Suppression of Out-of-Plane Thermal Deformation of CFRP Reflectors by Linear Actuators

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Carbon fiber reinforced plastic (CFRP) is considered to be one of the possible materials for reflectors of space-based astronomical observation systems because of its high specific elasticity and low coefficient of thermal expansion. However, it was demonstrated that non-negligible out-of-plane thermal deformation would be generated on CFRP reflectors. In this study, capability of a method of suppressing the out-of-plane thermal deformation was examined. In the proposed method, surface shape of the reflectors was controlled by applying external forces with linear actuators so that RMS error due to the thermal deformation was minimized. As a result, it was shown that the out-of-plane thermal deformation caused by fiber orientation error in manufacturing process could be suppressed significantly by the proposed simple method. Sensitivity analysis on location of actuators and input forces was also performed and it was seen that the location of actuators and the input forces need to be controlled strictly to realize high precision surface shape control of CFRP reflectors.

Key Words: Reflector, Composite Material, Shape Control, Thermal Deformation

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tr>
<td>D</td>
<td>defocusing value</td>
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<tr>
<td>E</td>
<td>elasticity modulus</td>
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<tr>
<td>f</td>
<td>input force on a pair of actuators</td>
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<td>G</td>
<td>shear modulus</td>
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<td>N</td>
<td>number of nodes of FEM model</td>
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<tr>
<td>RMS</td>
<td>root mean square of surface shape error</td>
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<tr>
<td>x, y, z</td>
<td>Cartesian coordinates</td>
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<tr>
<td>α</td>
<td>coefficient of thermal expansion</td>
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<tr>
<td>Δp</td>
<td>variation of location of actuators</td>
</tr>
<tr>
<td>ζ</td>
<td>shape error in z-direction generated by unit force or unit temperature variation</td>
</tr>
<tr>
<td>ν</td>
<td>Poisson’s ratio</td>
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<tr>
<td>i</td>
<td>the i th node</td>
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<tr>
<td>opt</td>
<td>optimal</td>
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<tr>
<td>T</td>
<td>unit temperature variation</td>
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1. Introduction

High-precision, large-size reflectors (mirrors) are necessary to realize high-resolution space observation systems. Reflectors of the space observation systems operated in space are required to be light and stiff to reduce cost of launching observation satellites into space. Reflectors are also required to maintain its shape against temperature variation because sometimes they are cooled cryogenic temperature to avoid their infrared radiation and exposed to an environment with large temperature variation. Accordingly, the material of reflectors of high-resolution space observation systems operated in space must have high specific stiffness and low coefficient of thermal expansion (CTE).

Carbon fiber reinforced plastic (CFRP), which is sometimes utilized as a material of space structures, has high specific stiffness and low CTE and accordingly, it is considered to be a suitable material for the reflectors of space observation systems. As an early research on utilization of the CFRP for space-based reflectors, Abusafieh et al. investigated the effects of moisture absorption and microcrack on dimensional stability of the CFRP and it was concluded that the CFRP can be utilized as a material of high-precision structures that require micron-order of dimensional stability.1) Arao et al. studied the effect of creep deformation, moisture absorption, self-shrinking2) and residual stress relaxation3) on long-term shape stability of CFRP reflectors. Yoon et al. studied the effect of thermal deformation and outgassing deformation of a composite structure on the performance of telescopes and it was concluded that thermal deformation degraded optical performance of telescopes significantly more than outgassing deformation.4) This result meant necessity of research on thermal deformation of CFRP reflectors.

In the previous studies, the present authors studied thermal deformation of reflectors experimentally and analytically.5,6) As a result, it was shown that surface coating which is essential to improve electric conductivity of the surface and the existence of error of carbon fiber orientation angle cause non-negligible out-of-plane thermal deformation. Therefore, to realize space-based CFRP reflectors, suppression of the out-of-plane thermal deformation is essential.

