Influence of Attack Angle on Magnetohydrodynamic Flow Control in Reentry Flight

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Three-dimensional magnetohydrodynamic (MHD) numerical simulation of MHD flow control is carried out to investigate the influence of the Hall effect on MHD flow control in Earth reentry flight with attack angles. Numerical results show that the Hall effect weakens the drag enhancement effect of MHD flow control in each attack angle case. Because the region with high electric current densities becomes confined to an area around the stagnation point. Moreover, when the vehicle has non-zero attack angles, the electric current occurs asymmetrically to the planes orthogonal to the entry direction of a vehicle. Consequently, a side force acts on the vehicle, although the side force is considerably weak compared with drag and lift forces. The direction of the side force depends on attack angles, and also the side force disappears at a certain attack angle.

Key Words: Magnetohydrodynamics, Hypersonic Flow, Thermal Protection

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>( B )</td>
<td>vector of magnetic flux density, T</td>
</tr>
<tr>
<td>( B_x, B_y, B_z )</td>
<td>components of magnetic flux density in the ( x, y, ) and ( z ) directions, T</td>
</tr>
<tr>
<td>(</td>
<td>B</td>
</tr>
<tr>
<td>( B_0 )</td>
<td>strength of magnetic flux density at the stagnation point, T</td>
</tr>
<tr>
<td>( C_{drag} )</td>
<td>drag force coefficient</td>
</tr>
<tr>
<td>( C_{lift} )</td>
<td>lift force coefficient</td>
</tr>
<tr>
<td>( C_{side} )</td>
<td>side force coefficient</td>
</tr>
<tr>
<td>( E )</td>
<td>electric field vector, V/m</td>
</tr>
<tr>
<td>( E )</td>
<td>total energy, J/kg</td>
</tr>
<tr>
<td>( e )</td>
<td>internal energy, J/kg</td>
</tr>
<tr>
<td>( F_{drag} )</td>
<td>drag force, N</td>
</tr>
<tr>
<td>( F_{lift} )</td>
<td>lift force, N</td>
</tr>
<tr>
<td>( F_l )</td>
<td>force attributed to reaction force of Lorentz force, N</td>
</tr>
<tr>
<td>( F_p )</td>
<td>force attributed to surface pressure, N</td>
</tr>
<tr>
<td>( F_{side} )</td>
<td>side force, N</td>
</tr>
<tr>
<td>( H )</td>
<td>total enthalpy, J/kg</td>
</tr>
<tr>
<td>( J )</td>
<td>tensor of viscous shear stress, N/m²</td>
</tr>
<tr>
<td>( \mathbf{n} )</td>
<td>unit vector of normal direction</td>
</tr>
<tr>
<td>( p )</td>
<td>static pressure, Pa</td>
</tr>
<tr>
<td>( p_\infty )</td>
<td>freestream pressure, Pa</td>
</tr>
<tr>
<td>( R_0 )</td>
<td>spherical nose radius, m</td>
</tr>
<tr>
<td>( S )</td>
<td>area of cell interface, m²</td>
</tr>
<tr>
<td>( S_{wall} )</td>
<td>area of wall surface of reentry vehicle, m²</td>
</tr>
<tr>
<td>( T )</td>
<td>temperature, K</td>
</tr>
<tr>
<td>( T_\infty )</td>
<td>freestream temperature, K</td>
</tr>
<tr>
<td>( u )</td>
<td>vector of flow velocity, m/s</td>
</tr>
<tr>
<td>( v_\infty )</td>
<td>flight velocity, m/s</td>
</tr>
<tr>
<td>( V )</td>
<td>cell control volume, m³</td>
</tr>
<tr>
<td>( V_{all} )</td>
<td>volume of computational area, m³</td>
</tr>
<tr>
<td>( x, y, z )</td>
<td>rectangular coordinates</td>
</tr>
</tbody>
</table>

\[ \alpha_A \] : attack angle of reentry vehicle, deg.
\[ \beta \] : Hall parameter
\[ \kappa \] : thermal conductivity, W/(m·K)
\[ \rho \] : total mass density, kg/m³
\[ \sigma \] : electrical conductivity, S/m
\[ \tau \] : tensor of viscous shear stress, N/m²
\[ \phi \] : cone diameter, m

