Spacecraft Line of Sight Maneuver Control Using Skew-arrayed Two Single-Gimbal Control Moment Gyros

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This paper considers line-of-sight (LOS) maneuver control of an underactuated spacecraft equipped with two skewed single-gimbal control moment gyros. To describe the spacecraft attitude, two parameters referred to as the W-Z parameters are used, the first of which represents the angle between the target LOS and the LOS of the mission-equipment, and the second of which represents the angle around the LOS. In order to stabilize the mission-equipment LOS to the target LOS, a two-step attitude control procedure is considered, which consists of feedforward control combined with feedback control based on a backstepping control method using the W-Z parameters. A numerical simulation is carried out to validate the proposed control procedure.

Key Words: W-Z Parameters, Control Moment Gyro, Line of Sight, Attitude Maneuver

1. Introduction

Control moment gyros (CMGs) can generate higher torque than reaction wheels (RWs) and are therefore used in the International Space Station (ISS) and other large spacecraft. Unlike RWs, CMGs contain motorized actuators known as gimbals. The presence of gimbals introduces a risk of failure, and additional redundancy is required in the attitude controller in order to keep controllability, since post-launch repair is generally impossible. The redundancy increases the fault tolerance, but also the spacecraft weight. Many studies have been performed on means of providing acceptable levels of fault tolerance without added weight, as a solution to the problem of maintaining 3-axis attitude control in the event of one or more actuator failures. In one form of the underactuated control problem, known as the two-torque problem,1) the goal is to damp out the rotational motion2–10) or/and to maintain 3-axis attitude control11–19) when torque can be generated only around two axes.

Most of the studies have envisioned the use of thrusters, two orthogonal RWs, or similar approaches, but a few have involved the use of two single-gimbal control moment gyros (SGCMGs). These include studies on linear parameter-varying (LPV) control by Kwong et al.20) and singular-value decomposition by Gui et al.21) which both assume two parallel SGCMG gimbal axes. So long as their angular momenta are not parallel, two parallel gimbal axes can generate 2-axis torque in a manner similar to that for attitude control using two orthogonal RWs. In the widely used pyramid-array SGCMGs, however, failure of two CMGs will not leave two functioning CMGs with parallel gimbal axes. In a skewed array, moreover, the independent degrees of freedom of the two SGCMGs are those of the two gimbals but the generated torque is three dimensional. With skewed-axis CMGs, it is therefore not possible to obtain control simply by applying 2-axis torque in the manner of two orthogonal RWs or two CMGs with parallel gimbal axes. Kasai et al.22) proposed a 3-axis attitude control method for two skewed CMGs, utilizing the corning effect of repeated 1-axis maneuvering with a built-in gimbal-rate limiter. However, its range of applications is lim-

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Nomenclature

- **A**: CMG Jacobian matrix
- **H**: CMG angular momentum vector
- **J**: Satellite inertia tensor (= diag(Jx, Jy, Jz))
- **k1, k2**: control gains
- **L**: target LOS direction in the inertial frame
- **Lb**: mission-equipment LOS direction in body frame
- **R**: transformation matrix from body frame to mission-equipment frame
- **(X, Y, Z)**: inertial coordinate axes
- **(Xb, Yb, Zb)**: satellite’s body axes
- **(Xe, Ye, Ze)**: CMG axes
- **w1, w2, z**: W-Z parameters
- **w**: W parameter vector (= [w1 w2]T)
- **β**: skew angle (= 54.73 deg)
- **δ**: gimbal angle vector (= [δ1 δ2]T)
- **δi**: gimbal angles (i = 1, 3)
- **ε**: LOS threshold for control-law switchover
- **Ωb**: spacecraft’s angular velocity in body frame
- **Ω**: spacecraft’s angular velocity in mission-equipment frame
- **ω**: two-dimensional spacecraft’s angular velocity in body frame
- **τ**: time derivative of CMG angular momentum vector
- *****: the complex conjugate of a complex number
ited because it is feedforward control. Gui et al.\textsuperscript{23)} therefore proposed feedback control using a generalized dynamic inverse (GDI) procedure for orientations departing substantially from the target attitude, together with backstepping control for stabilization to the target attitude. In all cases, however, 3-axis attitude control with two skewed SGCMGs using feedback control has invariably required complex nonlinear control techniques.

