station there (i.e., to maintain the position of S/Cs at SEL2).

This paper proposes the placement of a space port in a Sun–Earth distant retrograde orbit (SE-DRO). DROs arise as solutions to the circular restricted three-body problem (CR3BP) and are stable over long periods. DROs have been classified in the family $f$ of simple periodic orbits.4) These include the possibility that they be useful for certain applications, such as optical telescopes, radio imaging of the Sun, and warning systems for potentially hazardous asteroids approaching Earth.5,6) Because DROs usually have Lyapunov stability, station maintenance does not require highly accurate orbit control. However, the position of the space port changes relative to Earth.

Although several studies on the transition from Earth to SE-DROs have been conducted,5–8) escape trajectories from
these orbits have been studied little. In this study, escape trajectories from SE-DROs with one and two applied impulses $\Delta V$ were analyzed. The characteristics of the trajectories and the fuel consumption efficiency to obtain the necessary velocity at the boundary of Earth’s gravitational sphere of influence were investigated by conducting a parametric analysis assuming a space port existing on the SE-DRO. The connection between the escape trajectories and the $\Delta V$ Earth gravity-assist (DVEGA) trajectories were then examined. DVEGA trajectories can obtain high relative velocities to the Earth by increasing the eccentricity and accumulating orbit energy on the trajectories returning to Earth. DVEGA allows the S/C to achieve not only high efficiency but also various angles for the direction of the escape velocity. This is achieved by changing the angle of deflection during the swing-by phase. Additionally, electric $\Delta V$ Earth gravity-assist (EDVEGA), which is DVEGA using a low-thrust propulsion system, was analyzed. The feasibility of a practical mission to explore deep space using a space port was determined by investigating EDVEGA trajectories.

Section 2 describes the basic equations of motion of the CR3BP and confirms the stability of the DRO. The stability was evaluated using the eigenvalues of a monodromy matrix and compared with unstable and horizontal Lyapunov orbits. Section 3 describes one-impulse escape trajectories from the DRO. The efficiency of reaching the boundary of Earth’s gravitational sphere of influence was investigated from the viewpoint of the velocity at the infinity boundary $V_\infty$, the flight time, and the angle of escape. Section 4 shows the results of two-impulse DRO escape trajectories. After an explanation of the application of the second impulse, a comparison of the velocity at the infinity boundary $V_\infty$, the flight time, and the angle of escape with those in the one-impulse case is presented. In Section 5, the connection of DRO escape trajectories to DVEGA and EDVEGA trajectories is examined. The boundary conditions at the point connecting the DRO escape trajectories with the DVEGA and EDVEGA trajectories is described. The characteristics of a series of trajectories from the DRO to EGA trajectories were then evaluated. Section 6 concludes the paper.

2. Circular Restricted Three-Body Problem

2.1. Equations of motion

The dimensionless equations of motion for the CR3BP in the ecliptic plane are given as

$$\ddot{x} - 2 \dot{y} - x = \frac{1}{r_1^2} (x+\eta) - \frac{\eta}{r_2^2} (x-1+\eta)$$  
(1)

$$\ddot{y} + 2 \dot{x} - y = \frac{1}{r_1^2} y - \frac{\eta}{r_2^2} y$$  
(2)

where all masses, distances, and times are nondimensionalized by dividing them by the reference values of $m_1 + m_5$, 1 AU, and the orbital period of Earth divided by $2\pi$, $r_1$, $r_2$, and $\eta$ are given by

$$r_1 = \sqrt{(x+\eta)^2 + y^2}, \quad r_2 = \sqrt{(x-1+\eta)^2 + y^2}$$  
(3)

with $\eta = \frac{m_3}{m_1 + m_5}$.  
(4)

The motion of the S/C is described in a rotating frame ($x$, $y$, $z$) whose origin is the center of mass of the Sun–Earth system. The dynamics of the CR3BP give rise to an integral of motion called the Jacobi constant $C_j$, which is given by

$$C_j = \frac{1}{2} (x^2 + y^2) - \frac{1}{2} (x+\eta)^2 - \frac{1-\eta}{r_1} - \frac{\eta}{r_2}.$$  
(5)

DROs are one type of periodic orbit that satisfies these equations. In several conventional works, it has been considered that the target space port is on a periodic orbit around a Lagrangian point. The periodic orbits around a Lagrangian point are called horizontal Lyapunov orbits (HLOs). Figures 1(a) and (b) shows some DROs around Earth and some HLOs around SEL2, respectively. Earth and the Lagrangian points L1 and L2 are plotted as black dots in Fig. 1.

