On-board Orbit Determination using Sun Sensor and Optical Navigation Camera for Deep-Space Missions*

By Yosuke KAWABATA1) and Yasuhiro KAWAKATSU2)

1) Department of Advanced Energy, Graduate School of Frontier Sciences, The University of Tokyo, Tokyo, Japan
2) Institute of Space and Astronautical Science, JAXA, Sagamihara, Japan

On-board orbit determination (OD) using a Sun sensor and optical navigation camera (ONC) for autonomous navigation (AutoNav) is discussed in this paper. In low-Earth orbits, a global positioning system (GPS) is used for AutoNav. On the other hand, in deep space, the OD has been performed using range and range-rate (RARR), which is a traditional ground-tracking approach applying radio waves. RARR enables OD to have higher accuracy compared to other methods. However, such radio navigation has inevitable problems, such as the delay of radio waves, reduction in radio-wave strength and transmitter limitations. The influence of these problems becomes significant, especially for deep-space missions. Furthermore, it requires ground station staff to operate the spacecraft with full attention, which increases the operational cost considerably. Therefore there has been a growing interest in the AutoNav of the spacecraft in recent years because AutoNav can eliminate the aforementioned problems. The utilization of the AutoNav in deep space can reduce the complexity of operation at the ground station, and especially has a significant impact on reducing operational cost. This paper focuses on the configuration of observation objects and the sampling frequency for observation. Finally, as an example, the selection of observation and Earth-resonant trajectory are discussed.

Key Words: Autonomous, Navigation, Optical Navigation, Batch Sequential Filter, On-board

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$X$</td>
<td>state vector</td>
</tr>
<tr>
<td>$x$</td>
<td>state deviation vector</td>
</tr>
<tr>
<td>$y$</td>
<td>observation deviation vector</td>
</tr>
<tr>
<td>$\Phi$</td>
<td>state transition matrix</td>
</tr>
<tr>
<td>$r$</td>
<td>distance from the center of Body to spacecraft</td>
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<tr>
<td>$P$</td>
<td>covariance matrix</td>
</tr>
<tr>
<td>$H$</td>
<td>observation matrix</td>
</tr>
<tr>
<td>$R$</td>
<td>measurement noise covariance matrix</td>
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<tr>
<td>$a$</td>
<td>slope</td>
</tr>
<tr>
<td>$b$</td>
<td>intercept</td>
</tr>
<tr>
<td>$\mu$</td>
<td>gravitational parameter</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>axis headed for a bisector of an angle</td>
</tr>
<tr>
<td>$\eta$</td>
<td>axis vertical to $\zeta$</td>
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Subscripts

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>0</td>
<td>initial</td>
</tr>
<tr>
<td>$i$</td>
<td>index of body</td>
</tr>
<tr>
<td>$b0$</td>
<td>batch reference time</td>
</tr>
<tr>
<td>$F$</td>
<td>final</td>
</tr>
<tr>
<td>$p$</td>
<td>probe</td>
</tr>
<tr>
<td>$pi$</td>
<td>from probe to body $i$</td>
</tr>
<tr>
<td>body</td>
<td>body frame</td>
</tr>
<tr>
<td>-</td>
<td>a priori estimated value</td>
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© 2017 The Japan Society for Aeronautical and Space Sciences
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years, because the AutoNav can avoid the above-mentioned problems.\textsuperscript{1-3} Note that the application of AutoNav in deep space can also be conductive to reducing operational cost.\textsuperscript{4}

This paper proposes AutoNav using observations of bodies in the Solar System, such as planets, with optical navigation cameras (ONCs) and some on-board equipment. This is a kind of optical navigation. The Deep Space 1 (DS1) mission, the first mission in NASA’s New Millennium Program, has demonstrated AutoNav using the ONC. It is a representative example of AutoNav. DS1 observed the asteroid Braille and comet, Borrelly and performed AutoNav when DS1 flew by Braille and Borrelly. Although DS1 has experienced problems with the AutoNav algorithm, finally, it achieved success.\textsuperscript{5, 6}

This paper suggests the use of Sun sensors as on-board equipment to produce data for AutoNav. This is because the spacecraft usually have plural Sun sensors to detect the Sun’s direction; therefore, there is no extra cost and mass. Additionally, an attitude maneuver to observe the Sun is not necessary. To apply the ONC and Sun sensors as observation equipment, the following points are discussed in this paper.