LOS control is widely used in astronomical observations and communications, to point the mission-equipment LOS in a specified direction for astronomical body observation or communication. In many cases, the conditions for attitude stabilization around the LOS axis in a specified direction can be relaxed, and LOS control by two skewed SGCMGs may therefore be easier than 3-axis attitude control. However, it must be noted that in actual missions it would be desirable to include the option of setting the mission-equipment LOS to an arbitrary direction in the the satellite’s body coordinate system.

In this study, we assume such an arbitrary setting, and propose a LOS angle change control method using two skewed SGCMGs. We investigate the utilization of two-stage control to obtain a solution to the problem, with coarse feedforward utilization around the LOS axis in a specified direction for an astronomical body observation or communication. In many cases, the conditions for attitude stabilization, to point the mission-equipment LOS in a specified direction for an astronomical body observation or communication.

Let us now consider this in terms of the relevant equations. The attitude control torque generated by the CMGs (No. 2 and 4) in a pyramid array of four have failed and considered in this paper. It is assumed that two mutually opposing CMGs (No. 2 and 4) in a pyramid array of four have failed and possess no angular momentum. The angular momentum in the CMG system is then given by

\[ H = \sum_{i=1,3} h_i(\delta_i) = h_0 \begin{bmatrix} -c\beta \sin \delta_1 & c\beta \cos \delta_3 \\ c\beta \cos \delta_1 & -\sin \delta_2 \\ s\beta \cos \delta_1 & s\beta \cos \delta_3 \end{bmatrix} \]

(1)

where \( c\beta = \cos \beta \), and \( s\beta = \sin \beta \). Figure 2(a) shows a plot of the angular momentum in the form of a three-dimensional curved surface, and Figs. 2(b) and 2(c) show the cross sections at the \( Y_c = 0 \) and \( Z_c = 0 \) planes, respectively. As shown in Fig. 2(a), a hole is present along both gimbal axis directions. In Figs. 2(b) and 2(c), it can be seen that a linear angular momentum is present starting from the origin in the \( X_c \) and \( Z_c \) directions but not in the \( Y_c \) direction. This indicates that torque cannot be generated about the \( Y_c \)-axis alone with the two skewed SGCMGs, and confirms that attitude control cannot be performed independently for each axis.

Let us now consider this in terms of the relevant equations. Because the attitude control torque, which is generated by the CMGs and given to the spacecraft, is opposite to the time-derivative of the CMG angular momentum for the case of single spin, the attitude control torque generalized by the CMGs for the case of single spin can be written as

\[ \tau = -H = -\sum_{i=1,3} h_i = -h_0 \dot{\delta} \]

(2)

where \( A \) is the Jacobian matrix given by

\[ A = \begin{bmatrix} -c\beta \cos \delta_1 & c\beta \cos \delta_3 \\ -\sin \delta_1 & -\sin \delta_2 \\ s\beta \cos \delta_1 & s\beta \cos \delta_3 \end{bmatrix} \]

(3)

When the gimbals are actuated at \( \delta = [\delta \ 0]^T \) with \( \dot{\delta} = [-\dot{\delta} \ 0]^T \), the generated torque is then

\[ \tau = -\dot{H} = h_0 \begin{bmatrix} 0 & 0 & 2\dot{\delta} \cos \delta \end{bmatrix}^T \]

(4)

and torque can be generated only about the \( Z_c \)-axis. Conversely, if they are actuated at \( \delta = [0 \ \delta]^T \) with \( \dot{\delta} = [\dot{\delta} \ -\dot{\delta}]^T \), we then have

\[ \tau = -\dot{H} = h_0 \begin{bmatrix} 0 & 2\dot{\delta} \beta \cos \delta & 0 \end{bmatrix}^T \]

(5)

and torque can be generated only about the \( X_c \)-axis. The control near the target LOS utilizes this effect.