There have been some researches on controlling surface shape of reflectors to suppress shape misalignment caused by various factors (e.g. manufacturing error, dimensional error at deployment on orbit, aging) by applying external force. Chen et al. studied a method to reduce undesired deformation of...
polymer based reflector model with thin piezo-electric actuators attached on the surface.\textsuperscript{7} Hill et al. investigated optimal location of piezo-electric actuators which suppress thermal deformation of an inflatable polyimide-based reflector.\textsuperscript{8} In these studies, plate-shape actuators were attached on the surface of the reflectors so that they apply in-plane force. Datashvili et al. investigated controlling surface shape of a reflector composed of carbon fiber reinforced silicone to change its shape by applying out-of-plane forces by linear actuators.\textsuperscript{9} Tanaka developed a method to control surface shape of a 300 mm diameter and 1 mm thickness reflector model composed of aluminum alloy (A2024) by linear piezo-electric stack actuators and surface shape error of 0.9 mm in maximum was concluded to be controllable by six actuators.\textsuperscript{10} Wang et al. studied a method to suppress thermal deformation caused by the existence of out-of-plane thermal deformation due to the existence of surface coating and the fiber orientation error\textsuperscript{5,6} by several linear actuators. Additionally, moving the reflector models themselves rigidly, that is, defocusing was also introduced and optimal value of defocus was sought. Moreover, the deterioration of the efficiency of shape control due to the error of the actuator location and input forces was also investigated by performing sensitivity analysis.

2. Thermal Deformation on the CFRP Reflectors

In the previous study, out-of-plane thermal deformations of two types of CFRP reflector models were experimentally observed.\textsuperscript{5,6} The reflector models were composed of a quasi-isotropic CFRP laminate (Sanki composite Inc., XN60/NM31, \{0/−45/90/45\}) and cup-shaped. One had aluminum and polyurethane coating layers on the top surface and the other had no coating layers. The thickness of the reflector models with and without the aluminum and polyurethane layers were approximately 0.98 mm and 0.86 mm, respectively. The schematic view of the CFRP reflector models is shown in Fig. 1. The diameter and radius of curvature of the reflector models were 300 mm and 1000 mm, respectively. To observe the out-of-plane thermal deformation, the reflector models were placed inside a thermostatic chamber which had heat-resistant glass on the top and the out-of-plane thermal deformation was measured through the glass by a laser displacement sensor placed outside of the chamber. At that time, the reflector models were fixed on the experimental setup by a jig attached at the center. As a result of the observation, it was shown that non-negligible out-of-plane thermal deformation was generated and the out-of-plane displacement had a characteristic mode of distribution for each reflector model.

The observed result was also simulated by finite-element analysis (FEA) to find the reasons why the non-negligible out-of-plane displacement occurred. In the FEA, the engineering simulation software “ANSYS Mechanical APDL 14.5” was used. The analytical model of the reflector models was composed of 4-node shell elements (SHELL181) and the number of nodes and elements were 29453 and 29072, respectively. The origin of $xyC$-coordinate corresponded to the center of the reflector models and the projection of 0-degree directions of lamination of CFRP layers on the $xy$ plane corresponded to $x$-direction, as shown in Fig. 1. When practically laying up a prepreg along the curved surface, the direction of fibers will curve accordingly. However, the reflector model used in this study is sufficiently shallow and the variation of fiber direction is small. Thus, in the analysis, the effect of the curvature on the direction of fiber was ignored. The node in the center of the reflector models was fixed. Each layer of CFRP laminate was assumed to be an orthotropic material. Aluminum and polyurethane were assumed to be isotropic materials. The material constants of the CFRP, the aluminum layer, and the polyurethane layer are listed in Table 1. Thickness of one layer of the CFRP laminate, the aluminum, and the polyurethane layers were assumed to be 0.1075 mm, 0.02 mm, and 0.1 mm, respectively.

Calculated thermal deformation in $z$-direction caused by temperature variation of $+1$ K on CFRP reflector model with aluminum and polyurethane layers is shown in Fig. 2. Both positive and negative displacement were distributed in a circumferential direction of the reflector model. Maximum displacement was found in the direction of approximately $−23^\circ$ and $167^\circ$. Minimum displacement was found in the direction of approximately $−119^\circ$ and $61^\circ$. The maximum and minimum displacement were 4.03 $\mu$m and $−7.98$ $\mu$m, respectively. These results agreed with the experimental results in the order of magnitude.

