Constrained Adaptive Backstepping Control for Re-entry Vehicle

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This paper proposes a new guidance and control system for a re-entry vehicle. The vehicle flying through an extensive flight region can be expressed as a nonlinear system which has uncertain dynamic characteristics and physical constraints. In addition, to address considerations of mission abort and changes of landing site during a space transportation mission, sequentially generating flight trajectories from the guidance system is desirable. Therefore, in this study, we attempt to derive an online trajectory generation system by solving a two-point boundary value problem using the concept of flatness as an extension of exact linearization. Moreover, an attitude control system is designed to track a command signal corresponding to a generated trajectory. The system is employed as a constrained adaptive backstepping control method that can accommodate uncertain dynamic characteristics and physical constraints of a re-entry vehicle. The numerical simulations verify the effectiveness of the proposed guidance and control system.

Key Words: Space Transportation System, Online Trajectory Generation, Adaptive Backstepping Control, Anti-windup

Nomenclature

- \( R, \Theta, \Psi \) : altitude from the center of the earth, latitude, and longitude
- \( V_{TAS}, \gamma \) : true air speed, and flight path angle
- \( \chi_e \) : heading angle (reference to the east)
- \( \sigma, \alpha, \beta \) : bank angle, angle of attack, and sideslip angle
- \( P, Q, R \) : roll, pitch, and yaw rates
- \( \bar{R}, \dot{\Psi}_{\text{cr}} \) : earth’s radius and its rotational rate
- \( g \) : gravity acceleration
- \( S \) : representative area
- \( m \) : vehicle mass
- \( I_x, I_y, I_z \) : moment of inertia
- \( S_{\text{re}} \) : product of inertia
- \( J \) : inertia matrix
- \( q, b, \bar{C} \) : dynamic pressure, span, and mean aerodynamic chord
- \( C_L, C_D \) : lift and its coefficient
- \( C_Y, C_I \) : side force and its coefficient
- \( C_L, C_D \) : drag and its coefficient
- \( M, C_m \) : pitching moment and its coefficient
- \( N, C_N \) : yawing moment and its coefficient
- \( \delta_\alpha, \delta_\beta, \delta_\delta \) : aileron, elevator and rudder deflections
- \( \delta_s, \delta_b \) : speedbrake and bodyflap deflections
- \( x, u, u' \) : state and control input vectors
- \( f, G \) : nonlinear vector and matrix functions
- \( z, w \) : unknown and disturbance vectors
- \( e, \dot{e} \) : tracking and estimation error vectors
- \( \dot{\xi} \) : modified tracking error vector
- \( \xi \) : hedge signal vector
- \( K_i, K_n \) : control gain and observer gain matrices
- \( V \) : Lyapunov function

Subscripts

- \( \ell, (\cdot) \) : command and estimated values
- \( (\cdot)_n, (\cdot)_\Delta \) : nominal and uncertain values
- \( \mathfrak{L} \) : Laplace operator

1. Introduction

In this study, we present a new guidance and control system for re-entry vehicles. Especially, we focus on the terminal area energy management (TAEM) phase of the sub-orbital flight. In recent years, in order to establish the basic technology for advanced future space transportation systems, many studies have been conducted for sub-orbital flights. The sub-orbital flight considered in the present study consists of a climb to an altitude of 100 km at vertical launch, re-entry, energy management and approach and landing phases.

To address considerations of mission abort and changes of landing site during space transportation missions, sequentially generating a flight trajectory from the guidance system is desirable. In particular, sub-orbital flights involve no limitation with respect to aerodynamic heating, and the main constraints imposed on the vehicle are limitations regarding the load factor and dynamic pressure.

In 2005, Vernis and Ferreira proposed an online trajectory generation system in TAEM phase. In this method, by selecting one of the multiple heading alignment cylinders (HACs) depending on the flight condition, a trajectory for reducing the load factor is geometrically derived.

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However, in this method, it is not possible to generate a trajectory in the case of an unexpected landing site. Therefore, in the present study, we propose an online trajectory generation system based on the analytical solution of an optimal control problem combined with the concept of flatness as an extension of exact linearization.  

For the study of the guidance and control system using a concept of flatness, Morio et al. proposed a trajectory tracking control system for offline trajectories. 

In the online trajectory generation system proposed in this paper, the trajectory is generated on the basis of a performance index that minimizes the body acceleration to suppress the load factor and dynamic pressure. Since the proposed guidance law can generate the reference trajectory and wind axis angle commands from the current position and velocity of the vehicle, the trajectory tracking control system is not required. On the basis of these features, the proposed guidance law can cope with the deviation of the initial conditions after the re-entry phase and changes in the landing site.

On the other hand, for tracking the command signal in the attitude control system, feedback linearization combined with time-scale separation has received considerable attention. An advantage of this method is that the amount of time and effort spent on stability analysis can be significantly reduced compared with the conventional method.

However, guaranteeing the stability of the entire closed loop system is difficult because interference among different subsystems has not been considered. Moreover, because vehicle dynamics involve uncertainties such as wind disturbances and aerodynamic characteristics, the employment of an estimation mechanism is desirable in order to ensure good control performance.

To address these problems, we have previously proposed an adaptive backstepping control system using a disturbance observer. The stability of the entire system is thereby guaranteed by eliminating interference based on a backstepping controller. Because this method is an extension of feedback linearization combined with time-scale separation, it shares the above-mentioned advantages. Moreover, using a disturbance observer, the amount of prior information required for control system design can be reduced, and the robustness against unknown disturbances can be improved.