3. \textit{W-Z Parameters}

The attitude may be expressed in terms of Euler angles, quaternions, Rodrigues parameters, or various other parameters. In this study, we use the \textit{W-Z} parameters proposed by Tsiotras and Longuski.\textsuperscript{24)} Here we describe how these parameters are used for attitude expression and why they are suitable for LOS control.

The \textit{W-Z} parameters express the attitude in terms of two rotational transforms (see Fig. 3). The first is a rotation \( z \) about the \( Z \) axis. The second is a rotation \( \theta \) about a vector \( \mathbf{u} \) in the \( X-Y \) plane, to match the attitude. Let us denote the directional cosine vector of the axis \( Z \) with the body axes \( (X_b, Y_b, Z_b) \) as \( [a \ b \ c]^T \), as shown in Fig. 3. Defining the vector \( \mathbf{u} \) as

\[ \mathbf{u} = \left[ \frac{b}{\sqrt{c^2 + b^2}} \ \frac{c}{\sqrt{c^2 + b^2}} \right]^T = [w_1 \ w_2]^T \]

(6)

we then have the following relation between the time derivatives of the \textit{W-Z} parameters and the angular velocity of the satellite.

\[ \begin{bmatrix} \dot{w}_1 \\ \dot{w}_2 \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} \frac{1}{w_1^2 - w_2^2} \ w_1 w_2 \ w_2 \\ w_1 w_2 \ (1 - w_1^2 + w_2^2)/2 \ -w_1 \\ -w_2 \ w_1 \ 1 \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} \]

(7)

Rewriting \( \mathbf{u} \) in complex-number notation as

\[ \hat{\mathbf{u}} = w_1 + iw_2 \]

(8)
we can then rewrite Eq. (7) as
\begin{equation}
\dot{\hat{w}} = -i\omega_3 \hat{w} + \frac{\ddot{\hat{w}}}{2} + \bar{\hat{w}} \hat{w}^2 ,
\end{equation}
\begin{equation}
\dot{z} = \omega_2 + \text{Im}(\hat{w} \bar{\hat{w}}).
\end{equation}
In considering LOS control, if we take the direction of the mission equipment as the \(Z_b\) axis in the satellite's body coordinate system, it is then unnecessary to restrict the attitude about the \(Z_b\) axis, and consequently unnecessary to control the value of \(z\) in the \(W-Z\) parameters. The control problem is thereby reduced to a problem of stabilizing the two-dimensional vector \(\hat{w}\) to zero. Based on Eq. (10), the time derivative of the squared norm of \(\hat{w}\) is then
\begin{equation}
\frac{d}{dt} |\hat{w}|^2 = (1 + |\hat{w}|^2) \text{Re}(\hat{w} \bar{\hat{w}}) ,
\end{equation}
\begin{equation}
\frac{d}{dt} |w|^2 = (1 + |w|^2)(\hat{w} \cdot \bar{\hat{w}}) .
\end{equation}
The \(W-Z\) parameters are complex numbers, but as shown in Eq. (12) or Eq. (13), the time derivative of the squared norm of \(w\) can be expressed by only a real value. The \(W-Z\) parameters are therefore appropriate for LOS control, and in this study, we take Eq. (12) or Eq. (13) as the basis for designing a feedback controller that can be used near the target LOS. As noted in Section 2, the two skewed SGCMGs cannot independently generate torque about the \(Y_c\)-axis. On the other hand, the \(W-Z\) parameters are conceived as expressions for application to the two-torque problem in which torque cannot be generated about the \(Z_b\)-axis. To simplify the application of the \(W-Z\) parameters to the attitude maneuver problem with two skewed SGCMGs, hereinafter we assume that the \(X_c\)- and \(Z_c\)-axes of the CMG coordinate system coincide with the \(X_b\)- and \(Y_b\)-axes of the satellite's body coordinate system, respectively.
3.1. LOS control
The use of just one nonlinear feedback controller to perform attitude control (3-axis or LOS) is desirable since it eliminates the need to switch between two controllers, but nonlinear feedback control tends to require control quantities exceeding the actuator limit, with the attendant difficulty of designing a gain that would prevent limit overruns. Furthermore, the use of a star sensor as the attitude sensor would require some means of preventing it from pointing towards the sun, and it is no easy matter to design a nonlinear feedback control that would guarantee the necessary attitude restriction. In contrast, with feed-forward control, it is a relatively simple matter to design an attitude maneuver trajectory that incorporates actuator restriction or star-sensor LOS direction limiting. In this study, we therefore consider two-stage control, comprising a coarse control (feed-forward) stage and a fine control (feedback) stage.

