N₂O Flow History Prediction in an Oxidizer Feed Line of Hybrid Rockets

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(Received June 23rd, 2017)

Nitrous oxide is in a gas-liquid equilibrium state at room temperature, and its vapor pressure is very useful concerning devices such as a pressure-fed hybrid rocket engine systems. However, it is not easy to estimate the flow history of the equilibrium flow. The vapor pressure and the latent heat of the vaporization of nitrous oxide strongly depend on its temperature. Though many models have been proposed, we used a simple equilibrium model and examined important parameters and negligible parameters considering experimental results. Experimental results displayed a substantial difference between the top and bottom tank temperatures, and that the tank pressure greatly matched the saturated vapor pressure at the liquid-phase temperature. A simple analysis showed that the determination of initial temperature was the most important factor and the heat transfer from the tank wall to the nitrous oxide was negligible.

Key Words: Hybrid Rocket, Nitrous Oxide, Two-phase Flow, Self-pressurized Tank

Nomenclature

A_{ori} : orifice area, m²
 A_{surf} : liquid surface area of N₂O, m²
 A_{tank} : outside surface area of the tank, m²
 A_{wall} : contact surface area between the tank wall and N₂O liquid, m²
 C_{N2Ol} : specific heat of liquid N₂O, J/kg-K
 C_{ori} : orifice coefficient, -
 C_{tank} : specific heat of the tank, J/kg-K
 C_{vN2Og} : specific heat of gas N₂O at constant volume, J/kg-K
 H_v : latent heat of evaporation of N₂O per unit mass, J/kg
 h_{air} : heat transfer coefficient between air and the tank wall, W/m²-K
 h_{surf} : heat transfer coefficient between liquid and vapor N₂O, W/m²-K
 h_{wall} : heat transfer coefficient between N₂O and the tank wall, W/m²-K
 M_{evap} : mass of the evaporated N₂O, kg
 M_{gas} : mass of the gaseous phase N₂O, kg
 M_{init} : initial mass of the N₂O in the tank, kg
 M_{liq} : mass of the liquid phase N₂O, kg
 M_{out} : mass of the discharged N₂O, kg
 M_{tank} : mass of the tank, kg
 $\dot{m}$: mass flow rate, kg/s
 p_{N2O} : N₂O pressure in the tank, Pa
 p_{atm} : outside pressure (= 101.3 kPa)
 p_{inj} : N₂O pressure at the injector, Pa
 Q_{air} : heat from the air to the tank wall, J
 Q_{in} : heat transfer from the tank wall to liquid N₂O, J
 Q_{rob} : heat robbed from N₂O due to the evaporation, J
 Q_{surf} : heat transfer from gas to liquid N₂O, J
 T_{atm} : outside temperature (= 284 K)

T_{init} : initial temperature, K
 T_{inj} : N₂O temperature at the injector, K
 T_{N2O} : N₂O temperature in the tank, K
 T_{N2Og} : N₂O gas temperature in the tank, K
 T_{N2Ol} : N₂O liquid temperature in the tank, K
 T_{wall} : tank wall temperature, K
 T_0 : initial temperature, K
 t : time, s
 t_{end} : discharge end time, s
 V_{tank} : tank volume, m³
 Δt : time step, s
 ρ_{N2Og} : N₂O gas density in the tank, kg/m³
 ρ_{N2Ol} : N₂O liquid density in the tank, kg/m³

1. Introduction

Nitrous oxide (N₂O) has attracted attention in recent years as an oxidizer in small scale rocket engines.¹⁻³⁾ N₂O is in a gas-liquid equilibrium state at room temperature due to its own vapor pressure. Thus, it does not need a separate pressurant tank. The vapor pressure of N₂O reaches 5 MPa at 293 K, and the critical pressure is 7.2 MPa at 309 K, which is extremely practical with regard to pressure-fed rocket engine systems.

However, it is not easy to accurately estimate the flow characteristics such as pressure or temperature history during tank emptying. In the case of N₂O, as the temperature dependency of vapor pressure and the latent heat of evaporation are large, they significantly affect flow characteristics.