However, the effects of physical constraints, in addition to the range of feasible input values, have not yet been considered. Input saturation due to excessive commands was found to cause the control performance to deteriorate, thereby leading to instability.

In the present study, an anti-windup compensator (AWC) that generates a hedge signal on the basis of the difference between the desired and feasible inputs is added to the previously developed adaptive backstepping control system to prevent the deterioration of the control performance.

The outline of the remainder of this paper is as follows. First, the state equation for the control system design is discussed in Section 2. Sections 3 and 4 describe the concept of flatness and the online trajectory generation system. Section 5 presents the design of an attitude control system with a constrained adaptive backstepping control using an AWC. Section 6 shows the results of numerical simulations applied to a re-entry vehicle in the terminal area energy management (TAEM) phase.

2. State Equations

This section presents the nonlinear state equations for the guidance and control system designs. The state variables in each equation are defined by $x_i(t) = [x_{i1}, x_{i2}, x_{i3}, x_{i4}, x_{i5}, x_{i6}]^T$, $x_2(t) = [\sigma, \alpha, \beta]^T$ and $x_3(t) = [P, Q, R]^T$, where $T$ signifies the transpose. Figs. 1 and 2 illustrate the definitions of the variables and coordinates for Eq. (1) and Eqs. (2) and (3) respectively.

$$\dot{x}_1(t) = f_1(x_1) + G_1(x_1)h(x_2, \delta_e, \delta_o)$$
$$\dot{x}_2(t) = f_2(x_1, x_2) + G_2(x_2)x_2 + d_2$$
$$\dot{x}_3(t) = f_3(x_1, x_2, x_3) + G_3(x_1, x_2, x_3)u(t) + d_3$$

Here, $d(i = 2, 3)$ represents the effects of wind disturbance in each subsystem. The components of the nonlinear function vectors and matrices in each equation are given in Eq. (4), where, to simplify the representations, the expression $x_{i1}, x_{i2}, \ldots = x_{i1}, x_{i2}, \ldots$ is used.
of Eq. (2). Similarly, the command state 2 consist of five control surface deflections as follows:

\[
\begin{align*}
G_1(x_1) &= \begin{bmatrix} g_1(x_1), g_2(x_1), g_3(x_1) \end{bmatrix} \\
G_2(x_1) &= \begin{bmatrix} \cos \alpha / \cos \beta & 0 & \sin \alpha / \cos \beta \\
-\cos \alpha / \tan \beta & 1 & -\sin \alpha / \tan \beta \\
\sin \alpha & 0 & -\cos \alpha 
\end{bmatrix}
\end{align*}
\]

The control inputs \(\mathbf{u}^*\) consist of five control surface deflections as follows:

\[
\mathbf{u}^* = \begin{bmatrix} \delta_{a}, \delta_{e}, \delta_{r} \end{bmatrix}^T
\]

Here, in this control input, the speedbrake and bodyflap deflections \(\delta_{b}\) and \(\delta_{r}\) are used for the velocity control in Eq. (1), and the aileron, elevator and rudder deflections \(\mathbf{u} = \begin{bmatrix} \delta_{a}, \delta_{e}, \delta_{r} \end{bmatrix}^T\) are used for the attitude control in Eq. (3). Eq. (1) expresses the kinematics and dynamics of the translational motion of the center of gravity of the vehicle. Because the translational motion of the vehicle is controlled according to the bank angle and angle of attack in the reentry phase, the wind axis angles are selected as the states in Eq. (2). Eq. (3) expresses the dynamics of the rotational motion of the vehicle. Here, the second term of Eq. (2) is linear in relation to the state \(x_r\), and the command state \(x_{r_{c}}\) is handled as a pseudo-input in Eq. (1). In addition, Eqs. (2) and (3) are linear in relation to inputs; this indicates that Eqs. (2) and (3) constitute affine systems. On the other hand, \(h(x_2, \delta_{a}, \delta_{b})\) on the right-hand side of Eq. (1) are nonlinear functions of the state \(x_r\), thereby constituting non-affine systems.

3. Flatness and Exact Linearization

The concept of flatness is obtained as a technique that extends the exact linearization method. Flatness represents the characteristics of a nonlinear system using a structure similar to that of a linear system. In the exact linearization method, nonlinear mapping transforms an original system into a system suitable for the application of feedback linearization. In this study, the extended method is introduced to obtain the online trajectory generation algorithm analytically.

This algorithm is derived from the system obtained by applying the exact linearization method to the translational motion of the vehicle given by Eq. (1). A performance index based on the acceleration of the vehicle is employed under the constraint of this linearized system. To minimize this performance index, a guidance law can be analytically derived by solving a two-point boundary value problem.

