Development and Ground Evaluation of Fast Tracking Algorithm for Star Trackers

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(Received June 20th, 2017)

Fast tracking algorithm discussed in this paper is applied to star trackers for improving the performance of star identification. Space Robotics Laboratory (SRL) in Tohoku University has developed star trackers for micro-satellites so far. Since it has only a lost-in-space algorithm for star identification, the attitude update rate is limited up to 1 Hz. It was implemented to the Philippines’ 50 kg-class micro-satellite "DIWATA-1" released from the International Space Station on April 2016. Although on-orbit evaluation showed good results enough to output attitude autonomously, the performance of continuous attitude determination was worse than expected. Since quite a high access frequency of star catalog is required, timeout of the process for attitude calculation occurs frequently even if update rate is 1 Hz. Insufficient ground evaluation before launch is also one of the causes of operation failure. Tracking algorithm helps to calculate latest attitude faster than conventional methods by feeding back the previous attitude information. This algorithm includes two additional processes. First, future star positions on the image frame can be predicted according to the previous attitude and pre-identified star information. The sensor can find corresponding latest centroids compared to predicted star positions. Second, unidentified stars on the camera field of view (FOV) can be detected by referring to the star neighborhood catalog, which includes the list of some adjacent star IDs against each reference star. PC simulation shows that continuous attitude determination works effectively by keeping low catalog access frequency. The proposed algorithm is implemented to the real hardware. Then, ground evaluation is conducted using star simulator environment and satellite dynamics simulator. The result demonstrates that the processing speed in real situation becomes about 70 times faster compared to the previous method and it is successful to obtain much more stable 1 Hz attitude output.

Key Words: Star Tracker, Attitude Determination, Micro-satellite

Nomenclature

\[ A \] : direction cosign matrix
\[ b \] : star vector in sensor coordinate system
\[ C \] : star centroid position on image frame
\[ d \] : tracking radius [pixels]
\[ f \] : focal length of lens [pixels]
\[ g \] : luminance value of pixels
\[ h \] : star image height [pixels]
\[ I \] : inertia matrix of satellite
\[ P \] : predicted star position on image frame
\[ q \] : quaternion
\[ r \] : star vector in inertial coordinate system
\[ S \] : sum of luminance values in region
\[ w \] : star image width [pixels]
\[ x \] : horizontal position [pixels]
\[ y \] : vertical position [pixels]
\[ \Delta t \] : interval [sec]
\[ \omega \] : angular velocity [rad/sec]

Subscripts

\[ est \] : estimation
\[ i \] : index of star/centroid
\[ I \] : identity
\[ n \] : number of time step
\[ x \] : x coordinate

\[ y \] : y coordinate
\[ z \] : z coordinate
\[ * \] : inverse

1. Introduction

Star trackers are generally used for precise attitude determination of satellite. This sensor has a function to calculate and output spacecraft attitude in the sensor coordinate system autonomously. The calculation process is somewhat complicated. First, image sensor takes stellar sky image. Then, the processor detects bright pixels and calculates their centroids. By referring to the built-in star catalog and comparing star pattern in the camera field of view (FOV), each centroiding star is associated with corresponding star ID. Finally, attitude can be calculated by certain method.

Micro-satellites have great possibilities to achieve various advanced missions at low cost by applying precise attitude determination and control system. For example, 50-kg-class micro-satellite “RISESAT” has a high precision telescope and it is scheduled to conduct a fixed point observation by target pointing control. In addition, the optical communication technology requires much higher attitude control accuracy.\(^1\) In order to achieve them certainly, it is essential to use high performance and reliable star trackers while fulfilling low cost, small size and low power consumption. However, few star
Space Robotics Laboratory (SRL) has developed small star trackers by collaboration with Meisei Electric Co. shown in Fig. 1. Two camera heads are installed on Philippines’ micro-satellite called “DIWATA-1” (Fig. 2) which is planning to utilize for remote sensing over Philippines. DIWATA-1 was released from the International Space Station (ISS) on April 2016. Although its star trackers worked well to output attitude autonomously, the performance of on-orbit evaluation was worse than expected. One of the main problems was a low success rate of attitude calculation in continuous mode. This is because only a lost-in-space algorithm called Pyramid algorithm is implemented to the processors. This is a time-consuming method because a quite high access frequency of star catalog is required. If stars are “tracked” in a literal sense in the captured image based on previous attitude calculation results, catalog access frequency is supposed to be diminished. Several tracking techniques have been already proposed. This paper explains fast tracking algorithm for star identification in a wide FOV star camera.