Calculated thermal deformation in $z$-direction without the surface coating is shown in Fig. 3. In this case, the thermal deformation in $z$-direction is extremely small and it was seen by considering the in-plane displacement that this displacement was generated because the reflector model expanded uniformly.

| Table 1. Mechanical constants assumed in the calculation. |
|-----------------|-----------------|
| CFRP layer      | Aluminum layer  |
| $E_{11}$ 340 GPa | $E$ 70 GPa      |
| $E_{22}$ 5.2 GPa | $\nu$ 0.3       |
| $E_{12}$ 3.9 GPa | $\alpha$ 21$\times$10$^{-6}$/K |
| $\nu_{12}$ 0.35  | Polyurethane layer |
| $\nu_{23}$ 0.3   | $E$ 0.69 GPa     |
| $\sigma_{1}$ -0.7$\times$10$^{-6}$/K | $\nu$ 0.4 |
| $\sigma_{2}$ 35$\times$10$^{-6}$/K | $\alpha$ 100$\times$10$^{-6}$/K |

Fig. 1. Schematic view of the CFRP reflector models.
Fig. 2. Calculated thermal deformation in $z$-direction of the reflector model with surface coating.

Fig. 3. Calculated thermal deformation in $z$-direction of the reflector model without surface coating and fiber orientation error.

Fig. 4. Calculated thermal deformation in $z$-direction of the reflector model with fiber orientation error of $+1^\circ$ on the top surface.

However, characteristic out-of-plane thermal deformation was observed in the experiment. As a cause of this out-of-plane thermal deformation, the present authors assumed the asymmetry caused by the existence of fiber orientation error in lamination. Arao et al.\textsuperscript{12)} pointed out the fiber orientation error of 0.4 degrees in the standard deviation is inevitable on the hand-laminated CFRPs. To assess the effect of the fiber orientation error, fiber orientation error of $+1^\circ$ on the top surface (i.e., $\{1/(-45/90/45)/0\}$) was assumed and thermal deformation in $z$-direction generated on the CFRP reflector model with the laminate condition was calculated. Calculated thermal deformation in $z$-direction on CFRP reflector model with fiber orientation error is shown in Fig. 4. Both positive and negative displacement were distributed in a circumferential direction of the reflector model. Maximum displacement was found in the direction of approximately $-135^\circ$ and $45^\circ$. Minimum displacement was found in the direction of approximately $-45^\circ$ and $135^\circ$. The maximum and minimum displacement were 0.984 $\mu$m and $-0.950 \mu$m, respectively. The mode of distribution of the displacement agreed with the experimental result, however, the magnitude of the displacement was approximately two times larger than experimental result. Thus, this result agreed with the experimental result qualitatively.

When root mean square (RMS) of the surface shape error exceeds $1/14$ of a wavelength of a targeted electromagnetic wave, performance of a reflector is significantly degraded.\textsuperscript{13)} When a reflector for 30 GHz radio astronomical observation system is assumed, the RMS error should be less than 0.7 mm. If the utilization of 3-meter radius CFRP reflector and temperature variation of 300 K are assumed, RMS errors caused by experimentally observed thermal deformation of the reflector model with and without surface coating correspond with approximately 70 mm and 6 mm, respectively. These two exceed the criteria and accordingly, depending on the targeted wavelength, the thermal deformations mentioned above must be suppressed in some method.