First, features of the proposed AutoNav are discussed in terms of analytical approach and numerical approach, which clarifies the observation configurations are good or bad for the OD. Next, the observation frequency is presented. Finally, a case of an Earth-resonant trajectory with the proposed AutoNav is discussed as an application to actual missions.

2. Observation System and Most Likelihood Estimation Filter Method

2.1. Observation system

For AutoNav, the spacecraft is required to observe some bodies in the Solar System by mounted devices in this paper. This is one of the AutoNav methods without radio wave. The position and velocity of these celestial bodies are well-known because of long time observations from the ground. Then, the spacecraft can use such information of celestial bodies and actual observation information for the on-board OD.

Figure 1 shows an observation configuration on a spacecraft-centered coordinate frame. Each angle in Fig. 1 can be obtained from equipment as observables. The angles $\theta$ and $\phi$, direction information from spacecraft to a body, are in-plane and out-of-plane angles, respectively. The number of observation objects is limited to two to simplify the problems. The attitude maneuver can be neglected because the required time for the attitude maneuver is too short in comparison with the time to observe the objects for orbit determination. Therefore, the time for attitude maneuver is not considered in this paper. Moreover, the light-time equation is not taken into account, and observables are instantaneous values. In addition, the Sun exclusion region of equipment and eclipses are not taken into account. Each angle in Fig. 1 and an observation matrix, which is the derivative of observables with respect to the state vector, are expressed by the following equations:

$$\begin{bmatrix} \theta_1 \\ \phi_1 \end{bmatrix} = \begin{bmatrix} \arctan \frac{\frac{\mu_{Bi}}{p_i}}{\frac{\mu_{Bi}}{p_i} + \frac{\mu_{Bi}}{p_i}} \\ \arctan \frac{\frac{\mu_{Bi}}{p_i}}{\frac{\mu_{Bi}}{p_i} + \frac{\mu_{Bi}}{p_i}} \end{bmatrix} \left( -\pi \leq \theta_1 < \pi, -\frac{\pi}{2} \leq \phi_1 < \frac{\pi}{2} \right)$$

$$\hat{Y}_i = \begin{bmatrix} -y_{pi} \sqrt{x_{pi}^2 + y_{pi}^2} & x_{pi} \sqrt{x_{pi}^2 + y_{pi}^2} & 0 & 0 & 0 \\ -x_{pi} \sqrt{x_{pi}^2 + y_{pi}^2} & y_{pi} \sqrt{x_{pi}^2 + y_{pi}^2} & 0 & 0 & 0 \\ \frac{x_{pi}^2}{r_i^2} \sqrt{x_{pi}^2 + y_{pi}^2} & \frac{y_{pi}^2}{r_i^2} \sqrt{x_{pi}^2 + y_{pi}^2} & 0 & 0 & 0 \end{bmatrix}$$

where $r_i = \sqrt{x_{pi}^2 + y_{pi}^2 + z_{pi}^2}$, $i = 1, 2$.

2.2. Batch sequential filter\textsuperscript{7}

To estimate the spacecraft state vectors expressed by the position and velocity precisely, a certain type of filter is generally used. The Kalman filter and Batch Filter are often mentioned as typical filters. As this paper focuses on the interplanetary cruise, real-time estimation is insignificant. Therefore, the Batch Sequential Filter is selected, which is more robust than the Kalman filter.

The Batch Sequential Filter has two properties; namely, batch property and sequential property. The batch formulation provides an estimate of the state at a certain epoch using an entire batch or set of data. The sequence property enables the filter to cope with the uncertainties in the dynamical models.