By treating the state \(x_r\), speedbrake deflection, and bodyflap deflection as the command values, the state equation given by Eq. (6) is obtained. Here, the state command \(x_{r_{c}}\) is treated as the pseudo-input. The sideslip angle command that is a component of \(x_{r_{c}}\) is assumed to be zero to suppress the side force. Because the bodyflap deflection is utilized to assist in the velocity control by the speedbrake deflection, the bodyflap and speedbrake deflection commands are assumed to be regarded as a single control input. Therefore, the system given by Eq. (6) has three inputs: the bank angle command, the angle of attack command, and the synthesized value of the bodyflap and speedbrake deflection commands (hereinafter, \(\delta_{a, b, r}\)).

\[
\dot{x}_r(t) = f(x_r) + G_1(x_r) h(\sigma_r, \alpha_r, \delta_{a, b, r})
\]

In this system, the longitudinal motion has four states and two inputs. The lateral motion has two states and a single input. Then, by regarding all controllability indices of the three functions about input functions \(h_i(i=1, 2, 3)\) as two \(c_i = 2(i=1, 2, 3)\), exact linearization is performed. The nonlinear mapping \(\varphi(x) \in \mathbb{R}^{3\times 1}\) is assumed to exist from the system given by Eq. (6) to the feedback equivalent system. This assumption is also analytically obtained from the Frobenius theorem. The time derivative of this mapping function is described as follows:

\[
\frac{d\varphi(x)}{dt} = \frac{\partial\varphi(x)}{\partial x} \frac{dx}{dt} + \frac{\partial\varphi(x)}{\partial \xi} \frac{\dot{\xi}}{dt}
\]

In Eq. (7), the operator \(L\) represents the Lie derivatives and is
given by
\[
L_x^i \phi(x_i) = \frac{\partial \phi(x_i)}{\partial x_i} \mathbf{g}_i,
\]
(8a)
\[
L_z^i \phi(x_i) = \frac{\partial \phi(x_i)}{\partial x_i} \mathbf{g}_i.
\]
(8b)

At this point, to obtain a system for which feedback linearization is permissible so that the transformed system becomes a controllable canonical form, the condition that is obtained with respect to the input coefficient vector is as follows:
\[
L_x^i \phi(x_i) = \theta (i = 1, 2, 3)
\]
(9)

From Eq. (9), the conditions to be satisfied by the nonlinear mapping are obtained as follows:
\[
\frac{\partial \phi(x_i)}{\partial \psi_{\text{virs}}} = \frac{\partial \phi(x_i)}{\partial \psi_{\text{virs}}} = \frac{\partial \phi(x_i)}{\partial \psi_{\text{virs}}} = \theta_{\text{virs}}
\]
(10)

From this relation, it is clear that the mapping function must not be a function of the true air speed \(V_{\text{tas}}\), flight path angle \(\gamma\), and heading angle \(\chi_E\). Under this condition, \(\phi(x_i) \in \mathbb{R}^{3 \times 1}\) is determined as follows:
\[
\phi(x_i) = [R, R_0\Theta_{\text{w}}, R_0\psi_{\text{w}}]^T
\]
(11)

To unify the units of the state as the distance, the crossrange \(R_0\Theta_{\text{w}}\) and downrange \(R_0\psi_{\text{w}}\) are employed as the transformed states. From the condition given by Eq. (9), the time derivative of the mapping function becomes
\[
\frac{d \theta_{\text{virs}}}{dt} = L_x^i \phi(x_i).
\]

Next, the second time derivative of the mapping function is described as follows:
\[
\frac{d^2 \phi(x_i)}{dt^2} = \frac{dL_x^i \phi(x_i)}{dx_i} + \sum_{i=1}^{3} \frac{\partial \phi(x_i)}{\partial x_i} \mathbf{g}_i (x_i) h_i
\]
(12)

By representing the transformed state as \(\xi = [\phi(x_i), L_x^i \phi(x_i)]^T\), the transformation relation of the state can be expressed as follows:
\[
\left[ \begin{array}{c}
\xi_1 \\
\xi_2 \\
\xi_3 \\
\xi_4 \\
\xi_5 \\
\xi_6
\end{array} \right] = \left[ \begin{array}{c}
\phi(x_1) \\
L_x^1 \phi(x_1) \\
\phi(x_2) \\
L_x^2 \phi(x_2) \\
\phi(x_3) \\
L_x^3 \phi(x_3)
\end{array} \right] = \left[ \begin{array}{c}
R \\
R_0\Theta_{\text{w}} \\
R_0\psi_{\text{w}}
\end{array} \right] \xi

V_{\text{tas}} \sin \gamma

-R_0 V_{\text{tas}} \cos \gamma \sin \chi_E / R

R_0 V_{\text{tas}} \cos \gamma \cos \chi_E / (R \cos \Theta_{\text{w}})
\]
(13)

The transformed state equation and its components are described as follows:
\[
\frac{d}{dt} \left[ \begin{array}{c}
\phi(x_1) \\
L_x^1 \phi(x_1)
\end{array} \right] = \left[ \begin{array}{c}
L_x^1 \phi(x_1) \\
L_x^2 \phi(x_2) \\
L_x^3 \phi(x_3)
\end{array} \right] + \sum_{i=1}^{3} \frac{\partial \phi(x_i)}{\partial x_i} \mathbf{g}_i (x_i) h_i
\]
(14)

\[
L_x^i L_z^i \phi(x_i) = \frac{\partial \phi(x_i)}{\partial x_i} \mathbf{g}_i,
\]
(15a)

For the linearization feedback given by Eq. (16) is applied to the transformed state equation given by Eq. (14), and then, the exact linearized system given by Eq. (17) is obtained.
\[
h = [L_x^1 L_z^1 \phi(x_1), L_x^2 L_z^2 \phi(x_2), L_x^3 L_z^3 \phi(x_3)]'
\]
(16)