Fig. 1. Examples of (a) DROs around Earth and (b) HLOs around SEL2.

2.2. Stability

The stability of periodic orbits can be determined by analyzing the eigenvalues of the monodromy matrix of the orbit. The monodromy matrix is the state transition matrix for one period around the periodic orbit. The characteristic equation of the planar CR3BP has four eigenvalues that are related as

$$\lambda_2 = \frac{1}{\lambda_1}, \quad \lambda_4 = \frac{1}{\lambda_3}.$$  
(6)

If the magnitude of any of these eigenvalues is greater than one, the periodic orbit is unstable, and the state vector error from the initial orbit diverges. As a result of this condition and the inverse relationships between the two pairs of eigenvalues shown in Eq. (6), all magnitudes of these eigenvalues in a DRO must equal one. This indicates that DROs have Lyapunov stability and the state vector error does not diverge. The stability of DROs is compared to that of HLOs in Fig. 2. The horizontal axis represents the distance from the Sun crossing the $x$-axis, and the vertical axis represents the maximum magnitude of the monodromy matrix eigenvalues. A large $x$ means that the size of periodic orbit is large. Figure 2 shows that some DROs have Lyapunov stability and that the depicted HLOs are unstable.
3. One-impulse escape trajectories from a Sun–Earth distant retrograde orbit

In this section, the escape trajectories from an SE-DRO with one impulse $\Delta V$ are analyzed. The considered DRO crosses SEL2, making it the smallest possible DRO, and the characteristics and efficiencies of various escape trajectories were examined. The efficiency $\nu$ was calculated as the ratio of $\Delta V$ to the obtained velocity at the infinity boundary. The escape from a state of rest at SEL2 has already been determined. The DRO escape trajectories were compared to those from the SEL2.

3.1. Calculation conditions

Figure 3 shows the method used to define the departure velocity, which is tangential to the DRO. The conditions of the numerical calculation were as follows.

1) The boundary of Earth’s gravitational sphere of influence is defined as a distance of 3 million km from Earth. (The corresponding dimensionless value is 0.02.)

2) The S/C cannot fly at an altitude of less than 200 km from the surface of Earth.

Condition 1 represents the boundary where the gravitational force of Earth is 100 times smaller than that of the Sun. Earth’s gravity is negligible outside of this region. Condition 2 ensures the S/C does not collide with Earth.

3.2. Results of one-impulse escape trajectories

The parametric analysis results based on the magnitude of the departure velocity $\Delta V$ and departure position $\alpha$ of the S/C are shown in Figs. 4–6. And some cases of escape trajectories are shown in Fig. 7. The velocity at the infinity boundary $V_\infty$ is shown in Fig. 4. A negative $\Delta V$ means the S/C decreases its velocity on the SE-DRO. For $\Delta V = 0–20$ m/s, escape from the DRO is not possible, meaning periodicity around Earth is maintained, because DROs are stable orbits and require an impulse of $\Delta V > 20$ m/s to escape from the DRO. The white regions in Fig. 4 represent conditions under which the S/C collides with Earth or transfers to a complicated orbit that remains inside Earth’s gravitational sphere of influence for at least 20 years. For $\Delta V < 200$ m/s, an S/C departing at $\alpha = 0^\circ$ or $180^\circ$ can obtain larger $V_\infty$ than at $\alpha = 90$ or 270 deg. In Fig. 7(a) and (b), the obtained $V_\infty$ is 1190 m/s if the S/C departure angle is $\alpha = 0$ deg with $\Delta V = 100$ m/s, whereas $V_\infty = 762$ m/s for $\alpha = 90$ deg.

Figure 5 shows the flight time from the departure point on the DRO to the arrival at a point on the boundary of Earth’s gravitational sphere of influence. Solutions requiring more than 1500 days were excluded from the present analysis. It was found that small impulses $\Delta V$ require a very long flight time for the S/C to escape (e.g., Fig. 7(c)). Some of the small-impulse trajectories revolving several times around Earth could attain a high $V_\infty$ even if the S/C lost energy because its velocity decreased (e.g., Fig. 7(d)). However, this is not advantageous in comparison with accelerating with an equivalent amount of $\Delta V$.