The spacecraft dynamics and observation equation can be written as

$$\dot{X} = F(X, t) = -\mu \frac{r}{r^3}, \ X(t_k) = X_k$$

$$Y_i = G(X_i, t_i) + e_i, i = 1, \ldots, l$$

where $F$ means the equation of motion, and $G$ represents observables consisting of Eq. (1). $X_k$ is the unknown n-dimensional state vector at the time $t_k$, $e_i$ are errors in the observation. To clarify the properties of the suggested method,
a two-body problem is considered as the spacecraft dynamics. The nonlinear orbit determination problem is replaced by a linear orbit determination problem. Therefore, the state deviation vector, $x$, and the observation deviation vector, $y$, are defined by the following equations:

$$x(t) = X(t) - X_0(t)$$  \hspace{1cm} (5)  \\
y(t) = Y(t) - Y_0(t)$$  \hspace{1cm} (6)

where $\Theta^*$ indicates the reference solution. The linearized equation can be written as:

$$\dot{x}(t) = A(t)x(t)$$  \hspace{1cm} (7)  \\
y(t) = \tilde{H}_i x_i + e_i$$  \hspace{1cm} (8)

where

$$A(t) = \frac{\partial F(t)}{\partial X(t)}$$  \hspace{1cm} (9)  \\
$\tilde{H}_i = \frac{\partial G(i)}{\partial x(t)}$  \hspace{1cm} (10)

The initial condition correction factor $\hat{x}_{b0}$ is given by the following equation. When the process has not converged, the nominal trajectory is updated by $\hat{x}_{b0}$ until the process converges.

$$\hat{x}_{b0} = (H^T R^{-1} H + P_0)^{-1} (H^T R^{-1} Y + P_0^{-1} \hat{x}_{b0})$$  \hspace{1cm} (11)  \\
$X_{b0} = X_{b0} + \hat{x}_{b0}$  \hspace{1cm} (12)

where

$$H_i = \tilde{H}_i (t_i, \Psi_{b0}).$$  \hspace{1cm} (13)

$R_k$ is the $n \times n$ data weighting matrix. $\sigma_j$ is the weighted value based on the observation precision of observation equipment.

$$R = \begin{bmatrix} \sigma_1^{-2} & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & 1/\sigma_n^2 \end{bmatrix}$$  \hspace{1cm} (14)

3. Analytical Approach and Simulation Results

3.1. Analytical approach

To understand the optimum observation configuration of the optical navigation, the simple system shown in Fig. 2 is considered. This coordinate is a two-dimensional coordinate system containing Probe, Body 1, and Body 2. The direction of the $\xi$ axis is the direction of a bisector of two directions from the probe to each body in Fig. 1. The $\eta$ axis is normal to the $\xi$ axis on the two-dimensional plane.

A cross point of two lines pointing to the two bodies is computed using Eqs. (15) and (16).

$$x_p = \frac{b_2 - b_1}{a_1 - a_2}$$  \hspace{1cm} (15)  \\
y_p = a_1 x_p + b_1$$  \hspace{1cm} (16)

The angle $\Theta$ is an observable, and states of bodies are precisely known information because such bodies as planets are observed from the ground for long periods of time. The relationship between each slope and angle, and the relationship between each $\Theta$ is shown below.

$$a_1 = \tan \theta_1, a_2 = \tan \theta_2$$  \hspace{1cm} (17)  \\
$$\tan \theta = \tan (\theta_1 - \theta_2) = \frac{a_1 - a_2}{1 + a_1 a_2}$$  \hspace{1cm} (18)

From the above equations and the symmetric property, the position $x_p$ of the spacecraft can be computed using Eq. (19).

$$x_p = \frac{y_2 - y_1}{2 \tan \theta} + \frac{1}{2} (x_1 + x_2)$$  \hspace{1cm} (19)