3. Active Suppression of the Thermal Deformation by Linear Actuators and Defocusing

In this paper, the surface shape error was suppressed with two pairs of actuators, because two pairs of peak and valley of the displacement appeared in the circular direction of the reflector models as shown in Figs. 2 and 4. The forces applied by the actuators were represented by concentrated forces in $z$-direction. The location of actuators was varied along the edge of the reflector models. Two actuators of each pair were arranged on two nodes on the edge of the reflector models across the center. For each pair of the actuators, the same value of force was applied on the nodes where actuators were arranged. Sometimes defocusing, in which the reflectors are moved rigidly in the direction of the optical axis, improves the optical performance. Therefore, the effect of defocusing on suppressing the thermal deformation of the CFRP reflector model was investigated in addition to the surface shape control.

In this study, this RMS error was calculated by considering not only the displacement in $z$-direction but also the displacements in $x$ and $y$-direction of all nodes. When we assume $\zeta_i$ as the shape error in $z$-direction at the $i$th node position after the thermal deformation from its desired position, which is on the surface of R1000 mm, the RMS error can be calculated as,

$$\text{RMS} = \sqrt{\frac{1}{N} \sum_i \zeta_i^2}, \quad (1)$$

Moreover, the ratio of the RMS after to before shape control was also introduced to indicate the effect of the suppression.
Optimal location and input forces of actuators and defocusing value were calculated by the following procedures.

3.1. Without defocusing

The shape error in z-direction of the ith node \( \zeta_i \) generated by unit temperature variation, force with a magnitude of \( f_A \) from a pair of actuators, and force with a magnitude of \( f_B \) from another pair of actuators can be calculated by three results of the shape error generated by unit forces from two pairs of actuators, \( \xi_A \) and \( \xi_B \), and unit temperature variation, \( \xi_T \), by assuming superposition as,

\[
\zeta_i = f_A \xi_A + f_B \xi_B + \xi_T. \tag{2}
\]

Thus, RMS error of thermal deformation and applied forces \( \text{RMS}(f_A, f_B) \) can be calculated as,

\[
\text{RMS}(f_A, f_B) = \sqrt{\frac{1}{N} \sum_i (f_A \xi_A + f_B \xi_B + \xi_T)^2}. \tag{3}
\]

When we assume

\[
S = \sum_i (f_A \xi_A + f_B \xi_B + \xi_T)^2, \tag{4}
\]

the input forces \( f_A \) and \( f_B \) that minimize \( S \) are the input forces that minimize RMS error at the location of actuators. These values can be calculated by expanding (4),

\[
S = f_A^2 \sum_i \xi_A^2 + f_B^2 \sum_i \xi_B^2 + \sum_i \xi_T^2 + 2f_A f_B \sum_i \xi_A \xi_B + 2f_A \sum_i \xi_A \xi_T + 2f_B \sum_i \xi_B \xi_T, \tag{5}
\]

differentiating \( S \) by \( f_A \) and \( f_B \),

\[
\frac{\partial S}{\partial f_A} = 2f_A \sum_i \xi_A^2 + 2f_B \sum_i \xi_A \xi_B + 2 \sum_i \xi_A \xi_T, \tag{6}
\]

\[
\frac{\partial S}{\partial f_B} = 2f_B \sum_i \xi_B^2 + 2f_A \sum_i \xi_A \xi_B + 2 \sum_i \xi_B \xi_T, \tag{7}
\]

and calculating \( f_A \) and \( f_B \) that meet two formulas,

\[
\frac{\partial S}{\partial f_A} = 0, \quad \frac{\partial S}{\partial f_B} = 0. \tag{8}
\]

In the optimization, the process described above was repeated while location of actuators moved to a set of nodes to the adjacent set of nodes along the edge of the reflector model and input forces that minimize RMS error for various location of actuators were calculated. The set of location of actuators and input forces that minimize RMS error was sought from these results.

3.2. With defocusing

In addition to the shape control by linear actuators, the effect of defocusing was investigated. The optimal input forces and defocusing value can be obtained by adding the term of defocusing value \( D \) in Eq. (4) as,

\[
S = \sum_i (f_A \xi_A + f_B \xi_B + \xi_T + D)^2, \tag{9}
\]

and calculating \( f_A \), \( f_B \) and \( D \) that meet three formulas,

\[
\frac{\partial S}{\partial f_A} = 0, \quad \frac{\partial S}{\partial f_B} = 0, \quad \frac{\partial S}{\partial D} = 0. \tag{10}
\]

The set of optimal location of actuators, input forces and defocusing value that minimize RMS error was sought in the same method described in the previous subsection.