From the determinant of the inverse matrix \(-qR_0 \cos \Theta_{\text{w}}\) and nonlinear term \(L_x^i \phi(x_i)\) in Eq. (16), in the case where the dynamic pressure \(q\) and the distance from the earth center \(R\) is zero, and when the latitude becomes \(\Theta_{\text{w}} = \pm \pi / 2\) rad (i.e. north and south poles), this linearization feedback has a singular point. In the case of the flight region being discussed in this study, for flying in the atmosphere near the equator, it is assumed that the realizable values are generated from Eq. (16).
\[
\frac{d}{dt} \left[ \begin{array}{c}
\phi(x_1) \\
L_x^1 \phi(x_1)
\end{array} \right] = \left[ \begin{array}{c}
L_x^1 \phi(x_1) \\
L_x^2 \phi(x_2) \\
L_x^3 \phi(x_3)
\end{array} \right] + \sum_{i=1}^{3} \frac{\partial \phi(x_i)}{\partial x_i} \mathbf{g}_i (x_i) h_i
\]
(17)

In the next section, a guidance law, which can minimize the acceleration of the vehicle, is derived based on this exact linearized system. The transformed state equation will hereinafter be represented as follows:
\[
\xi(t) = A\xi(t) + bv(t)
\]
(18)

Here, when the cancelation of the nonlinear term is completely achieved by Eq. (16), the body acceleration coincides with the new control input \(a(t) = \frac{d}{dt} \xi(t) / dt = v(t)\).

4. Guidance System

In this section, a guidance law is introduced analytically on the basis of a two-point boundary value problem. To obtain the guidance law, the performance index is defined as follows:
\[
J = \int_{t_i}^{t_f} \left[ \frac{1}{2} \mathbf{v}^T \left( \mathbf{a}^*( \tau ) \mathbf{a}( \tau ) \right) \right] d\tau
\]
(19)
values, respectively. The weighting parameter \( r \) is introduced to give flexibility to the trajectory design, and the flight time can be adjusted. The initial and final conditions for the state, i.e., \( \xi(t_0) \) and \( \xi(t_f) \), are treated as arbitrary values. By solving this two-point boundary value problem, the guidance law which is analytically obtained is as follows: \[ v_i(t) = -2[\xi_i^2(t) + \xi_i^3(t)] \dot{t}_p + 6[\xi_i^2(t) - \xi_i^3(t)] \ddot{t}_p \quad (i = 1, 2, 3) \tag{20} \]

In Eq. (20), the parameter \( t_p \) denotes the remaining time between the present time and final time \( (t_p = t_f - t) \), and is given numerically by solving the following equation derived from the principle of optimality. The guidance law Eq. (20) stops computing when the remaining flight time \( (t_p \rightarrow 0) \) becomes 1 second, thereby avoiding a division by zero.

\[
\Gamma_p^r = -2[\xi_i^2(t) + \xi_i^3(t)] \dot{t}_p + 6[\xi_i^2(t) - \xi_i^3(t)] \ddot{t}_p + \xi_i^2(t) \dot{t}_p + \xi_i^3(t) \ddot{t}_p \\
\quad + 12[\xi_i^2(t) \dot{t}_p + \xi_i^3(t) \ddot{t}_p] \left[ \xi_i(t) - \xi_i(t_f) \right] \ddot{t}_p + \frac{12}{2} \left[ \xi_i(t) - \xi_i(t_f) \right] \left[ \xi_i^2(t) - \xi_i^3(t) \right] \ddot{t}_p \\
\quad - 18 \left[ \xi_i(t) - \xi_i(t_f) \right]^2 + \left[ \xi_i(t) - \xi_i(t_f) \right]^3 + \left[ \xi_i(t) - \xi_i(t_f) \right]^4 = 0
\]

From Eqs. (20) and (21), the control input \( v_i(t) \) and the remaining flight time \( t_p \), for generating the reference trajectory are determined by the current and final values of the position and velocity of the vehicle. Because the online trajectory is sequentially generated while flying, the proposed system can cope with flight to unexpected regions and changes in the landing site. In the simulation presented later, the vehicle is shown to function in accordance with the adopted constraints by minimizing the acceleration and adjusting the flight time.

5. Constrained Adaptive Backstepping Control

This section presents the design of the constrained adaptive backstepping control system for the attitude controller based on the state equations given by Eqs. (2) and (3). Eqs. (2) and (3) have uncertain nonlinear terms \( f_i(x) \) and \( G_i(x) \) that involve uncertainties, such as the moment of inertia, aerodynamic forces, and aerodynamic moments. These uncertain dynamics are expressed as the additive representation as follows:

\[
\begin{align*}
\dot{x}(t) &= \underbrace{f_i(x(t)) + \Delta f_i}_{f_i(x(t))} + \underbrace{G_i(x(t)) + \Delta G_i}_{G_i(x(t))} \\
(22)
\end{align*}
\]

Furthermore, the tracking error between the state value and its command value is defined as follows:

\[
\zeta = x - x_c
\tag{23}
\]

Using Eqs. (22) and (23), Eqs. (2) and (3) are rewritten in terms of error dynamics, and new variables \( \zeta \) are defined as follows:

\[
\begin{align*}
\dot{\zeta}_i &= f_i(x_{ci}) + \Delta G_i(x_i) x_i(t) + d_i - \dot{x}_i(t) \quad (i = 1, 2, 3) \\
\end{align*}
\tag{24a}
\]

\[
\begin{align*}
\dot{\zeta}_i &= f_i(x_{ci}) + \Delta G_i(x_i) u(t) + d_i - \dot{x}_i(t) \\
\end{align*}
\tag{24b}
\]

In Eq. (24), \( \zeta \) include each uncertain function, wind disturbances, and the derivatives of the command state corresponding to each subsystem. By estimating \( \zeta \) using a disturbance observer, prior knowledge in the control system design can be simplified. Thus, the error dynamics are rewritten as follows:

\[
\begin{align*}
\dot{\zeta}_i &= z_i(x_{ci}, u, \dot{\zeta}_i, \dot{x}_i, d_i) + G_{z_i}(x_i) x_i(t) \\
\end{align*}
\tag{25a}
\]

\[
\begin{align*}
\dot{\zeta}_i &= z_i(x_{ci}, u, \dot{\zeta}_i, \dot{x}_i, d_i) + G_{z_i}(x_{ci}) u(t) \\
\end{align*}
\tag{25b}
\]

5.1. Control law

In this subsection, we describe the control system design by backstepping control in consideration of input saturation.