1. Introduction

MHD flow control, which is an active thermal protection technique in earth reentry flight using magnetohydrodynamic (MHD) technology, has been proposed in the 1950s to 60s.⁴⁻⁶ In MHD flow control, a magnetic field is applied to a plasma flow around a reentry vehicle, which leads to inducing electric currents in a shock layer. The interaction between the magnetic field and the electric current induces the Lorentz force decelerating the plasma flow in the shock layer. This deceleration results in the expansion of shock layer and the mitigation of convective heat transfer to the vehicle. Furthermore, since a reaction force of the Lorentz force acts on a magnet installed into the vehicle, the utilization of MHD flow control for active control of flight dynamics in reentry flights has been proposed.

A lot of numerical studies on MHD flow control have been conducted. Most of the numerical studies adopted two-dimensional or axisymmetric two-dimensional approximation. However, strong three-dimensional magnetohydrodynamic fields are anticipated in the situation where MHD flow control is operated for reentry vehicles with non-zero attack angles or tilted magnetic fields. Recently, Nagata et al.⁷ examined the influence of tilted magnetic fields on the fluid dynamics and the flight dynamics of a blunt body utilizing MHD flow control by means of three-dimensional numerical simulation. The numerical results demonstrated for the first time that MHD flow control induces a side force on the blunt body due to influences of the Hall effect when tilted magnetic fields are utilized.
However, influences of the Hall effect on MHD flow control in reentry flights with non-zero attack angles have not been revealed so far. The purpose of the present study is, therefore, to examine the influences of the Hall effect on MHD flow control in reentry flights with non-zero attack angles by means of three-dimensional numerical simulation.

2. Numerical Method and Numerical Condition

2.1. Basic equations for gasdynamics

The basic equations for the gasdynamics are composed of the mass conservation equation, the momentum conservation equation with the Lorentz force, and the total energy conservation equation with the Joule heating and the work by the Lorentz force. The Earth atmosphere is assumed to be composed of 78% N\textsubscript{2}, 21% O\textsubscript{2}, and 1% Ar by mass. The gas is assumed to be in thermochemical equilibrium state, and also the following 13 chemical species are taken into account: N, O, Ar, N\textsubscript{2}, O\textsubscript{2}, NO, O\textsuperscript{+}, Ar\textsuperscript{+}, N\textsubscript{2+}, O\textsubscript{2+}, NO\textsuperscript{+}, and e.\textsuperscript{5} The thermodynamic properties and the transport coefficients are formulated as a function of temperature and pressure.\textsuperscript{6-8} The Hall parameter \( \beta \) at the position where the electrical conductivity is less than unity is artificially set to zero. The basic equations for the gasdynamics are written as follows.

The mass conservation equation:

\[
\frac{\partial}{\partial t} \iiint_V \rho \, dV + \iint_S \rho u \cdot n \, dS = 0
\]  

The momentum conservation equation:

\[
\frac{\partial}{\partial t} \iiint_V \rho u \, dV + \iint_S \{\rho u (u \cdot n) + p n\} \, dS = \iint_S \vec{T} \cdot n \, dS + \iiint_V \vec{J} \times \vec{B} \, dV
\]  

The total energy conservation equation:

\[
\frac{\partial}{\partial t} \iiint_V \rho E \, dV + \iint_S \rho Hu \cdot n \, dS = \iint_S \kappa (\nabla T \cdot n) \, dS + \iint_S (\vec{T} \cdot \vec{u}) \cdot n \, dS + \iiint_V \left\{ \frac{P}{\sigma} + u \cdot (\vec{J} \times \vec{B}) \right\} dV
\]  

where the total energy and the total enthalpy are defined as

\[
E = e + \frac{u^2}{2}
\]  

\[
H = E + \frac{P}{\rho}
\]  

2.2. Basic equations for electrodynamics

The basic equations for the electrodynamics are composed of the steady Maxwell equations and the generalized Ohm's law as follows.

\[
\nabla \times \vec{E} = \vec{0}
\]  

\[
\nabla \cdot \vec{J} = 0
\]  

\[
\vec{J} = \sigma (\vec{E} + \vec{u} \times \vec{B}) - \frac{\beta}{|\vec{B}|} (\vec{J} \times \vec{B})
\]  