4. Results and Discussion

4.1. With surface shape control

Calculated result of distribution of displacement in z-direction and RMS error with surface shape control of the reflector model with aluminum and polyurethane layers are shown in Fig. 5 and Table 2, respectively. Control forces of
−7.470 mN and 12.252 mN were applied on nodes marked with × (actuators A1 and A2) and + (actuators B1 and B2), respectively. The maximum and minimum displacement were 0.475 μm and −1.787 μm, respectively. As a result of surface shape control, RMS error was reduced to 0.332 μm, which is 14.34% of that without surface shape control.

Calculated result of distribution of displacement in z-direction and RMS error with surface shape control of the reflector model with fiber orientation error of +1° on the top surface are shown in Fig. 6 and Table 3, respectively. Control forces of −0.852 mN and 0.708 mN were applied on nodes marked with × (actuators A1 and A2) and + (actuators B1 and B2), respectively. The defocusing value was −33.644×10⁻³ μm. The maximum and minimum displacement were 0.453 μm and −1.775 μm, respectively, and the RMS error was reduced to 0.331 μm, which is 14.29% of that without surface shape control. Comparing to the result with only shape control, the RMS error was slightly reduced.  

Calculated result of distribution of displacement in z-direction and RMS error with surface shape control and defocusing of the reflector model with fiber orientation error of +1° on the top surface are shown in Fig. 8 and Table 3, respectively. Control forces of −0.895 mN and 0.700 mN were applied on nodes marked with × (actuators A1 and A2) and + (actuators B1 and B2), respectively. The defocusing value was 2.489×10⁻³ μm. The maximum and minimum displacement were 0.023 μm and −0.019 μm, respectively, and the RMS error was reduced to 0.00548 μm, which is 1.35% of that without surface shape control. It was not significantly different from that with only surface shape control.

4.3. Discussion on results  

From the results shown above, it is seen that thermal deformation caused by the existence of fiber orientation error can be suppressed more efficiently than that caused by the existence of surface coating. This is because the mode of distribution of the out-of-plane deformation of the reflector model generated by applying out-of-plane external forces with actuators arranged along the edge agreed well with that generated by thermal deformation caused by the existence of fiber orientation error. Accordingly, it is seen that in manufacturing process of CFRP reflectors, certain degree of fiber orientation error can be allowed if surface shape of the reflectors is controlled properly.

On the other hand, thermal deformation generated on the reflector model with surface coating cannot be suppressed significantly by the simple method. This indicates that another method of controlling surface shape are needed if surface coating as discussed in this paper is required, or the laminate condition should be made symmetric by adding another surface coating on the opposite side of CFRP reflectors.

With respect to the defocusing, it was made clear that defocusing has limited effect on suppressing RMS error caused by thermal deformation of CFRP reflector models due to the surface coating and the fiber orientation error if the surface shape is controlled properly.

5. Sensitivity Analysis on Shape Control

To clarify the sensitivity of RMS error with respect to the location of actuators and control forces, sensitivity analysis was also performed. In the sensitivity analysis, the RMS errors when the location of actuators or input forces varied from their optimal values were calculated.

5.1. Location of actuators

The RMS errors when some actuators were moved to the adjacent node along the edge of the reflector models were calculated. In this condition, the input forces were unchanged. The center angle between each optimal actuator location and its adjacent node are listed in Table 4. The positive direction of the angle is counter-clockwise.