First, new signal vectors \( x_{ci} \) and \( u \) that do not consider the physical limits are defined as original commands. These original commands are regarded as the pseudo-inputs of each subsystem. Here, a command filter is introduced to constrain the original command under physical limits such as magnitude, rate, and bandwidth. The command filter generates the feasible commands \( x_c \) and \( u \) from \( x_{ci} \) and \( u \). The error dynamics given by Eq. (25) are rewritten using these original and feasible command variables as follows:

\[
\begin{align*}
\dot{\zeta}_i &= z_i + G_{z_i}(x_i) x_i(t) \\
&+ G_{z_i}(x_i) (x_i(t) - x_{ci}(t)) + G_{z_i}(x_{ci}) (x_{ci}(t) - x_{ci}(t)) \\
\end{align*}
\tag{26a}
\]

\[
\begin{align*}
\dot{\zeta}_i &= z_i + G_{z_i} u(t) + G_{z_i} (u(t) - u(t)) \\
\end{align*}
\tag{26b}
\]

The second term on the right hand side of Eq. (26a) reflects the original command \( x_{ci} \). The third term on the right hand side of Eq. (26a) shows the interference between Eqs. (2) and (3). The last term on the right-hand side of Eq. (26a) and the third term on the right-hand side of Eq. (26b) present the error due to the effects of the constraints. The hedge signal \( \zeta \) for the anti-windup is generated by the compensators as follows:

\[
\begin{align*}
\hat{\zeta}_i &= -K \hat{\zeta}_i(t) + G_{z_i}(x_{ci}(t) - x_{ci}(t)) \\
\end{align*}
\tag{27a}
\]

\[
\begin{align*}
\hat{\zeta}_i &= -K \hat{\zeta}_i(t) + G_{z_i}(u(t) - u(t)) \\
\end{align*}
\tag{27b}
\]

These compensators are configured as a linear filter whose gains determine the cutoff frequency. Using \( \xi_i \), the modified tracking error \( \tilde{e} \) is introduced as follows:

\[
\tilde{e}_i(t) = e_i(t) - \xi_i(t)
\tag{28}
\]

The modified tracking error reflects the influence of the input saturations. Thus, excessive control and pseudo-input values can be suppressed. By differentiating \( \tilde{e} \) and substituting Eqs. (26) and (27), the modified error dynamics are obtained as follows:

\[
\begin{align*}
\dot{\tilde{e}}_i(t) &= z_i + G_{z_i} x_{ci}(t) + G_{z_i} e_i(t) + K \xi_i(t) \\
&= z_i + G_{z_i} u(t) + K \xi_i(t) \\
\end{align*}
\tag{29a}
\]

\[
\begin{align*}
\dot{\tilde{e}}_i(t) &= z_i + G_{z_i} u(t) + K \xi_i(t) \\
\end{align*}
\tag{29b}
\]

In this study, the pseudo-input \( x_{ci} \) is determined as follows:

\[
\begin{align*}
\dot{x}_{ci}(t) &= \hat{\zeta}_i(t) \\
\end{align*}
\tag{30}
\]

In this equation, \( x_{ci} \) is a virtual pseudo-input that can be determined arbitrarily and is modified by the hedge signal \( \xi \), which, in turn, is generated from the input error \( u(t) - u(t) \). In other words, while input saturation occurs, \( x_{ci} \) is suppressed using \( \xi \) to correct \( x_{ci} \). Eq. (29a) is rewritten using Eq. (30) as follows:

\[
\begin{align*}
\dot{\tilde{e}}_i(t) &= z_i + G_{z_i} x_{ci}(t) + G_{z_i} \hat{\zeta}_i(t) + K \xi_i(t) \\
\end{align*}
\tag{31}
\]

Here, \( x_{ci} \) and \( u(t) \) are determined for satisfying the below mentioned control laws with respect to the error dynamics of
Eqs. (29b) and (31).
\[ x'_i(t) = G_{z_i}^{-1}(-\hat{z}_i(t) + v_i) \]  \hspace{1cm} (32a)
\[ u'(t) = G_{z_i}^{-1}(-\hat{z}_i(t) + v_i) \]  \hspace{1cm} (32b)

These control laws comprise the estimation signal \( \hat{z}_i \) and new control input \( v_i \). When Eq. (32) is substituted into Eqs. (29b) and (31), the modified tracking error dynamics can be described as follows:
\[ \dot{\hat{e}}_i(t) = -e_i(t) + v_i + K_e \hat{e}_i(t) + G_{z_i}(x_i) \hat{e}_i(t) \]  \hspace{0.5cm} (33a)
\[ \dot{\hat{e}}_i(t) = -e_i(t) + v_i + K_e \hat{e}_i(t) \]  \hspace{0.5cm} (33b)