3.2. Coarse (feedforward) control
In this section, we first show that the LOS of mission equipment having a designed arbitrary direction set in the body coordinate system can be pointed in an arbitrary direction in the inertial coordinate system using two single-axis rotations ($X_b$-axis then $Y_b$-axis or $Y_b$-axis then $X_b$-axis) about the body axes. We then discuss the conditions for rotational axis selection and the method for calculating the angle of rotation.

3.2.1. Two consecutive attitude maneuvers about the $x$-y or $y$-$x$ axes
Since a 180 deg change in the angle of rotation about any given axis simply reverses the LOS direction, the target LOS direction and the mission-equipment LOS direction can be covered, without loss of generality, by considering just one eighth of the global points comprising the 1st quadrant of the northern hemisphere as shown in Fig. 4. Let us refer to this quadrant as domain A. For simplicity the attitude maneuver is referred to as $x$-$y$ axes rotation when performed in the order $X_b$- then $Y_b$-axis rotation about the body axis, and as $y$-$x$ axes rotation when performed in the order $Y_b$- then $X_b$-axis rotation about the body axis. Let us refer to the mission-equipment LOS direction in the inertial coordinate system as $L$, and the target LOS direction in the inertial coordinate system as $L_b$ (Fig. 5). The angles $\theta_x$ and $\theta_y$ of $L_b$ with the $X_b$- and $Y_b$-axes are then, respectively
\[
\theta_x = \cos^{-1}(L_b \cdot X_b),
\]
\[
\theta_y = \cos^{-1}(L_b \cdot Y_b).\]
When the first rotation is performed about the body’s $X_b$-axis, the body’s $Y_b$-axis moves in a circle in the $Y$-$Z$ plane of the inertial coordinate system. The second rotation about the body’s $Y_b$-axis can then encompass the complete range of angles $\theta_y$ between the target LOS direction and the $Z$-$Y$ plane of the inertial coordinate system (Fig. 6). The $y$-$x$ axes rotation can similarly encompass the complete range of angles $\theta_x$ between the target LOS direction and the $X$-$Z$ plane of the inertial coordinate system. If the angles between $L_b$ and the $X_b$- and $Y_b$-axes are equal such that $\theta_x = \theta_y = \frac{\pi}{2}$ rad, then the range traversed by the two rotations is minimal. The difficulty of attaining the direction of the target LOS is maximal when the target LOS is furthest from the $X_b$- and $Y_b$-axes and thus in the direction of the $Z$-axis in the inertial coordinate system. Even in this case, however, the LOS can be pointed in the target LOS direction by first performing a rotation of $\frac{\pi}{2}$ rad about one axis and then a rotation of a $\frac{\pi}{2}$ rad about the other axis. These considerations, in summary, confirm that the mission-equipment LOS can be pointed in any arbitrary direction in domain A by $x$-$y$ axes or $y$-$x$ axes rotation.