2.3. Numerical procedure

The basic equations for the gasdynamics are discretized by the finite volume method on the rectangular coordinate system. The numerical fluxes of convection terms are evaluated by the advection upstream splitting method (AUSM)-DV scheme.\textsuperscript{9} The first derivatives of velocities and temperatures included in diffusion terms are calculated by using Green's theorem. The time integration is performed by the data-parallel lower-upper relaxation (DP-LUR) implicit scheme.\textsuperscript{10} The no-slip wall condition is used on the wall boundary, and the wall temperature is set to 1500 K.

A second-order partial differential equation on the electric potential are derived from Eqs. (6) \textendash (8), and then it is discretized by the Galerkin finite element method. A set of linear equations obtained by the discretization is solved by the GP-BiCG-Safe2 method.\textsuperscript{11}

In this simulation, a multi-block and overlapping structured grid system is used, in which a computational domain is split into five blocks. The flow and electric properties in overlapping regions are complemented by a linear interpolation formula, in which the interpolation coefficients are obtained by solving the simultaneous equations for grid positions in the overlapping regions.

2.4. Flight condition and externally applied magnetic field

Figure 1 shows the forebody shape of a test body, which is composed of a spherical nose and a cone. The spherical nose radius is 1.5 m, and the cone diameter is 5.0 m. Figure 2 shows the computational domain. The number of computational grid points is about \( 1.25 \times 10^6 \). The flight velocity \( u_{\infty} \), the freestream pressure \( p_0 \), and the freestream temperature \( T_0 \) are 6500 m/s, 21.9 Pa, and 247 K, respectively. An amount of attack angle \( \alpha_0 \) is parametrically varied in a range of \( 0^\circ \textendash 25^\circ \). Figure 3 shows the distribution of an externally applied magnetic field assigned in this study. The distribution is assumed to be produced by a
2.5. Definition of force coefficient

In the coordinate system shown in Fig. 4, a drag force \( F_{\text{drag}} \), a lift force \( F_{\text{lift}} \), and a side force \( F_{\text{side}} \) are formulated by

\[
F_{\text{drag}} = F_{\text{drag},p} + F_{\text{drag},l}
\]

\[
F_{\text{lift}} = F_{\text{lift},p} + F_{\text{lift},l}
\]

\[
F_{\text{side}} = F_{\text{side},p} + F_{\text{side},l}
\]

\[
F_{\text{drag},p} = -\iint_{S_{\text{wall}}} (pm)_x \, dS \cos \alpha_A
\]

\[
-\iint_{S_{\text{wall}}} (pm)_y \, dS \sin \alpha_A
\]

\[
F_{\text{lift},p} = -\iint_{S_{\text{wall}}} (pm)_x \, dS \sin \alpha_A
\]

\[
+\iint_{S_{\text{wall}}} (pm)_y \, dS \cos \alpha_A
\]

\[
F_{\text{side},p} = -\iint_{S_{\text{wall}}} (pm)_y \, dS
\]

where the total force \( F_i \) (\( i = \text{drag}, \text{lift}, \text{or side} \)) consists of the force attributed to the surface pressure \( F_{\text{pp}} \) and the force attributed to the reaction force of the Lorentz force \( F_{\text{ll}} \).

A drag force coefficient \( C_{\text{drag}} \), a lift force coefficient \( C_{\text{lift}} \), and a side force coefficient \( C_{\text{side}} \) are defined, respectively, as

\[
C_{\text{drag}} = \frac{8F_{\text{drag}}}{\rho U^2 \pi \phi^2},
\]

\[
C_{\text{lift}} = \frac{8F_{\text{lift}}}{\rho U^2 \pi \phi^2},
\]

\[
C_{\text{side}} = \frac{8F_{\text{side}}}{\rho U^2 \pi \phi^2}.
\]