Moreover, \( e_i \) provides the estimation error, which is given as the difference between the unknown vector \( z_i \) and estimation value \( \hat{z}_i \) as follows:
\[ e_i(t) = \hat{z}_i(t) - z_i(t) \]  \hspace{0.5cm} (34)

5.2. Redesigned disturbance observer

In this subsection, a redesigned disturbance observer is presented to estimate the unknown vector \( z_i \) and to guarantee stability. The proposed disturbance observer is given in the form as follows:
\[ \hat{z}_i(s) = (sI + K_{z_i})^{-1} \{ K_{z_i}z_i(s) + \hat{e}_i(s) \} \]  \hspace{0.5cm} (35)

The observer gain \( K_{z_i} \) becomes a cutoff frequency by which the sensitivity of the disturbance observer can be determined. Moreover, the time derivative of the unknown vector \( z_i \) is assumed to be negligible relative to that of the response of the observer \( \hat{z}_i \), such that \( \dot{\hat{z}}_i \equiv \hat{z}_i \), and consequently,
\[ \dot{e}_i(t) = \hat{z}_i(t) - \hat{z}_i(t) = -K_e e_i + \hat{e}_i(t) \]  \hspace{0.5cm} (36)

This representation cannot be implemented in the control system because the unknown vector \( z_i \) is required to estimate \( \dot{\hat{z}}_i \). To realize the disturbance observer, Eq. (25) is rewritten for \( z_i \) and rearranged as follows:
\[ A[z_i(x)] = (sI + K_{z_i})e_i(s) - K_{x_i}e_i(s) - A[G_{z_i}(x_i) x_i(t)] \]  \hspace{0.5cm} (37a)
\[ A[z_i(x)] = (sI + K_{z_i})e_i(s) - K_{x_i}e_i(s) - A[G_{z_i}(x_{i,2}) u(t)] \]  \hspace{0.5cm} (37b)

Next, the transfer function of the disturbance observer from Eq. (35) is similarly rearranged.
\[ A[z_i(x)] = K_{z_i}^{-1} (sI + K_{z_i}) \hat{z}_i(s) - K_{x_i}^{-1} \hat{e}_i(s) \]  \hspace{0.5cm} (38)

The right-hand sides of Eqs. (37) and (38) are set equal and solved with respect to the estimated vectors \( \hat{z}_i \):
\[ \hat{z}_i(s) = K_{z_i} e_i(s) - (sI + K_{z_i})^{-1} \times \{ K_{x_i} e_i(s) - \hat{e}_i(s) + K_{x_i} A[G_{z_i}(x_i) x_i(t)] \} \]  \hspace{0.5cm} (39a)
\[ \hat{z}_i(s) = K_{z_i} e_i(s) - (sI + K_{z_i})^{-1} \times \{ K_{x_i} e_i(s) - \hat{e}_i(s) + K_{x_i} A[G_{z_i}(x_{i,2},) u(t)] \} \]  \hspace{0.5cm} (39b)

In these representations, the vector \( \hat{z}_i \) can be estimated by known values. Therefore, the control system is implemented using these equivalent representations of the redesigned disturbance observer given by Eq. (39).

5.3. Stability analysis

In this subsection, we discuss the stability of the proposed attitude control system using the Lyapunov stability theorem. The quadratic forms of the modified tracking errors and estimation errors are employed as candidates of the Lyapunov function \( V \) as follows.
\[ V(t) = \frac{1}{2} (e_i^T e_i + e_i^T e_i) + \frac{1}{2} (e_i^T e_i + e_i^T e_i) \]  \hspace{0.5cm} (40)

By substituting Eqs. (29) and (36) into the time derivative of Eq. (40), the equation obtained is as follows:
\[ \dot{V}(t) = \dot{\hat{e}}_i^T \hat{e}_i + \hat{e}_i^T \hat{e}_i + \hat{e}_i^T \hat{e}_i + \hat{e}_i^T \hat{e}_i + \hat{e}_i^T \hat{e}_i + \hat{e}_i^T \hat{e}_i \]  \hspace{0.5cm} (41)

\[ \dot{V}(t) = \hat{e}_i^T \hat{e}_i - \hat{e}_i^T \dot{K}_e \hat{e}_i - \hat{e}_i^T \hat{e}_i + \hat{e}_i^T \hat{e}_i + \hat{e}_i^T \hat{e}_i \]  \hspace{0.5cm} (42a)

Here, the new control input \( v_i \) is determined as follows:
\[ v_i(t) = -\hat{K}_e e_i \]  \hspace{0.5cm} (42a)

\[ v_i(t) = -K_e e_i - G_{z_i}(x_i) \hat{e}_i \]  \hspace{0.5cm} (42b)

\( \hat{K}_e \) is the same feedback gain employed in the compensator given by Eq. (27). The second term on the right-hand side of Eq. (42b) is determined on the basis of the backstepping method to eliminate the interference among subsystems. Eq. (42) is substituted in Eq. (41) to obtain the equation as follows:
\[ \dot{V}(t) = \hat{e}_i^T \hat{e}_i - \hat{e}_i^T \dot{K}_e \hat{e}_i - \hat{e}_i^T \hat{e}_i - \hat{e}_i^T \hat{e}_i - \hat{e}_i^T \hat{e}_i \leq 0 \]  \hspace{0.5cm} (43)

By appropriately selecting each feedback gain, the time derivative of the Lyapunov function \( \dot{V} \) always maintains a negative value and the stability of the attitude control system can be guaranteed. The synthesized control law is then given.
by

\[ x'(t) = G_2(x(t))\{ -\dot{z}_c(t) - K_e e(t) \} \quad (44a) \]

\[ u'(t) = G_1(x(t))\{ -\dot{z}_c(t) - K_e e(t) - G_2(x(t))\dot{e}(t) \} \quad (44b) \]

Fig. 3 shows the block diagram of the proposed guidance and control system.