3.2.2. Rotational axis selection and calculation of consecutive attitude maneuver angles
As described above, to point the mission-equipment LOS in an arbitrary direction, it is necessary to select the axes of rotation in accordance with the following size relations of the angles between $L_b$ and $L$ and the $X$- and $Y$-axes.

\begin{align*}
\text{condition 1:} \quad & \theta_y \geq \psi_y = \frac{\pi}{2} - \cos^{-1}(L \cdot X) \quad (16) \\
\text{condition 2:} \quad & \theta_x \geq \psi_x = \frac{\pi}{2} - \cos^{-1}(L \cdot Y) \quad (17)
\end{align*}

If condition 1 is satisfied, then $x$-$y$ axes rotation should be chosen, while $y$-$x$ axes rotation should be chosen if condition 2 is satisfied. Here, let us consider only the calculation under condition 1. The maneuver angles under condition 2 can be obtained by a similar calculation. If both are satisfied, then we select the one involving the smaller rotation angle and maneuvering time. Defining the vectors and angles as shown in Fig. 7, we can calculate the rotational angles $\phi_x$ and $\phi_y$ for $x$-$y$ axes rotation using the formulae for spherical triangles shown in Eq. (18) and (19).

\begin{align*}
\phi_x &= \frac{\pi}{2} - \cos^{-1}\left(\frac{\cos \theta_x}{\cos \psi_y} \right) \pm \cos^{-1}\left(\frac{\cos \theta_y}{\cos \psi_x} \right) , \quad (18) \\
\phi_y &= \arctan\left(\frac{\text{atan2}(y', (h_{L_b} \times h_L))}{h_{L_b} \cdot h_L} \right) , \quad (19)
\end{align*}

\begin{align*}
h_{L_b} &= \frac{y' \times (L_b \times y')}{\|y' \times (L_b \times y')\|} , \quad (20) \\
h_L &= \frac{y' \times (L \times y')}{\|y' \times (L \times y')\|} , \quad (21) \\
L_b' &= C_x(\phi_x) L_b , \quad (23) \\
C_x(\phi_x) &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \phi_x & -\sin \phi_x \\ 0 & \sin \phi_x & \cos \phi_x \end{pmatrix} . \quad (25)
\end{align*}
Overall system is zero. Near the target LOS attitude, we assume

\[ H \approx h_0 \begin{bmatrix} -c\beta (\sin \delta_1 - \sin \delta_3) \\ s\beta (\sin \delta_1 + \sin \delta_3) \\ 0 \end{bmatrix}. \]  

(26)

Because we assume that the angular momentum of the overall system is zero, if the angular momentum of the CMG about the \( Y_s \)-axis is zero, then the angular velocity of the satellite about the \( Z_s \)-axis is also zero. In such a case, the equation of motion can be approximated to describe two-axis rotation about the \( X_b \)- and \( Y_b \)-axes as

\[
\begin{bmatrix}
J_x & 0 \\
0 & J_y
\end{bmatrix}
\begin{bmatrix}
\omega_x \\
\omega_y
\end{bmatrix}
= -h_0
\begin{bmatrix}
-c\beta c\delta_1 & c\beta c\delta_3 \\
s\beta c\delta_1 & s\beta c\delta_3
\end{bmatrix}
\begin{bmatrix}
\delta_1 \\
\delta_3
\end{bmatrix}
\]  

(27)

which can be rewritten

\[
\tilde{J}\dot{\omega}_b = -h_0\tilde{\lambda}\delta
\]  

(28)

where

\[
\tilde{\lambda} = \begin{bmatrix}
-c\beta c\delta_1 & c\beta c\delta_3 \\
s\beta c\delta_1 & s\beta c\delta_3
\end{bmatrix}.
\]  

(29)

If we let \( v = |\tilde{\omega}|^2 \), the differential equation Eq. (12) can be rewritten as

\[
v = (1 + v)(\dot{\omega} \cdot \omega).
\]  