The calculated RMS errors of the controlled reflector model with surface coating and that with fiber orientation error when...
The ratio of the variation of RMS error in each condition the input forces varied are listed in Tables 7 and 8, respectively. The ratio of the variation of RMS error per 1° of location variation in each condition

\[
\frac{|\text{RMS} - \text{RMS}_{\text{opt}}|}{\text{RMS}_{\text{opt}} \cdot \Delta \rho}, \quad (11)
\]
is also listed in these tables. From these tables, it was found that when the center of a pair of actuators missed the center of the reflector model, the RMS error significantly increased. On the other hand, when a pair of actuators moved on the same direction, that is, center of the two actuators was still on the center of the reflector model, the variation of RMS error is relatively small. Accordingly, the location of each pair of actuators should be accurately on the opposite side of each other. It was also found that the variation of RMS error of the controlled reflector model with fiber orientation error by location variation of actuators is significantly high more than that of the reflector model with surface coating. It means that to realize high precision surface shape control of the CFRP reflectors, the location of actuators should be controlled accurately in accordance with their fiber orientation error.

### 5.2. Input forces

The RMS errors when the input forces of some actuators varied by +1% or −1% from their optimal values were calculated. In this condition, the location of actuators were unchanged. The calculated RMS errors of the controlled reflector model with surface coating and with that with fiber orientation error when the input forces varied are listed in Tables 7 and 8, respectively. The ratio of the variation of RMS error in each condition

\[
\frac{|\text{RMS} - \text{RMS}_{\text{opt}}|}{\text{RMS}_{\text{opt}}}, \quad (12)
\]
is also listed in these tables. From these tables, it was found that when the input forces of a pair of actuators were biased, the RMS error significantly increased. Accordingly, the input forces of a pair of actuators should be controlled strictly to maintain their balance. It was also found that the variation of RMS error of the controlled reflector model with fiber orientation error by variation of input forces is significantly high more than that of the reflector model with surface coating, although the RMS error of controlled reflector model with fiber orientation error itself is much smaller than that of reflector model with surface coating. It means that to realize high precision surface shape control of the CFRP reflectors, the input force should also be controlled accurately in accordance with their fiber orientation error.

### 6. Conclusion

In this study, capability of suppressing the out-of-plane thermal deformation generated on CFRP reflector models by simply applying the external out-of-plane forces was investigated. As a result, it was demonstrated that the out-of-plane thermal deformation caused by the existence of fiber orientation error which occurred in lamination process can be suppressed to 1.45% in RMS error. Accordingly, if a surface shape of the CFRP reflector is appropriately controlled, fiber orientation error in lamination can be permitted to some extent depending on a target wavelength of the reflector.

It was also demonstrated that capability of suppressing the...
out-of-plane thermal deformation caused due to the surface coating is limited to 14.3% in RMS error, which is approximately 10 times larger than that caused by the existence of the fiber orientation error. Accordingly, to realize space-based CFRP reflectors with surface coating, more number of actuators or a more complicated method for controlling the surface shape should be introduced, or laminate condition should be made symmetric by adding another surface coating on the opposite side of CFRP reflectors. Additionally, the effect of defocusing in addition to surface shape control was also investigated. As a result, it was made clear that no significant effect can be expected for defocusing on suppression of thermal deformation of CFRP reflectors for the two types of thermal deformations investigated in this study.

Sensitivity analysis on the effect of variations of the location of actuators and the input forces were also performed. It was seen from the result that the RMS error would significantly increases when the locations and the input forces of a pair of actuators are unbalanced. It was also shown that the surface shape control to suppress the out-of-plane thermal deformation due to the existence of the fiber orientation error is more difficult than that due to the existence of surface coating layers. Considering that fiber orientation error of a certain extent is unavoidable, to realize high precision shape control of CFRP reflectors, the location and the input forces of actuators should be controlled strictly.

In this study, the analytical model of the reflector models was composed of shell elements. Thus, the effects of the out-of-plane stress distribution were ignored. If these effects are included, the mode of distribution of the out-of-plane thermal deformations must be different from the mode without the effects. However, the former modes are expected to be observed as superposition of the latter modes and those generated by the effects of out-of-plane stress distribution without the effects of surface coating or fiber orientation error. Accordingly, the result obtained from this study is still useful to suppress the modes generated by the existence of surface coating or fiber orientation error. Of course the effects of the out-of-plane stress distribution should be investigated in the future.

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