Taking account of the small error $\delta \theta_1$ in angle $\theta_1$ and ignoring the higher-order terms, the position error $\delta x_p$ is given by Eq. (20):

$$\delta x_p \approx -\frac{r_2 - r_1}{2 \sin \theta_1} \delta \theta$$  \hspace{1cm} (20)

where $y_i = r_i \sin \theta_1$ and $r_i = \sqrt{(x_i - x_p)^2 + (y_i - y_p)^2}$.

$$\delta y_p = \tan \theta_1 \delta x_p$$  \hspace{1cm} (21)  \\
$$= -\frac{r_2 - r_1}{2 \cos \theta_1} \delta \theta_1$$  \hspace{1cm} (22)

The good or bad angles for observation between two lines of sight from the spacecraft to bodies are given by Eq. (22). Equation (22) includes three parameters: the relative distances from the spacecraft, the angle between two lines of sight, and the observation uncertainty. To pay attention to these parameters, Eq. (22) also indicates how to minimize the uncertainty of the OD. In other words, the coefficient of $\delta \theta_1$ is related to OD accuracy. This analytical approach is the two dimensional approach. However, observables from the ONC
are direction information and insensitive to observation direction. This means that the maximum error of an error ellipsoid is in the plane consisting of two observation direction. The knowledge of this analytical approach can be extended to three dimensions.

3.2. Characteristic observation and its features

In the previous section, the analytical equation, Eq. (22), is derived. To acquire and confirm features of the proposed AutoNav method, some simulations are performed with the following setting. To clarify the influence of the observation configurations, the error covariance matrix $P$ is not updated in the sequential estimation algorithm in this section. This simulation condition sheds light on the observation properties. We assume that the spacecraft follows the Earth’s trajectory. The spacecraft observes the Sun and Mars. The other conditions are shown in Table 1 and Table 2. To make the computation time short and to distinguish the variation clearly, the observation frequency of 4,320 s was chosen. As for the batch interval, 8,640 s was chosen to shorten the time propagating the covariance matrix.

Table 1. Observation accuracy of each piece of equipment.

<table>
<thead>
<tr>
<th>Equipment</th>
<th>Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>ONC</td>
<td>0.1 deg</td>
</tr>
<tr>
<td>Sun sensor</td>
<td>0.1 deg</td>
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Table 2. Computational conditions.

<table>
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<tr>
<th>Condition</th>
<th>Setting</th>
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<tbody>
<tr>
<td>Observation frequency</td>
<td>4,320 s</td>
</tr>
<tr>
<td>Batch Interval</td>
<td>8,640 s</td>
</tr>
<tr>
<td>Observation objects</td>
<td>Sun, Mars</td>
</tr>
</tbody>
</table>

Fig. 3. Time variation of Standard deviation of position.

Fig. 4. Magnified view of Fig. 3.

Fig. 5. Parallel observation directions at time (a) (heliocentric J2000EC coordinate).

Fig. 6. Orthogonal observation directions at time (b) (heliocentric J2000EC coordinate).
The time variation of standard deviation indicates the estimation quality as shown in Fig. 3 and Fig. 4. Figure 4 is a close-up view of Fig. 3. The largest error part (a) suggests the worst observation geometry, and the smallest error part (b) means the best observation geometry in Fig. 3 and Fig. 4. The error is also larger at part (c). Figure 5, Fig. 6 and Fig. 7 show the locations of the spacecraft, the Sun, and Mars at each time. From these figures, the worst configuration is a conjunction; that is, the spacecraft and bodies lie in a line as shown in Fig. 5. In addition to this, a case like Fig. 7, in which Mars is at opposition to the Sun, is also not desirable. The spacecraft cannot determine its trajectory precisely using optical information from the ONC under these configurations. To begin with, the spacecraft cannot observe Mars in configuration (a) because the ONC cannot be pointed in the direction of the Sun. Fortunately, the spacecraft can know when these configurations occur before the launch, and it can cope with these problems. There are two simple coping techniques to avoid prospects in these cases. First, changing the observation objects to determine the trajectory more precisely. Second, propagating the accurate state of the spacecraft and the error covariance matrix. On the other hand, the best configuration occurs when two lines of sight from the spacecraft to bodies cross at right angles. These results indicate times when the spacecraft should observe objects as priority and otherwise. These simulation results match the analytical approach result, which indicates the geometry is better when $\sin 2\theta_1$ in Eq. (22) is smaller. In other words, Eq. (22) can guide the spacecraft on how to select observation objects. In addition, Eq. (22) indicates that the closer observation objects are, the better the OD accuracy. From Fig. 3, Fig. 5, and Fig. 7, this is confirmed.