(30)

Introducing the virtual angular velocity

\[
\dot{\omega}_b = -k_1 v \quad \text{(or } \dot{\omega}_b = -k_1 \omega), \quad (k_1 > 0),
\]  

(31)

and substituting \( \dot{\omega}_b \) into \( \dot{\omega} \) in Eq. (30), Eq. (30) then becomes

\[
v = -k_1(1 + v)v \quad (< 0).
\]  

(32)

The solution of the above differential equation is then

\[
v = |\dot{\omega}|^2 = |\omega|^2 = 1/(\gamma e^{k_1 t} - 1)
\]  

(33)

where \( \gamma \) is an integration constant. As shown in Eq. (33), the parameter \( \gamma(=|\dot{\omega}|^2) \) is asymptotically stable. The virtual angular velocity (Eq. (31)) is input to stabilize the body axis \( Z_b \) to the inertial \( Z \)-axis. In this paper, we consider the task of pointing the mission-equipment LOS in the target direction starting from an arbitrary direction in the satellite’s body coordinate system. In the previous section, we showed that the LOS can be placed in any arbitrary direction, and for simplicity we therefore take the \( Z \)-axis of the inertial coordinate system as the target LOS direction without loss of generality. We denote as \( R \) the matrix of coordinate transformation from the satellite’s body coordinate system to the mission-equipment LOS reference coordinate system.

In order to derive a steering control law that is intended to follow the virtual angular velocity input (Eq. (31)), by introducing the difference between the current angular velocity vector and the virtual angular velocity input as \( \sigma = \dot{\omega} - \dot{\omega}_b \), a Lyapunov function candidate is chosen as

\[
V(v, \sigma) = v + \frac{1}{2} \sigma^T \sigma = v + \frac{1}{2} |\dot{\omega} - \dot{\omega}_b|^2.
\]  

(34)

The satellite angular acceleration due to the CMG-generated torque is transformed to the mission-equipment LOS coordinate

3.3. Fine (feedback) control

In this study, we assume that the angular momentum of the overall system is zero. Near the target LOS attitude, we assume that if gyration has sufficiently slowed, the two gimbal angles of the two skewed SGCMGs are both close to 0 or 180 deg. The angular momentum of the CMG system as expressed in the satellite’s body coordinate system is then

\[ H \approx h_0 \begin{bmatrix} -c\beta (\sin \delta_1 - \sin \delta_3) \\ s\beta (\sin \delta_1 + \sin \delta_3) \\ 0 \end{bmatrix}. \]  

(26)
always possible to calculate the gimbal steering law (Eq. (42)),

$$\dot{\omega} = \ddot{R}\omega_b = -h_0\ddot{R}J^{-1}\dot{A}\delta$$  \hspace{1cm} (35)

where

$$\ddot{R} = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \end{bmatrix} , \hspace{1cm} (36)$$

$$\ddot{R}' = \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} . \hspace{1cm} (37)$$

In addition, note that

$$\dot{\omega} = \ddot{\omega}_d + \sigma \cdot$$  \hspace{1cm} (38)

$$\dot{\omega}_d = -k_1\dot{\omega} = -k_1\dot{W}\omega = -k_1\dot{W}R\omega_b$$  \hspace{1cm} (39)

where

$$\dot{W} = \begin{bmatrix} \frac{1}{2} (w_1^2 - w_2^2) / 2 & w_1w_2 \\ w_1w_2 & \frac{1}{2} (1 - w_1^2 + w_2^2) / 2 & -w_1 \end{bmatrix} . \hspace{1cm} (40)$$

By taking the time derivative of Eq. (34) and substituting Eqs. (35), (38) and (39), we have

$$\dot{V} = -k_1(1 + v)w + \sigma \cdot (1 + v)w + (k_1\dot{W}R\omega_b - h_0\ddot{R}J^{-1}\dot{A}\delta) . \hspace{1cm} (41)$$