3. Results and Discussion

Figure 5 depicts the stream lines of electric current and the distributions of electric current density on the \( x-z \) plane in the cases with and without taking the Hall effect into account for the two cases of \( \alpha_A = 0^\circ \) and \( \alpha_A = 25^\circ \). When the Hall effect is not taken into account, a circular electric current, which has larger densities near the shock front than near the wall surface, occurs symmetrically with respect to the \( x-z \) plane for any attack angle. In the cases with non-zero attack angles, however, the center of the circular electric current shifts to the negative direction of the \( z \)-axis. When the Hall effect is taken into account, a spiral-like electric current is induced by applying the magnetic field regardless of attack angles. In contrast to the electric current density without the Hall effect, the one with the Hall effect becomes larger near the wall surface than near the shock front. Furthermore, the region with higher electric current densities becomes confined to the area around the stagnation point. It can be also found from Fig. 5-(c) that when the attack angle is zero, the Hall effect induces the symmetric spiral-like electric current with respect to the \( x-z \) plane. In the
cases of non-zero attack angles, on the other hand, the Hall effect shifts the center of the spiral-like electric current to the positive direction of y-axis, as can be seen from Fig. 5-(d).

Figure 6 shows the distributions of the electric potential in the case without taking the Hall effect into account for the two cases of $\alpha_d = 0^\circ$ and $\alpha_d = 25^\circ$. In the zero attack angle case, the electric potential difference is not induced in space if the Hall effect is not considered, as can be seen from Fig. 6-(a). For the non-zero attack angle cases, in contrast, the electric potential difference is generated by the difference in Faraday’s electromotive force between the upper (+z) and the lower (−z) sides, even when the Hall effect is neglected, as can be seen from Fig. 6-(b). However, the generated electric potential has a distribution symmetric with respect to the $x$-$z$ plane.

Figure 7 depicts the distributions of the electric potential in the case with taking the Hall effect into account for the two cases of $\alpha_d = 0^\circ$ and $\alpha_d = 25^\circ$. In the case of the zero attack angle, the Hall effect induces an electric potential distribution axisymmetric with respect to $x$-axis, and also the induced electric potential is constant in the azimuthal direction on any plane parallel to the $y$-$z$ plane, as can be seen from Fig. 7-(a). When the attack angle is not zero, the Hall effect leads to an electric potential distribution asymmetric with respect to all of $x$, $y$, and $z$-axes, as shown in Fig. 7-(b). The asymmetric distribution of electric potential is generated by the difference in Hall electromotive force between the upper (+z) and the lower (−z) sides, and also the asymmetric distribution of electric potential results in shifting the center of the spiral-like electric current to the positive direction of y-axis, as shown in Fig. 7-(b).

Figure 8 shows the effect of MHD flow control on the distributions of the convective wall heat flux in the cases with and without taking the Hall effect into account for the two cases of $\alpha_d = 0^\circ$ and $\alpha_d = 25^\circ$. From this figure, one can see that the convective wall heat flux is mitigated by MHD flow control for both $\alpha_d = 0^\circ$ and $\alpha_d = 25^\circ$ cases regardless of whether the Hall effect is taken into account or not. However, the mitigation amount in the case with the Hall effect is larger than the one in the case without it. This is probably because the Hall effect strengthens the electric current densities near the wall surface, so that the deceleration of the flow near the wall surface

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**Fig. 5.** Stream lines of electric current and distribution of electric current density on $x$-$z$ plane ($y = 0$ m) in the cases with and without taking the Hall effect into account for the two cases of $\alpha_d = 0^\circ$ and $\alpha_d = 25^\circ$.

**Fig. 6.** Distribution of electric potential in the case without taking the Hall effect into account for the two cases of $\alpha_d = 0^\circ$ and $\alpha_d = 25^\circ$.

**Fig. 7.** Distribution of electric potential in the case with taking the Hall effect into account for the two cases of $\alpha_d = 0^\circ$ and $\alpha_d = 25^\circ$.
by the Lorentz force arises stronger in the case with the Hall effect than the case without the Hall effect. For $\alpha_A = 25^\circ$ in the case with the Hall effect, the distribution of electric current is asymmetrical about $x-z$ plane (Fig. 5), while the distribution of convective wall heat flux is almost symmetric about $x-z$ plane, as can be seen from Fig. 8-(f). However, the authors expect that the convective wall heat flux with the Hall effect would have a asymmetric distribution about $x-z$ plane for non-zero attack angle cases, if MHD flow control is operated under the conditions with stronger MHD interaction. This is because the stronger the asymmetry of electric current about $x-z$ plane becomes, the larger the difference in Lorentz force between the right and the left side of the test body becomes.