Akiba et al.⁴⁾ displayed a simple tank-emptying model that did not consider the heat capacity of the tank or the real gas effect. The calculated temperature was far lower than the experimental result. They concluded that the heat capacity of the tank was the cause of the difference. On the other hand,

detailed emptying models were also advocated by Zilliac et al.⁵⁾ This model gives very good explanation for their experimental result but requires several physical parameters that are hard to implement. In this paper, we try to explain the experimental N₂O flow characteristics with as simple of a model as possible.

2. N₂O Flow Experiment

2.1. Experimental setup

N₂O flow experiment was conducted to determine the pressure, temperature and mass flow rate history during the tank emptying process. We used a rocket sled propulsion module which consists of four hybrid motors, a single run-tank and a main valve.^{6,7)} Figures 1 and 2 show the side view and the piping diagram of the flow test. A photo of the flow test is shown in Fig. 3. N₂O was fed to the run tank from the mother tank through the fill valve. Usually the pressure of the run tank was lower than that of the mother tank due to the existence of the small vent tube (1/16 inch). Sometimes we need to open the pneumatic vent valve (1/4 inch) to keep a sufficient pressure difference between them, especially at the end of the tank filling process.

Fig. 1. Side view of the run tank and the oxidizer feed line.

Fig. 2. A piping diagram of the rocket sled propulsion module.

Fig. 3. N₂O flow test.

The weight loss of the mother tank was monitored and was considered to be the total mass of the N₂O fed to the run tank. The amount of N₂O remaining in the feeding line and the loss from the vent tube was considered to be about 3 kg at maximum. This was accounted for as an error, which should be subtracted. Pressure sensors (KYOWA PHB-A-10MP) were equipped at the top of the tank and upstream of the injectors. Thermocouples were mounted at the top and the bottom of the tank and upstream of the injectors. The measurement error of the pressure and the temperature was less than 2 %.

2.2 Experimental results

Generally, it is difficult to determine the accurate mass flow rate of the N₂O flow at equilibrium condition with orifice or Venturi flow sensors, as a slight increase of the flow velocity in the feed line easily causes cavitation and effective orifice coefficient changes. One of the best ways would be to measure the weight change or the liquid level of the run tank directly. However, as we only knew the initial total mass in this experiment, the following calculation process was adopted to estimate the mass flow rate history. The mass flow rate at a specific time is obtained from Eq. (1), where the density or the pressure drop would be determined from p_{inj} and T_{inj} . We do not yet know the exact value of C_{ori} , but it can be solved by imposing that its time integral must satisfy Eq. (2). Here we assumed that C_{ori} was constant during tank emptying.

$$\dot{m}(t) = C_{ori} A_{ori} \sqrt{2\rho(p_{inj} - p_{atm})} \quad (1)$$

$$M_{init} = \int_0^{t_{end}} \dot{m}(t) dt \quad (2)$$

Resultant C_{ori} was found to be approximately 0.14 for this case. In our experience, C_{ori} determined in this manner was in the range of 0.1 to 0.25. We assumed liquid density at Eq. (1), but actually the vapor phase may be dominant. This is the reason why C_{ori} is unusually small.

Figures 4 and 5 show the pressure and temperature history of the N₂O flow test when the initial mass of N₂O was 9.6 kg and the outside temperature was 284 K. At $t = 4$ s, the liquid phase N₂O is considered to be almost exhausted. It is considered that about 0.98 kg of N₂O remains at this time. There is a substantial difference between the tank top temperature and the tank bottom temperature. This means the inside of the tank did not reach thermal equilibrium. Figure 6

shows the estimated N_2O density and Fig. 7 shows the mass flow rate history for the total flow rate to match the experimental result.

Fig. 4. Pressure history of the N_2O flow test. P_{inj1} to P_{inj4} showed almost the same trend. The difference between the tank pressure and the injector pressure was small.

Fig. 5. Temperature history of the N_2O flow test. There is a substantial difference between the tank top temperature and the tank bottom temperature.