If a steering control law is designed as

$$\delta = \frac{1}{h_0}\ddot{A}'J^{-1}\ddot{R}' \left\{ k_1\dot{W}R\omega_b + k_2(\dot{R}\omega_b + k_1w) + (1 + v)w \right\} \hspace{1cm} (42)$$

then

$$\dot{V} = -k_1(1 + v)w - k_2|\sigma|^2 \leq 0 . \hspace{1cm} (43)$$

As we assume the application of feedback control near the target LOS, and consider $\delta_1 \approx \delta_2 \approx 0$ or $\sigma$, then $h_0 \delta_1 \approx 0$ and $h_0 \sigma \neq 0$. We then have $|\dot{A}| \neq 0$, and so long as $\ddot{R}$ is regular it is always possible to calculate the gimbal steering law (Eq. (42)), in short, the steering control law (Eq. (42)) does not encounter singularities.

Consequently, $\dot{V}$ is radially unbounded, $\dot{V} < 0 \forall v, \sigma^T \in \mathbb{R}^3 \setminus \{0\}$, and $V(0) = 0$. This proves that the steering control law employed near the target LOS can asymptotically stabilize the mission-equipment LOS to the target LOS.

4. Numerical Simulation

4.1. Simulation parameters

The reverse timing for the gimbal steering direction in the feedforward control was determined by the method given in Ref. 22). Switchover from the coarse control to the feedback control is determined based on a threshold deviation between the current attitude and the target LOS, $\varepsilon$, and the control gains are set to avoid overrun of the gimbal angular-velocity limit. Table 1 shows the satellite inertia tensor, CMG wheel angular momentum, control gains, and the other calculation parameters applied in the numerical simulation to test the validity of the coarse (feedforward) and fine (feedback) control procedure described in the previous section.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J$</td>
<td>diag(0.5, 0.5, 0.5) [kgm²]</td>
</tr>
<tr>
<td>$h_0$</td>
<td>0.02 [Nms]</td>
</tr>
<tr>
<td>$w_1(0), w_2(0), z(0)$</td>
<td>(-0.366, 0.366, 0.0)</td>
</tr>
<tr>
<td>$L_0$</td>
<td>$\begin{bmatrix} \frac{1}{\sqrt{3}} &amp; \frac{1}{\sqrt{3}} &amp; \frac{1}{\sqrt{3}} \end{bmatrix}^T$</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>0.02 [rad]</td>
</tr>
<tr>
<td>$k_1, k_2$</td>
<td>0.5, 0.5</td>
</tr>
</tbody>
</table>

4.2. Simulation results

Figures 8 to 10 show the time responses of the simulated attitude, LOS angular error, and gimbal angle, respectively. As can be seen in Figs. 9 and 10, the coarse control continued for approximately 38 s, with 1-axis maneuvers resulting from reverse-phase and then in-phase steering of the two gimbal angles. At the second 1-axis maneuver termination, the LOS an-
gular error reached the control-law switchover threshold, and feedback control was then applied. The results shown in Fig. 9 confirm that the LOS angular error is stabilized to zero. These results demonstrate that the mission-equipment LOS can be pointed and stabilized to a target LOS direction by two skewed SGCMGs with the control technique described in this paper.

5. Conclusion

In this study, we have proposed a control method for mission-equipment LOS angle maneuvering using two skewed SGCMGs from an arbitrary initial LOS direction in the satellite-body coordinate system to an arbitrary direction in the inertial coordinate system. To simplify the solution, we considered a two-stage control procedure consisting of a feedforward coarse-control stage followed by a feedback fine-control stage. In the coarse-control stage, we converted the problem to a geometrical control stage followed by a feedback fine-control stage. In the feedback-control stage we used the W-Z parameters to express the attitude, and designed a backstepping controller that can stabilize the mission-equipment LOS to the target LOS. The validity of the proposed control method was confirmed by numerical simulation.

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