6. Numerical Simulation

In this section, numerical simulations in the TAEM phase are conducted to verify the effectiveness of the proposed system. The flight model uses the six-degree-of-freedom nonlinear flight simulation model of the highly maneuverable space vehicle used by the Institute of Space and Astronautical Science of the Japan Aerospace Exploration Agency. 14)

6.1. Parameters

The parameters, limitations, and boundary conditions used in the numerical simulations are listed in Tables 1–3. Table 1 lists the boundary conditions, gains, and weighting factor \( \Gamma \) used to adjust the time of flight. In the simulation, the runway is assumed to lie at a latitude and longitude of 0 rad. The initial conditions are set as downrange 10 km, crossrange 20 km, and altitude 25 km. The final conditions are set as downrange –20 km, crossrange 0.0 km, and altitude 5.0 km with respect to the runway. Tables 2 and 3 list the physical limitations of the vehicle’s actuator and the limitations of roll, pitch, and yaw rates.

Table 1. Boundary conditions and parameters.

<table>
<thead>
<tr>
<th>Initial states</th>
<th>Final states</th>
<th>Gains</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R(t_i) = 0.0 [m] ) ( \theta(t_i) = 20.0 [\text{deg}] )</td>
<td>( R_f = 5000.0 [m] ) ( \theta_f = 0.0 [\text{deg}] )</td>
<td>( K_e = 2.5, K_i = 6.25, K_o = 8.0, K_q = 8.0 )</td>
<td>( R_e = 637 [\text{km}] ) ( \gamma = 7.292 \times 10^{-3} [\text{rad/s}] ) ( \Gamma = 20[-] )</td>
</tr>
</tbody>
</table>

Table 2. Physical limitations of the actuator.

<table>
<thead>
<tr>
<th>Input variable</th>
<th>Magnitude min/max(deg)</th>
<th>Rate max/(deg/s)</th>
<th>Frequency (rad/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elevon</td>
<td>–30/30</td>
<td>100</td>
<td>62.83</td>
</tr>
<tr>
<td>Rudder</td>
<td>–30/30</td>
<td>100</td>
<td>62.83</td>
</tr>
<tr>
<td>Speedbrake</td>
<td>0/90</td>
<td>30</td>
<td>25.13</td>
</tr>
<tr>
<td>Bodyflap</td>
<td>–30/30</td>
<td>30</td>
<td>25.13</td>
</tr>
</tbody>
</table>

Table 3. Limitations of roll, pitch and yaw rates.

<table>
<thead>
<tr>
<th>Command variable</th>
<th>Magnitude min/max (deg/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roll rate ( P )</td>
<td>–30/30</td>
</tr>
<tr>
<td>Pitch rate ( Q )</td>
<td>–30/30</td>
</tr>
<tr>
<td>Yaw rate ( R )</td>
<td>–20/20</td>
</tr>
</tbody>
</table>

6.2. Nominal case simulation

First, to evaluate the basic control performance of the proposed control system, a simulation was conducted for the nominal case. In Figs. 4–10, the dashed, dotted, and solid lines represent final, command, and actual values, respectively.

The top figure of Fig. 4 represents a three-dimensional flight trajectory, whereas the second and third figures represent two-dimensional trajectories projected on the horizontal and vertical planes. The vehicle performs an arched flight path at 75–150 s and reaches the target point while smoothly descending. Fig. 5 represents the time history of the air speed, which decelerates gradually to the vicinity of the target value.
Figs. 6 and 7 show time histories of the flight path and heading angles, respectively. Although these flight parameters exhibit minor overshoots, both angles converge to the target values. Fig. 8 shows time histories of the wind-axis angles and their commands. Although these commands exhibit slight oscillations, the actual values follow these commands with a reasonable accuracy under the proposed control system. Fig. 9 shows time histories of the roll, pitch, and yaw rates. Although the pitch rate caused some tracking error, the actual value finally converges to the command value. Fig. 10 shows time histories of the dynamic pressure (black line) and load factor.
(gray line). The dynamic pressure exhibits a maximum value of 8.0 kPa at 150 s and finally decreases to less than 5 kPa. The maximum value of the load factor is 3.5 G, which is a sufficiently small value for this vehicle. Fig. 11 shows time histories of the deflection angles. The aileron, elevator, and rudder deflections generated using an AWC are controlled within their respective limits. On the other hand, speedbrake and bodyflap deflections generated by the online trajectory generation system are partly saturated. Fig. 12 shows the remaining flight time $t_{po}$. The initial value is 160 s, and $t_{po}$ reduces linearly to less than 1.0 s. These results verify that a feasible trajectory was generated by the proposed system. In addition, the results show that attitude control has been performed satisfactorily.

Next, numerical simulations were employed to verify whether the proposed system can accommodate changes in a landing point during the course of the flight. In these simulations, the vehicle begins from an equivalent initial state, but three final conditions are provided according to three cases, as listed in Table 4. In Case 1, the landing site is maintained according to that given in Table 1, and in Cases 2 and 3, the landing site is changed once the vehicle attains an altitude of 15 km. The final conditions changed are $R_p\theta_\psi(t_f), R_p\psi(t_f)$, and $\chi_\psi(t_f)$. The simulation results are shown in Figs. 13 and 14.