Fig. 6. Density history of the N_2O flow test. Density is determined from $T_{tankBTM}$ and p_{tank} with the NIST database.⁹⁾

Fig. 7. Estimated mass flow rate history for the total flow rate to match the experimental result.

3. Modeling

3.1. Single temperature model

The simplest form of the tank-emptying model is described as follows. In the single temperature model, we assume that the gas-phase and liquid-phase temperatures are the same.

$$T_{N2O l} = T_{N2O g} = T_{N2O} \quad (3)$$

In the single temperature model, we set the initial temperature T_{init} to be the outside temperature T_{atm} . The effect of the initial temperature is described later. P_{N2O} is given as the saturated vapor pressure at T_{N2O} as shown in Eq. (4). As P_{N2O} is defined at the equilibrium state, density can be defined both for the liquid and gas phase as shown in Eqs. (5) and (6). The state of N_2O is very close to the critical point and we must not use the ideal gas state equation anywhere in this calculation. Physical properties are always referred from NIST database software REFPROP.⁹⁾

$$P_{N2O} = P_{sat}(T_{N2O}) \quad (4)$$

$$\rho_{N2O l} = \rho(P_{N2O}, T_{N2O}) \quad (5)$$

$$\rho_{N2O g} = \rho(P_{N2O}, T_{N2O}) \quad (6)$$

We know the total loading N_2O mass M_{init} and the tank volume V_{tank} . From these values, we can set the initial mass of the gas and liquid phases of N_2O as follows:

$$M_{liq} = \frac{\rho_{N2O l}(M_{init} - V_{tank}\rho_{N2O g})}{\rho_{N2O l} - \rho_{N2O g}} \quad (7)$$

$$M_{gas} = M_{init} - M_{liq} \quad (8)$$

At each time step, the N_2O mass that is blown out from the tank is determined as follows:

$$\dot{m} = C_{ori} A_{ori} \sqrt{2\rho(p_{inj} - p_{atm})} \quad (9)$$

$$M_{out} = \dot{m} \Delta t \quad (10)$$

Then, liquid N_2O evaporates to compensate for the volume

of the tank, and the N_2O mass of the new time step is updated.

$$M_{evap} = \frac{M_{out}}{\rho_{N2Og} \left(\frac{1}{\rho_{N2Og}} - \frac{1}{\rho_{N2Ol}} \right)} \quad (11)$$

$$M_{liq}^{t+\Delta t} = M_{liq}^t - M_{out} - M_{evap} \quad (12)$$

$$M_{gas}^{t+\Delta t} = M_{gas}^t + M_{evap} \quad (13)$$

The latent heat of the vaporization of N_2O is also determined from the NIST database. The Clapeyron-Clausius formula should not be used to obtain the latent heat of vaporization because the state of N_2O is very close to the critical point and it would provide the wrong value (higher than true value).

$$Q_{rob} = M_{evap} H_v(P_{N2O}, T_{N2O}) \quad (14)$$

$$Q_{in} = h_{wall} A_{wall} (T_{wall} - T_{N2O}) \Delta t \quad (15)$$

$$Q_{air} = h_{air} A_{tank} (T_{atm} - T_{wall}) \Delta t \quad (16)$$

$$T_{N2O}^{t+\Delta t} = T_{N2O}^t - \frac{(Q_{rob} - Q_{in})}{M_{liq} C_{N2Ol}} \quad (17)$$

$$T_{wall}^{t+\Delta t} = T_{wall}^t - \frac{(Q_{in} - Q_{air})}{M_{tank} C_{tank}} \quad (18)$$

Fig. 8. Comparison between the calculated (dashed) and experimented (solid) N_2O pressure with various heat transfer coefficient.

Fig. 9. Comparison between the calculated (dashed) and experimented (solid) N_2O temperature with various heat transfer coefficient.

Fig. 10. Comparison between the calculated (dashed) and experimented (solid) N_2O mass flow rate (for 4 engines) with various heat transfer coefficient.