3.3. Sampling frequency

In actual operation, the sampling frequency of each sensor is higher than the previous section’s assumption. The order of frequency is generally a few hertz. Therefore, the spacecraft should observe celestial bodies to estimate its state precisely when possible.

During a short time, orbital motion can be negligible. Therefore, the standard deviation, in this case, obeys the rule of root N raw shown in Eq. (23).

$$\sigma_N = \frac{\sigma_0}{\sqrt{N}}$$  \hspace{1cm} (23)

The Batch Sequential Filter and analytical results are similar in Fig. 8. The sampling frequency is 1 Hz, and the number of data points is 144,000. The accuracy of OD is generally relative to the number of data points. Depending on the mission needs, the necessary data points can be computed previously.

3.4. Earth-resonant trajectory

The Earth-resonant trajectory is often applied to the trajectory design, which allows the spacecraft to cruise back to the Earth for Earth gravity assistance. This can reduce the $\Delta V$. There are some merits for the proposed AutoNav method. For example, the distance between the spacecraft and Earth is always close. Furthermore, the distance between the spacecraft and Sun is kept at almost 1 AU during the Earth-resonant trajectory phase. This is desirable for operation of the Sun sensor and power generation. The effect of keeping the distance between the spacecraft and an observation object close is particularly important in terms of Eq. (22) because the coefficient of $\delta \theta_1$ becomes small.

In this section, the initial position of the spacecraft was set to a position a little farther from the Earth because the dynamics of the trajectory is a two-body problem, taking into account only the gravity of the Sun. The other computational conditions are shown in Table 3 and Table 4. Though the batch interval is 50, this is not significant because a two-body problem is considered here and the gravity field is uniform.

To deal with the possibility of actual operation, repetitions of the observation period and Sun observation period are assumed. This is because it’s not true, due to some troubles, mission requirements and so on, that the spacecraft always can observe the bodies by the ONC. Then, observation time and Sun observation time are set in this simulation as shown in Fig. 9. The spacecraft can use Sun sensors to observe the
During period other than observation time because the spacecraft has Sun sensors and can observe the Sun anytime. An initial observation time is assumed to be 12 h and the other observation time is 5 h. This is because the spacecraft should observe celestial bodies for a longer time during the initial observation to determine its trajectory more precisely when the covariance is large. All Sun observation times are set to be 24 h.

Figure 10 shows the estimated trajectory and true trajectory. Figure 11 and Fig. 12 show the error between two trajectories and the distance of three standard deviations, respectively. As for Fig. 10, the error from the true trajectory is less than 50km over the entire trajectory, as shown in Fig. 11. The spacecraft can determine its trajectory using the proposed method. As a benchmark, we added the corresponding value of half-beam width of the Usuda Deep Space Center (UDSC) antenna to Fig. 12. The red line in Fig. 12 is criterion that the spacecraft can communicate with a ground station. If the distance between the spacecraft and the Earth is small, the criterion of the error angle becomes strict. Therefore, the precise OD during initial observation is more important than at other times. The estimated values in Fig. 12 are less than this criterion at the initial observation. Therefore, the result of estimation is good for these results. If Sun sensors are not used for OD during the Sun observation time, the error will be larger than the case using Sun sensors as shown in Fig. 12. From this result, Sun sensors give the spacecraft useful observation data for the state estimation.