Figure 6 shows a map of the escape velocity angle $\phi$ of the escape velocity for one-impulse trajectories. The angle $\phi$ of the escape velocity and that $\xi$ of the escape position are defined in Fig. 8. Figure 8 demonstrates that the S/C can escape with various escape velocity angles given an impulse of $\Delta V > 0$ m/s because any departure position can be selected on the DRO. $\phi$ is sensitive to changes in $\Delta V$ for $-100$ m/s $< \Delta V < 0$ m/s, unlike in the case where $\Delta V > 0$ m/s. The reason $\phi$ is sensitive to small differences in $\Delta V$ of approximately 10 m/s is that such changes cause the S/C to transfer to complicated trajectories that revolve several times around Earth (e.g., Fig. 7(c)).

Figure 9 shows the efficiencies $\nu = V_\infty / \Delta V$ of escaping from a DRO and from a state of rest at SEL2 at a maximum $\Delta V$ of 1500 m/s. The efficiency $\nu$ of escaping from a DRO is much higher than that of escaping from SEL2 for almost all $V_\infty$ except $V_\infty = 600–700$ m/s. Escaping from a state of rest at SEL2, the S/C can achieve a small $V_\infty$ given a negligible $\Delta V$ because SEL2 is an unstable equilibrium point. However, to achieve $V_\infty > 700$ m/s, escaping from a DRO is more reasonable than from SEL2 because the S/C can take advantage of the high energy of the DRO.
Fig. 4. Map of velocity at infinity for one-impulse trajectories.

Fig. 5. Map of flight time for one-impulse trajectories.

Fig. 6. Map of angle of the escape velocity for one-impulse trajectories.

Fig. 7. Escape trajectories with one impulse $\Delta V$.

Fig. 8. Definitions of the angle $\phi$ of the escape velocity and that $\xi$ of the escape position.

Fig. 9. Efficiency of escaping from a DRO and from SEL2.
4. Two-impulse Escape Trajectories from Sun–Earth Distant Retrograde Orbit

This section discusses the examination of two-impulse escape trajectories. Because Eq. (5) indicates that the Jacobian increases in proportion to the square of the velocity, an impulse should be applied to the S/C at periapsis to efficiently increase the Jacobian of the escape trajectories (Fig. 10). Thus, the second impulse $\Delta V_{esc2}$ was applied at the point at which the S/C was closest to Earth.

**Fig. 10.** Method of escaping from SE-DRO with two impulses $\Delta V_{esc1}$ and $\Delta V_{esc2}$.

**4.1. Condition of calculation**

After the first impulse $\Delta V_{esc1}$ was applied in the direction opposite the S/C velocity on the DRO and the spacecraft was made to approach Earth, the second impulse $\Delta V_{esc2}$ was applied in the direction tangential to the S/C path at the position where the S/C most closely approached Earth for the first time. The escape trajectories considered in this study had total impulses $\Delta V_{tot} = \Delta V_{esc1} + \Delta V_{esc2}$ ranging from 100 to 1000 m/s at 100 m/s increments. The same conditions as in Section 3 were used to investigate these trajectories.

**4.2. Results of two-impulse escape trajectories**

The $\Delta V_{tot}$ and flight time required to satisfy several given values of $V_{\infty}$ are plotted against the angle $\phi$ of the escape velocity in Fig. 11. The blue points represent the solution of the two-impulse trajectory and the minimum $\Delta V_{tot}$ in the solutions that satisfy a given $V_{\infty}$. The red points represent the solution of the one-impulse trajectory, and the empty red points indicate that the one-impulse trajectory requires a larger $\Delta V$ than the two-impulse trajectory. As $V_{\infty}$ increased, the two-impulse case became increasingly beneficial in comparison with the one-impulse case regarding the required impulse. For $V_{\infty} > 2000$ m/s, the two-impulse trajectory was more efficient for all $\phi$, as shown in Fig. 10. A $V_{\infty}$ of more than 4000 m/s could be attained by applying a total impulse $\Delta V_{tot}$ of less than 1000 m/s.