Fig. 11. Latent heat of vaporization of N_2O .

Here, we can update the temperature of N_2O . In addition, the wall temperature is updated. The calculation was terminated when M_{liq} became less than 1 kg because this model is valid only for liquid phase discharge.

We assumed a wide range of heat transfer coefficient to evaluate the effect on the flow characteristics. The calculated results are shown in Figs. 8 to 10. If nucleate boiling is occurring, the heat transfer coefficient can be very large. According to Ref. 8), it can reach the order of thousands $W/m^2 \cdot K$. Thus, the maximum assumption was set to be $h_{wall} = 10^4 W/m^2 \cdot K$. However, the simulated temperature history or the mass flow rate was not changed regardless of h_{wall} . This is because the latent heat of the vaporization of N_2O is much higher than the estimated heat transfer from the tank wall to the oxidizer. When we assume the contact area between the tank wall and oxidizer to be $0.5 m^2$, the heat transfer coefficient between them to be $10^4 W/m^2 \cdot K$ and the temperature difference between them to be 100 K as maximum possible values, the heat transfer from the tank wall to the oxidizer becomes 50 kW. However, the latent heat of evaporation of nitrous oxide reaches 200 kJ/kg at $T_{N2O} = 284 K$ as shown in Fig. 11. The N_2O mass flow rate (for 4 engines)

is approximately 2 kg/s, resulting total latent heat is about 400 kW. Thus the heat transfer is almost negligible in this case. Actually the temperature drop of T_{wall} in the calculation was only 2 K even in the case of $h_{\text{wall}} = 10^4 \text{ W/m}^2\cdot\text{K}$. For the same reason, we can neglect the effect of h_{air} for all cases.

Next, the effect of the initial temperature is discussed. As already shown in Fig. 5, the temperature of the liquid part and gas part of N_2O are not the same from the beginning. The initial temperature of the liquid part N_2O ($=290 \text{ K}$) is higher than the outside temperature (284 K) and the initial temperature of the gas part N_2O ($=276 \text{ K}$) is lower than the outside temperature. Such divergence usually occurs but sometimes it does not happen. It may depend on the propellant loading process.

We used these three different temperatures as an initial condition, and evaluated the effect. Figures 12 and 13 clearly show that the initial temperature has a great effect on the pressure and temperature history.

Fig. 12. Comparison between the calculated (dashed) and experimented (solid) N_2O pressure with various initial temperature.

Fig. 13. Comparison between the calculated (dashed) and experimented (solid) N_2O temperature with various initial temperature.

When $T_{\text{init}} = 276 \text{ K}$, corresponding to the initial value of $T_{\text{tankBTM}}(\text{exp})$, the estimated temperature and pressure history agreed well with the experiment. However, after $t=3 \text{ s}$, the estimated curve deviated from the experimental history. Once

the estimated temperature becomes lower than the actual temperature, the temperature decrease is accelerated due to the large latent heat of vaporization.

3.2. Double temperature model

In the experiment, there was a significant difference between the tank top temperature and the bottom temperature. This implies that the temperature of N_2O in the tank is far from the equilibrium condition. Here, we modify the flow model, that is, we assumed gas and liquid phase temperature separately. As an initial condition, we give experimentally obtained values for both of $T_{\text{N}_2\text{Og}}$ and $T_{\text{N}_2\text{Ol}}$. $P_{\text{N}_2\text{O}}$ is determined from the liquid-phase temperature as its saturated pressure, because this treatment gives very good agreement.

$$T_{\text{N}_2\text{Og}}^{t=0} = 290 \text{ K} \quad (19)$$

$$T_{\text{N}_2\text{Ol}}^{t=0} = 276 \text{ K} \quad (20)$$

$$P_{\text{N}_2\text{O}} = P_{\text{sat}}(T_{\text{N}_2\text{Ol}}) \quad (21)$$

From Eq. (3) to Eq. (13), the calculation proceeds as same as the single temperature model. For Eq. (14), we use $T_{\text{N}_2\text{Og}}$ to obtain the latent heat of vaporization assuming the evaporation occurs near the liquid surface layer.