Fig. 13 represents the three-dimensional flight trajectories and corresponding two-dimensional trajectories projected on the horizontal and vertical planes. From an altitude of 25 km to 15 km, all trajectories move equivalently toward the target point of Case 1. At an altitude of 15 km, the vehicle trajectories change toward the respective target points of Cases 2 and 3. Fig. 14 represents the remaining flight time $t_{po}$, which is reduced to 3 s or less in all cases. In Cases 2 and 3, these values increase at approximately 110 s (i.e., at an altitude of 15 km). The remaining time is adjusted to correspond to the change in the target point. These results verify that the proposed system can accommodate changes in the landing point during the course of the flight.

### Table 4. Boundary conditions for changing final states.

<table>
<thead>
<tr>
<th>Initial states</th>
<th>Final states Case 1</th>
<th>Final states Case 2</th>
<th>Final states Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R(t_i) = R_g + 25000$ [m], $R_e \theta_\psi(t_i) = 20$ [km]</td>
<td>$R(t_i) = R_g + 500$ [m], $R_e \theta_\psi(t_i) = 0$ [km]</td>
<td>$R(t_i) = R_g + 500$ [m], $R_e \theta_\psi(t_i) = -10$ [km]</td>
<td>$R(t_i) = R_g + 500$ [m], $R_e \theta_\psi(t_i) = -20$ [km]</td>
</tr>
<tr>
<td>$R_p\psi(t_i) = 10$ [km], $V(t_i) = 650$ [m/s]</td>
<td>$R_p\psi(t_f) = -20$ [km], $V(t_f) = 200$ [m/s]</td>
<td>$R_p\psi(t_f) = -30$ [km], $V(t_f) = 200$ [m/s]</td>
<td>$R_p\psi(t_f) = -40$ [km], $V(t_f) = 200$ [m/s]</td>
</tr>
<tr>
<td>$\gamma(t_i) = -10$ [deg], $\chi_\psi(t_i) = 180$ [deg]</td>
<td>$\gamma(t_i) = -30$ [deg], $\chi_\psi(t_i) = 45$ [deg]</td>
<td>$\gamma(t_i) = -30$ [deg], $\chi_\psi(t_i) = 90$ [deg]</td>
<td>$\gamma(t_i) = -30$ [deg], $\chi_\psi(t_i) = 90$ [deg]</td>
</tr>
</tbody>
</table>

### 6.2. Monte Carlo Simulation

Finally, to evaluate the robustness and flexibility of the proposed system, Monte Carlo Simulation (MCS) with 500 repetitions was performed. In this simulation, judgment conditions are defined such that final downrange, crossrange and altitude distance have converged on the target point to within ±500 m. Table 5 shows the variation range of the initial conditions. In addition to Table 5, aerodynamic characteristics, actuator characteristics and vehicle specifications have been varied to values of 3$\sigma$ standard deviations.

### Table 5. Variation range of initial conditions for MCS.

<table>
<thead>
<tr>
<th>Uncertainties</th>
<th>Variation range min/max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Altitude $R$</td>
<td>-3000/0 (m)</td>
</tr>
<tr>
<td>Crossrange $R_p\theta_\psi$</td>
<td>-2500/2500 (m)</td>
</tr>
<tr>
<td>Downrange $R_p\psi$</td>
<td>-2500/2500 (m)</td>
</tr>
<tr>
<td>True air speed $V_{cas}$</td>
<td>-2.5/2.5 (m/s)</td>
</tr>
<tr>
<td>Flight path angle $\gamma$</td>
<td>-2.5/2.5 (deg)</td>
</tr>
<tr>
<td>Heading angle $\chi_\psi$</td>
<td>-5.0/5.0 (deg)</td>
</tr>
</tbody>
</table>

Fig. 15 shows the flight trajectories of 500 repetitions. Depending on the respective initial condition, the control system generates an appropriate trajectory. Fig. 16 shows the distribution of distance at final conditions. The dashed line...
translates the range of judgment conditions of this simulation. Under these conditions, the success rate attains a value of 94.2%. Particularly, the final downrange could not converge to the acceptable range in many of the cases including the failure case. Moreover, the distribution has concentrated around an altitude of 4.5 km because the simulation was stopped when reaching this altitude. These results indicate that the proposed method has good robustness and flexibility for trajectory generation.

\[
\begin{align*}
\text{Altitude } h = & R - R_0 \quad \text{[km]} \\
\text{Crossrange } R = & R_0 \quad \text{[km:N]} \\
\end{align*}
\]

Fig. 15. Flight trajectories in MCS.

\[
\begin{align*}
\text{Altitude } h = & R - R_0 \quad \text{[km]} \\
\text{Downrange } R = & R_0 \quad \text{[km:E]} \\
\end{align*}
\]

Fig. 16. Distribution of distance at the end of simulations.

7. Conclusions

In this paper, we proposed a guidance and control system that is synthesized by an online trajectory generation system and a constrained adaptive backstepping control system. The online trajectory generation algorithm was analytically derived by combining flatness and a two-point boundary value problem. Moreover, to avoid the deterioration of the control performance due to input saturation, an AWC was added to the conventional adaptive backstepping control system. On the basis of the two types of numerical simulation results, the proposed system can accommodate variations in the target point during the course of the flight and suppress excessive command signals to the system. Furthermore, MCS verified that the proposed system demonstrates flexibility and robustness against various parameter changes.

Acknowledgments

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Reference