Figure 9 shows the influence of MHD flow control on the relationship between the attack angle and the drag force coefficient for the cases with and without taking the Hall effect into account. When the Hall effect is neglected, MHD flow control leads to slightly increasing drag force coefficients for any attack angle. For the case with taking the Hall effect into account, however, MHD flow control has hardly effect on the drag force coefficients. The difference between the impacts of MHD flow control on drag forces with and without the Hall effect is attributed to the fact that the region with higher electric current densities becomes confined to the area around the stagnation point by taking the Hall effect into account, and therefore the volume integration of the Lorentz force in the case with the Hall effect is considerably small compared to the one in the case without it.

Figure 10 presents the influence of MHD flow control on the relationship between the attack angle and the lift force coefficient for the cases with and without taking the Hall effect into account. When the Hall effect is neglected, a lift force coefficient is slightly decreased by MHD flow control for non-zero attack angle cases because the center position of circular electric current is located in the negative side of $z$-axis, and consequently a reaction force of the Lorentz force acts on the vehicle in the opposite direction of lift force. For the case with taking the Hall effect into account, however, the effect of MHD flow control on the lift force coefficients is negligibly small compared to that in the case without the Hall effect, as can be seen Fig. 10. This is because the Hall effect narrows the region with strong Lorentz force.

Figure 11 depicts the influence of MHD flow control on the relationship between the attack angle and the side force coefficient for the cases with and without taking the Hall effect into account. When the Hall effect is neglected, no side force is induced by MHD flow control. In contrast, MHD flow control in the case with taking the Hall effect into account generates a...
that the direction and strength of side forces strongly depend on attack angles, and the side force becomes zero at a certain attack angle.

The side force \( F_{\text{side}} \) consists of the following two components: one is the side force attributed to the surface pressure \( F_{\text{side,p}} \) and the other is the side force attributed to the reaction force of the Lorentz force \( F_{\text{side,l}} \). Figure 12 shows the contributions of \( F_{\text{side,p}} \) and \( F_{\text{side,l}} \) to the side force coefficient \( C_{\text{side}} \) for each attack angle. It can be seen from this figure that the side force induced by the surface pressure acts on the body in the positive \( y \)-direction for all of the attack angles covered in this study. In contrast, the side force induced by the reaction force of the Lorentz force acts on the body in the negative \( y \)-direction for all of them. Furthermore, the side force induced by the surface pressure has a peak value at about \( \alpha_4 = 15^\circ \), because the area of wall surface where the large surface pressure acts becomes narrow with increasing attack angles. The side force by the reaction of the Lorentz force, however, becomes negatively larger with increasing attack angles. This is because the asymmetry of electric potential about \( x-z \) plane becomes larger with increasing it, and so the center of the spiral-like electric current shifts to the positive \( y \)-direction. It can be also predicted from Fig. 12 that there is an attack angle where the sum of these two components is zero, i.e. the attack angle where the side force becomes zero. The attack angle is about 17 to 18 degree.

### 4. Conclusion

The authors examined the influence of the Hall effect on MHD flow control in reentry flight with non-zero attack angles by three-dimensional magnetohydrodynamic (MHD) numerical simulation.

The numerical results indicate that MHD flow control can effectively function as a thermal protection scheme in reentry flight with non-zero attack angles as well as with the zero attack angle, even under the flight conditions where the Hall effect is not negligible.

However, the Hall effect weakens a drag enhancement effect of MHD flow control in reentry flight with non-zero attack angles as well as with the zero attack angle. This is because the region with high electric current densities becomes confined to an area around the stagnation point by taking the Hall effect into account.

Furthermore, the numerical results suggest that when the MHD flow control is operated for reentry vehicles with non-zero attack angles, the side force is generated due to the distortion of electric currents on the planes orthogonal to the moving direction of the vehicle by the Hall effect. The strength and direction of side force induced by MHD flow control depend on attack angles, and also the side force disappears at a certain attack angle.

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### References