In order to evaluate the performance of the proposed algorithm compared to conventional one, ground evaluation system is established. This paper also presents the overview of this system including actual star tracker hardware, star simulator and satellite dynamics simulator. The ground test of star tracker applied tracking algorithm is conducted and the performance of this algorithm is verified.

Fig. 1. Two camera heads are installed on Philippines’ micro-satellite called “DIWATA-1” (©JAXA).

2. On-orbit Evaluation of Current Star Trackers

SRL has developed star trackers for DIWATA-1 shown in Fig. 1. Various improvements were added from the previous generations of star trackers installed on micro-satellite “RISING-2” which was launched on May 2014. The sensor of old generation didn’t performed well due to a large amount of false detection of stars. Almost all of them were white spots caused by radiation effect against CCD image sensor. New star tracker hardware includes white spot avoidance algorithm. Also, it can synchronize attitude output timing with the pulse per second (PPS) signal transmitted from a GPS receiver. It has a baffle which helps to avoid sun light effectively. The shape of baffle was also re-designed. This sensor has built-in star catalog stored up to 6-class magnitude of stars into Flash ROM which is used to calculate attitude online.

2.1. Algorithm for attitude determination

The sequence of attitude determination in each calculation step is processed in the following order: image acquisition, star detection, centroiding, star identification and attitude calculation.

First of all, CCD image sensor captures stellar sky images by pointing towards the dark space direction. Short focal length of lens helps to expand the FOV and find more stars although the resolution of the image is decreased. In the star trackers developed by SRL, the focal length of the lens is 12.3 mm and the FOV is 27.8 deg.

The captured image is transmitted to FPGA and star detection process is executed. A 5x5 square examination region is scanned sequentially on the whole image. If the luminance value of the center pixel in the region exceeds both that of other pixels in the same region and pre-determined threshold value, it can be recognized as a candidate star. White spots caused by radiation effect are distinguished and excluded by the filters expressed in Ref. 14).

After that, each star position is obtained by the following centroid calculation.

\[
C_x = \sum_{x=-2}^{x=2} \sum_{y=-2}^{y=2} x g(x,y) / S \tag{1}
\]
\[
C_y = \sum_{x=-2}^{x=2} \sum_{y=-2}^{y=2} y g(x,y) / S \tag{2}
\]

Then, star vectors pointing from the center of the lens to star centroid positions on the focal plane are calculated.

\[
b_i = \frac{1}{\sqrt{f^2 + C_{ix}^2 + C_{iy}^2}} \begin{bmatrix} f & -C_{ix} \\ 0 & C_{iy} \end{bmatrix} \tag{3}
\]

The next process is star identification and matching. Star centroids detected by the previous process are associated with corresponding star IDs one by one referring to the built-in star catalog. Pyramid algorithm developed by Mortari et al. is applied considering the ease of implementation. In this method, the inner product values of 6 pairs of star vectors derived from 4 star centroids are calculated to search four corresponding stars on the star catalog, which stores pairs of star IDs in the descending order of inner product value. Since it is a lost-in-space algorithm, this process works properly regardless of previous attitude calculation result. Some accurate star positions are necessary for matching these detected stars correctly. In the pyramid algorithm, four accurate star positions are required to obtain the reliable matching result.

In the final process, satellite attitude is calculated by TRIAD method. Two out of four matched stars are selected, and then star vectors in inertial coordinate system are obtained from star catalog. The definitions of star vectors in both inertial and sensor coordinate systems are described in Fig. 3.
The third vectors perpendicular to first and second vectors are calculated in both coordinate systems as follows.

\[ b_3 = \frac{b_1 \times b_2}{\|b_1 \times b_2\|} \]  
\[ r_3 = \frac{r_1 \times r_2}{\|r_1 \times r_2\|} \]

Then, attitude in sensor coordinate system is obtained in the form of direction cosine matrix (DCM).

\[ A = [b_1 \ b_2 \ b_3][r_1 \ r_2 \ r_3]^T \]

It can be transformed into quaternion in certain calculation.

\[
\begin{align*}
\text{Right ascension} & : 0.0820 \\
\text{Declination} & : 0.0712 \\
\text{Rotational angle} & : 0.849
\end{align*}
\]

### 2.2. On-orbit evaluation results

Two star trackers developed by SRL are mounted on DIWATA-1 in different direction relatively for several reasons. Firstly, it can compensate for attitude determination accuracy around the optical axis. Secondly, even if one sensor is failed to determine attitude due to sunlight, the other is possible to succeed to obtain attitude.