From the viewpoint of Eq. (22), the spacecraft can select the optimal bodies to observe. Especially, the spacecraft can always use the Sun’s direction obtained by Sun sensors as an observable object. In this case, the optimal second observed object can be selected using the following selection factor, which is defined as the reciprocal of the coefficient of \( \delta \theta_1 \) in Eq. (22). It is generally believed that the optical information, like the observation using an ONC, has good sensitivity to the direction perpendicular to the line of sight, and the line of sight error is the largest, which is on the plane consisting of two directions from the spacecraft to two bodies. This means that the selection factor can offer an indirect measure of OD accuracy.

\[
\text{Selection Factor} = \frac{\sin \theta_s}{r_s} \tag{24}
\]

where \( r_s \) is the sum of relative distances from the spacecraft to the bodies, which is nondimensionalized by the astronomical unit. \( \theta_s \) is the angle between two observation directions.

Figure 13 shows the selection factor for each body in the case when the spacecraft uses the Sun’s direction, which indicates the best second body is the Earth during the entire period.

On the other hand, to use the property of the Earth-resonant trajectory, we can consider the case that the first body to observe is the Earth. In this case, the spacecraft needs two ONCs to observe two celestial bodies. The selection factor to select the second body is shown in Fig. 14. This figure shows the selection factors of Mercury and Venus are higher than that of the Sun over a period of time. Namely, if the spacecraft has two ONCs and needs precise OD, there are other observation choices to improve OD accuracy.

| Table 3. Observation accuracy of each piece of equipment. |
|----------|-----------|
| ONC      | 0.001 deg |
| Sun sensor| 0.001 deg |

| Table 4. Computational condition. |
|-----------------|-----------|
| Observation frequency | 1 s |
| Batch interval     | 50 s     |
| Observation objects | Sun, Earth |

Objects: Sun, Earth

Duration: 12 [h] 24 [h] 5 [h] 24 [h] ... Time

Fig. 9. Observation objects and observation time.
the selection of observation objects is important for the spacecraft to observe according to the mission requirements. Estimation results are given when the spacecraft is aligned during observations conducted for a short period of time. This is obvious from these results that accurate estimating the state of the spacecraft.

If the size of an observation object is taken into account, there is the possibility that the spacecraft can achieve higher observation accuracy to estimate the object’s centroid precisely, which leads to higher-state estimation results.

4. Conclusions

The characteristics and one application of a suggested autonomous navigation method were discussed in this paper. The following results are revealed.

First, the optimum observation configuration was found using analytical and numerical approaches. As a result, the best observation configuration occurs when two observation directions are perpendicular and the sum of the distance to observation bodies is small. On the other hand, the worst estimation results are given when the spacecraft is aligned with observation objects. It is obvious from these results that the selection of observation objects is important for the accurate estimating the state of the spacecraft.

Next, the observation frequency obeys the root $N$ law during observations conducted for a short period of time. This means we can calculate how many data points the spacecraft should observe according to the mission requirements.

Finally, with regard to actual missions, an Earth-resonant trajectory is given as an example. Moreover, the efficient use of Sun sensors for orbit determination is proposed. Generally, plural Sun sensors are equipped with to obtain the Sun’s direction, which results in no extra cost or weight for the autonomous navigation. Under this assumption, the Earth-resonant trajectory has some merits for the autonomous navigation method proposed with regards to Earth and Sun distances. That is to say, the Earth distance is close and the Sun distance is kept to almost 1 AU. The analytical equation also states the method how to select the observation objects, and proves the effectiveness of the Earth-resonant trajectory under the assumption of using Sun sensors for orbit determination.

The proposed autonomous navigation method can reduce operation frequency, which leads to the reduction of operation cost. In the future, several assumptions to simplify problems in this paper also need to be discussed in more detail.

References