In Fig. 12, $V_{\infty}$ is plotted against $\phi$ and $\xi$ in a three-dimensional plot at $\Delta V_{tot} = 600$ m/s. The red and blue plots represent one- and two-impulse trajectories. Figure 12 shows that changing the ratio of the two impulses can extend the solution space. It should be noted that the ranges of the angle $\xi$ of the escape position and that $\phi$ of the escape velocity are extended in the two-impulse escape in comparison with those in the one-impulse case.

To connect the escape trajectories that extend from the SE-DRO to the boundary of Earth’s gravitational sphere of influence ($3 \times 10^9$ km) to interplanetary trajectories beyond this boundary, the positions and velocities on the two trajectories must be equal at the connection point. If the escape trajectories inside Earth’s gravitational sphere of influence are connected to DVEGA trajectories, it is generally known that the spacecraft must escape by traveling in the $\pm x$-direction in the Earth-fixed frame. Both the position and velocity on the boundary of Earth’s gravitational sphere of influence must be in the $\pm x$-direction ($\phi = \xi = 0, 180, 360$ deg).

The next section discusses the analysis of the connection to DVEGA trajectories. Therefore, two-impulse trajectories from a DRO not only achieve a high $V_{\infty}$ but also facilitate the connection of escape trajectories to interplanetary trajectories.
Fig. 12. Velocity at the infinity boundary plotted against the angles of the escape velocity and position for one- and two-impulse trajectories with $\Delta V_{\text{tot}} = 600\, \text{m/s}$.

5. Analysis of Connection to Earth Gravity-Assist Trajectories

In this section, DVEGA and EDVEGA trajectories were connected to the two-impulse escape trajectories obtained in the previous sections. The velocity at the infinity boundary $V_\infty$ for the DRO that has been discussed thus far was redefined as $V_{\text{Dep}}$ to distinguish it from the velocity at the infinity boundary relative to Earth at the end of the DVEGA trajectory, which is denoted $V_{\text{Arr}}$. The efficiency of the Earth gravity-assist (EGA) is denoted $\rho$. After the connection was analyzed, the efficiencies $\rho$ of a series of trajectories from their departure from a DRO to the end of DVEGA and EDVEGA trajectories were investigated. Finally, the characteristics, such as the efficiency $\rho$ and the flight time, of the escape trajectory from the DRO were compared with those of the escape trajectory from SEL2 including the connections to DVEGA and EDVEGA.

5.1. Connection to $\Delta V$ Earth gravity-assist

This subsection explains how escape trajectories from DROs are connected to DVEGA trajectories and presents the derivation of the efficiency $\rho$ of a DVEGA trajectory. The method of connecting the DRO and DVEGA escape trajectories is as follows.

1) Obtain the first $\Delta V_{\text{DVEGA1}}$ and second $\Delta V_{\text{DVEGA2}}$ impulses applied on the DRO escape trajectory with respect to $V_{\text{Arr}}$ that maximize $\rho$ by varying the flight time in Lambert’s problem.

2) Obtain the boundary conditions $(V_{\text{Dep}}, \phi)$ and $\xi$ at the point of intersection between the DVEGA trajectory and the boundary of Earth’s gravitational sphere of influence.

3) Obtain the initial conditions $(\alpha, \Delta V_{\text{esc1}}, \Delta V_{\text{esc2}})$ for the DRO escape trajectory that satisfy the boundary conditions required to connect it to the DVEGA trajectory using the differential correction method.

Generally, to attain the ideal DVEGA trajectory, the first impulse $\Delta V_{\text{DVEGA1}}$ is applied to the S/C in the direction of Earth’s revolution, and the second $\Delta V_{\text{DVEGA2}}$ at the apoapsis in the direction opposite Earth’s revolution, as shown in Fig. 13(a). This DVEGA trajectory is called the outbound trajectory, and the case in which the $\Delta V$ directions are opposite those described for the outbound trajectory is called the inbound trajectory. Given $\Delta V_{\text{DVEGA1}}$, $\Delta V_{\text{DVEGA2}}$ can be obtained by solving Lambert’s problem to encounter Earth considering only the Sun and S/C gravities.