$$Q_{\text{rob}} = M_{\text{evap}} H_v(P_{\text{N}_2\text{O}}, T_{\text{N}_2\text{Og}}) \quad (22)$$

The heat transfer from the gas part to the liquid part of N_2O should be taken into consideration. As a result of parametric survey, h_{surf} did not affect the pressure and temperature history as well as h_{wall} . Here we set h_{surf} to $1000 \text{ W/m}^2\cdot\text{K}$ and $h_{\text{wall}} = 100 \text{ W/m}^2\cdot\text{K}$ as representative values.

$$Q_{\text{surf}} = h_{\text{surf}} A_{\text{surf}} (T_{\text{N}_2\text{Ol}} - T_{\text{N}_2\text{Og}}) \Delta t \quad (23)$$

Gas and liquid temperatures are updated as follows:

$$T_{\text{N}_2\text{Ol}}^{t+\Delta t} = T_{\text{N}_2\text{Ol}}^t - \frac{(Q_{\text{rob}} - Q_{\text{in}} - Q_{\text{surf}})}{M_{\text{liq}} C_{\text{N}_2\text{Ol}}} \quad (24)$$

$$T_{\text{N}_2\text{Og}}^{t+\Delta t} = T_{\text{N}_2\text{Og}}^t - \frac{Q_{\text{surf}}}{M_{\text{gas}} C_{\text{N}_2\text{Og}}} \quad (25)$$

Figures 14 to 17 show the calculated results of the double temperature model. Double temperature model gave higher value at pressure and mass flow rate history. In Fig. 15, both the T_{liq} and T_{gas} history agreed with the experimental results.

Fig. 14. N_2O pressure obtained from single temperature model and double temperature model.

Fig. 15. N_2O temperature obtained from single temperature model and double temperature model.

Fig. 16. N_2O mass flow rate obtained from single temperature model and double temperature model.

Fig. 17. Heat robbed from N_2O due to vaporization obtained from single temperature model and double temperature model.

Fundamental difference between the both models is based on the estimation of the latent heat of vaporization. In Fig. 17, the significant difference exists on Q_{rob} between single and double temperature model. This is because H_v is calculated with $T_{\text{N}_2\text{Og}}$ in the case of double temperature model. $T_{\text{N}_2\text{Og}}$ is always higher than $T_{\text{N}_2\text{Ol}}$ and H_v is smaller at higher

temperature as shown in Fig. 11.

In the Ziliac model,⁵⁾ the temperature at which they defined the latent heat of vaporization is considered to be one of the most important factors. The temperature used to have H_v will be between $T_{\text{N}_2\text{Og}}$ and $T_{\text{N}_2\text{Ol}}$ but the way to find the correct value is not found yet. In order to know this, more complex model must be used. In this study, we had almost good approximation that agrees well to the experimental result with relatively simple model, as initially intended. The most important point is to set the initial temperature correctly. This implies that accurate modelling is also required for tank filling process.

4. Conclusion

With an experimental flow test and a simple analysis on nitrous oxide flow history, the following conclusions are obtained:

- 1) There was a significant difference between the top and the bottom temperature of the run tank in the flow experiment.
- 2) The most important point is to use the correct initial temperature in the calculation. Incorrect initial temperature gives very erroneous history. This implies that careful modelling is also required for tank filling process.
- 3) The heat transfer rate between the tank wall and N_2O was considered to be negligible compared to the latent heat of the evaporation. For the same reason, heat transfer from the air to the tank is also negligible in this experiment. Note that the tank emptying time is relatively short in our experiment, the situation may be different at the other cases⁴⁾. At any rate, one can easily predict the influence of the heat transfer from the tank wall comparing to the latent heat of evaporation of N_2O .
- 4) Double temperature model gives accurate temperature history for both for gas phase and liquid phase N_2O . In the case of the double temperature model, the temperature used for the determination of latent heat should be chosen carefully.

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