In step 1, optimized DVEGA trajectories, which maximize $\rho$, are derived for various values of $V_{\text{Arr}}$. $\rho$ can be expressed as

$$\rho = \frac{V_{\text{Arr}}}{\Delta V_{\text{DVEGA1}} + \Delta V_{\text{DVEGA2}}}.$$  

(7)

The value of $\rho$ depends on the flight time of the DVEGA trajectory. Figure 14(a) shows $\rho$, which has a peak of approximately 2, plotted against the flight time. The value of $\Delta V_{\text{DVEGA2}}$ that maximizes $\rho$ for each $\Delta V_{\text{DVEGA1}}$ was selected; these values are plotted as the blue line in Fig. 14(a). These sets of $\Delta V_{\text{DVEGA1}}$ and $\Delta V_{\text{DVEGA2}}$ values can realize ideal DVEGA trajectories. $V_{\text{Arr}}$ is plotted as the red line in Fig. 14(b). Some $\Delta V_{\text{DVEGA1}}$ and $\Delta V_{\text{DVEGA2}}$ values are given in the upper part of Table 1 for target DVEGA velocities at the infinity velocities of $V_{\text{Arr}} = 3000, 4000, 5000$, and $6000\, \text{m/s}$.

In step 2, the boundary conditions required for connecting the DRO escape trajectories and the DVEGA trajectories obtained in step 1 are derived. When the DVEGA trajectories obtained in step 1 are drawn in the Sun–Earth-fixed frame, the point of intersection of the DVEGA trajectories and the boundary of Earth’s gravitational sphere of influence indicates the boundary conditions, as shown in Fig. 13(b). The middle of Table 1 gives three parameters $(V_{\text{Dep}}, \phi, \xi)$ for each considered $V_{\text{Arr}}$. The escape trajectories from the DRO must be designed to satisfy these values on the infinity boundary.

Finally, in step 3, the initial conditions of the escape trajectories are derived. When the initial conditions are defined as $\phi_0$, $\Delta V_{\text{esc10}}$, and $\Delta V_{\text{esc20}}$, the escape trajectories are propagated, and the final state on the infinity boundary can be described by the parameters $V_{\text{Dep0}}$, $\phi_0$, and $\xi_0$. The errors $\epsilon_{\phi, \Delta V_{\text{esc1}}}$, $\epsilon_{\phi}$, and $\epsilon_\xi$ between the required and actual parameters are given by

$$\epsilon_{\phi} = V_{\text{Dep0}} - V_{\text{Dep}},$$

$$\epsilon_\phi = \phi_0 - \phi,$$

$$\epsilon_\xi = \xi_0 - \xi.$$  

(8)

These errors are functions of variables $\alpha$, $\Delta V_{\text{esc01}}$, and $\Delta V_{\text{esc02}}$ because $V_{\text{Dep}}$, $\phi$ and $\xi$ can be changed depending on $\alpha$, $\Delta V_{\text{esc01}}$, $\Delta V_{\text{esc02}}$. Solving the following nonlinear simultaneous equations forms a connection between the DRO and DVEGA escape trajectories:

$$e_{\phi}(\alpha, \Delta V_{\text{esc01}}, \Delta V_{\text{esc02}}) = 0,$$

$$e_{\phi}(\alpha, \Delta V_{\text{esc01}}, \Delta V_{\text{esc02}}) = 0,$$

$$e_\xi(\alpha, \Delta V_{\text{esc01}}, \Delta V_{\text{esc02}}) = 0.$$  

(9)

The differential correction method is an effective method of solving Eq. (9). In this method, errors were made to converge to 0 by numerically iterating the following equations:
where δ indicates an infinitesimal change and \( \partial \) represents a partial differential.

As a result, the initial conditions \((\alpha, \Delta V_{esc1}, \Delta V_{esc2})\) were obtained for the considered arrival velocities at infinity, as given in the lower part of Table 1.

The efficiency of the series of connected escape trajectories from the DRO to the end of DVEGA is denoted by \( \rho_{DRO} \).

\[
\rho_{DRO} = \frac{V_{eArr}}{\Delta V_{Arr} + \Delta V_{DVEGA}}.
\]

\( \rho_{DRO} \) was found to increase with increasing \( V_{eArr} \). The reason for this is that a small \( \Delta V_{tot} = \Delta V_{esc1} + \Delta V_{esc2} \) can achieve a large \( V_{eArr} \) if the phase \( \alpha \) of the position where the S/C escapes from DRO is selected appropriately.

### Table 1. Conditions on the boundary of Earth’s gravitational sphere of influence that satisfy several target \( V_{eArr} \) values

<table>
<thead>
<tr>
<th>( V_{eArr} ) [m/s]</th>
<th>3000</th>
<th>4000</th>
<th>5000</th>
<th>6000</th>
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</thead>
<tbody>
<tr>
<td>( \Delta V_{DVEGA} ) [m/s]</td>
<td>650</td>
<td>860</td>
<td>1060</td>
<td>1250</td>
</tr>
<tr>
<td>( \Delta V_{DVEGA} ) [m/s]</td>
<td>820</td>
<td>1110</td>
<td>1400</td>
<td>1690</td>
</tr>
<tr>
<td>( V_{eDep} ) [m/s]</td>
<td>1130</td>
<td>1140</td>
<td>1250</td>
<td>1350</td>
</tr>
<tr>
<td>( \Phi ) [deg]</td>
<td>21</td>
<td>0</td>
<td>343</td>
<td>331</td>
</tr>
<tr>
<td>( \zeta ) [deg]</td>
<td>334</td>
<td>316</td>
<td>307</td>
<td>300</td>
</tr>
<tr>
<td>( \alpha ) [deg]</td>
<td>253</td>
<td>247</td>
<td>238</td>
<td>236</td>
</tr>
<tr>
<td>( \Delta V_{esc1} ) [m/s]</td>
<td>295</td>
<td>270</td>
<td>266</td>
<td>264</td>
</tr>
<tr>
<td>( \Delta V_{esc2} ) [m/s]</td>
<td>22</td>
<td>50</td>
<td>53</td>
<td>73</td>
</tr>
<tr>
<td>( \rho_{DRO} )</td>
<td>2.64</td>
<td>2.84</td>
<td>2.91</td>
<td>2.96</td>
</tr>
</tbody>
</table>

#### 5.2. Application to Electric \( \Delta V \) Earth gravity-assist

The optimization of DVEGA was then analyzed. The procedure of designing EDVEGA escape trajectories is as follows.

1. The EDVEGA trajectory is optimized starting from the infinity boundary.
2. The escape trajectory from the DRO to the boundary of Earth’s gravitational sphere of influence is made to satisfy the initial conditions of the EDVEGA trajectory.

In this study, direct collocation with nonlinear programming (DCNLP), which is a numerical calculation method of optimizing low-thrust trajectories, was used for step 2.

#### 5.2.1. Formulation

The optimization problem was solved using the following constraint conditions, where \( u_x \) and \( u_y \) are the input thrusts in the x- and y-directions, respectively, and \( t_f \) is the flight time of the EDVEGA trajectory.

1. Objective function:
   \[
   J = \int_0^{t_f} \sqrt{u_x^2 + u_y^2} \, dt
   \]
2. Initial position: on the infinity boundary
3. \( V_{eDep} \): less than 1000 m/s
4. Terminal position: on the surface of Earth’s sphere of influence (930,000 km from Earth)
5. \( V_{eArr} \): Designated
6. Terminal time: Free
7. Upper limit of thrust:
   \[
   u_{\text{max}} = \sqrt{u_x^2 + u_y^2} = 1.5 \times 10^{-4} \, [\text{m/s}^2]
   \]

The thrust in this investigation was equal to 75 mN for a 500-kg S/C. Under these conditions, the sum of the thrust is minimized.

#### 5.2.2. Optimization results of Electric \( \Delta V \) Earth gravity-assist trajectories

An example of an optimized EDVEGA trajectory in the rotating and inertial frames and its thrust profile is shown in Figs. 15, 16, and 17, respectively, for \( V_{eArr} = 4 \) km/s. Such outbound trajectories were investigated for \( V_{e} = 3, 4, 5, \) and 6 km/s. The efficiencies \( \rho \) and flight times from the departure of the S/C from the DRO to the terminal positions of the EDVEGA trajectories are shown in Figs. 18 and 19, respectively. The efficiencies \( \rho \) were calculated by...
substituting $\Delta V_{\text{EDVEGA}}$ with the sum of all thrusts $u$ in the EDVEGA trajectory. In Figs. 18 and 19 the efficiencies and flight times from the departure from the DRO to the terminal ends of the EDVEGA trajectories are also compared with the ideal efficiency and the corresponding flight time obtained in the previous subsection and to the results for SEL2 departures obtained in a previous study. Matsumoto and Kawaguchi investigated two typical cases of SEL2 departures. In case 1, the SEL2 escape trajectory was directly connected to an EDVEGA trajectory, and in case 2, it was connected to an EDVEGA trajectory after the S/C had traveled once around Earth, as shown in Fig. 20.

If the S/C has a sufficiently high thrust and departs from Earth, the ideal efficiency of the EDVEGA trajectory is 2.7) Figure 18 shows that all efficiencies obtained in this study were greater than 2 because the S/C already had a high energy and could take advantage of it when departing from both the DRO and the SEL2, unlike in the case of a departure from Earth. Figure 18 shows that the efficiency $\rho$ of the escape from the DRO is less than the ideal efficiency of the DVEGA trajectory. The reason for this is that an impulse cannot be applied to the S/C at apoapsis and the S/C is thus unable to take advantage of the gravity of the Sun because of its low thrust propulsion. However, the efficiency of escaping from the DRO is larger than that of escaping from the SEL2 in case 1 for $V_{\infty,\text{Arr}} = 4$, 5, and 6 km/s and tends to increase over time depending on the value of $V_{\infty,\text{Arr}}$, whereas that of escaping from SEL2 tends to decrease over time. Thus, escape from the DRO is more useful when $V_{\infty,\text{Arr}}$ is high than when it is low.

Matsumoto and Kawaguchi revealed that the escape trajectory from SEL2 in case 1 has a low efficiency and a rapid escape and that in case 2 has a high efficiency and a slow escape. This is the result of the tradeoff relationship between efficiency and flight time. Escaping from a DRO realizes a trajectory with the high efficiency of the SEL2 case 2 trajectory and the short escape time of the SEL2 case 1 trajectory at large $V_{\infty,\text{Arr}}$. The efficiencies of the escape trajectories from the departure of the S/C from a DRO to the end of the EDVEGA trajectories is also large and advantageous as well as the efficiency $\nu$ of only escaping from the DRO.

![Fig. 15. EDVEGA trajectory at $V_{\infty,\text{Arr}} = 4$ km/s in rotating frame.](image1)

![Fig. 16. EDVEGA trajectory at $V_{\infty,\text{Arr}} = 4$ km/s in inertial frame.](image2)

![Fig. 17. Timeline of thrust of EDVEGA at $V_{\infty,\text{Arr}} = 4$ km/s.](image3)

![Fig. 18. Efficiencies $\rho$ of a series of trajectories from the departure of the S/C from the DRO and SEL2 to the end of DVEGA and EDVEGA trajectories.](image4)

![Fig. 19. Flight time of a series of trajectories from the departure of the S/C from the DRO and SEL2 to the end of DVEGA and EDVEGA trajectories.](image5)
6. Conclusion

The Lyapunov stability of DROs was confirmed, and the characteristics of escape trajectories from SE-DROs were examined, considering the presence of a space port on the orbit. Although escaping from a DRO requires an impulse $\Delta V$ of at least 20 m/s because of the stability of the DRO, the achieved efficiency of increasing the velocity of the S/C is much larger than that achieved using a space port located in the vicinity of the L2 Lagrangian point for a target $V_\infty$ of greater than 700 m/s.

Two-impulse escape trajectories on which the spacecraft is given the second impulse $\Delta V_2$ at periapsis were also investigated. The largest energy increase can be obtained using such trajectories ($V_\infty = 4000$ m/s with $\Delta V_{\text{tot}} < 1000$ m/s). In addition, two-impulse maneuvers increase the number of solutions on the infinity boundary that can be easily connected to interplanetary DVEGA trajectories.

It was demonstrated that one- and two-impulse escape trajectories can also be connected to EDVAGA trajectories, which are advantageous for use with low-thrust propulsion systems. The overall efficiency from the departure of the S/C from the DRO to the end of the EDVEGA trajectory was compared with similar departures from SEL2. Even if EDVEGA trajectories are involved, escape from the DRO is much more advantageous than escape from SEL2 in terms of obtaining both a high efficiency and a short flight time.

The fact that escape trajectories from an SE-DRO can be connected to DVEGA and EDVEGA trajectories means that this trajectory design could be used to accomplish various interplanetary missions because both allow the angle of deviation to be selected and can therefore connect to various interplanetary trajectories. In summary, this work demonstrates the overall utility and feasibility of trajectory design from a space port located on an SE-DRO for interplanetary exploration missions.

References