After its launch, on-orbit verification test of star trackers was conducted. During the test, inertial-fixed attitude control was performed and the slew rate of the satellite was brought close to zero for avoiding the enlargement of star images. In the beginning, camera parameters were adjusted and it was clarified that optimal exposure time was 300 milliseconds when the camera gain was set to 30 dB. Then, sensor worked properly for calculating attitude in single operation mode. Also, white spots could be distinguished and excluded from captured image by implemented filters. One of captured star images is shown in Fig. 4, which points to the direction near the false cross and represents attitude quaternion: \( \{0.438252, -0.271275, -0.789119, -0.334121\} \). Over 30 images were taken and transmitted to the ground for analyzing attitude determination accuracy.

### Table 1. Analysis result of attitude determination accuracy.

<table>
<thead>
<tr>
<th>Angle</th>
<th>Accuracy (3σ) [deg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Right ascension</td>
<td>0.0820</td>
</tr>
<tr>
<td>Declination</td>
<td>0.0712</td>
</tr>
<tr>
<td>Rotational angle</td>
<td>0.849</td>
</tr>
</tbody>
</table>

However, it became obvious that success rate of attitude determination in 1 Hz attitude output mode was significantly low despite the fact that the enlargement of the star image did not occur by keeping the state of no rotation during the measurement. Fig. 5 describes the result of on-orbit analysis in one-minute measurements for multiple cases. As shown in the graph, the maximum rate is 62.7 %. On average, it is only 33.0 %.

It often takes too much time for star identification and timeout of the calculation process occurs frequently. This is because processor is required to access a significant deal of times to built-in star catalog in Pyramid algorithm. The satellite needs continuous and latest attitude measurements from star trackers to calculate the most probable attitude using filtering technique and to apply the attitude control. For this reason, it is essential to improve the success frequency of attitude determination. Therefore, the total processing time should be shortened by applying a method to reduce the frequency of access to the star catalog.
tracker, the operation mode is set to “initial attitude acquisition” phase at first. The conventional lost-in-space algorithm is applied for calculating attitude in this phase. Therefore, fully processing time is required.

If sensor detects attitude twice in succession, it shifts to “star tracking” phase. Processor estimates attitude in the next time step by simple calculation under the assumption of constant angular velocity. The estimated attitude information provides the assumption of future star pattern on the image frame because attitude value corresponds to star pattern one by one. The star IDs are assigned to predicted star positions using the previous star information and the star neighborhood catalog. Then, they are also associated to star centroids measured from the image by comparing the position between measured centroids and estimated ones. The detailed methods are expressed by following sections.

3.1. Prediction of future star positions

Let \( q_{n-1} \) and \( q_n \) be the attitude of the latest two consecutive time steps and \( \Delta t_n \) be the interval between them. The angular velocity at this moment can be calculated as follows:

\[
\omega_n = \frac{2}{\Delta t_n} (q_{n-1} \otimes q_n - q')
\]

(7)

Where

\[
q \otimes p = \begin{bmatrix} p_4 & p_3 & -p_2 & p_1 \\ -p_3 & p_4 & p_1 & -p_2 \\ p_2 & -p_1 & p_4 & -p_3 \\ p_1 & p_2 & p_3 & p_4 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}
\]

(8)

\[
q' = \begin{bmatrix} -q_1 \\ -q_2 \\ -q_3 \\ q_4 \end{bmatrix}
\]

(9)

\[
q' = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}
\]

(10)

Under the assumption that angular velocity is constant in a short period, estimated quaternion in the next time step can be obtained by following equation:

\[
q_{n+1}^{\text{est}} = q_n + \frac{\Delta t_{n+1}}{2} (q_n \otimes \omega_n)
\]

(11)

Where

\[
\omega_n = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}
\]

(12)

This quaternion should be normalized and transformed into direction cosign matrix (DCM).

Next, new star positions are predicted. These positions indicate where star centroids are assumed to appear on the image in the next time step. As advanced preparation, identified star IDs in the last time step is stored. Star vectors in inertial coordinate system corresponding to these star IDs are obtained from the star catalog. By multiplying these vectors by DCM, estimated star vectors in sensor coordinate system are obtained.

\[
b_{i,x}^{\text{est}} = A_{\text{DCM}}^{\text{est}} b_i
\]

(13)

Then, estimated centroid positions on the image frame can be calculated.

\[
P_{i,x} = -f b_{i,x}^{\text{est}}
\]

(14)

\[
P_{i,y} = f b_{i,y}^{\text{est}}
\]

(15)

Eqs. (14) and (15) describe inverse calculation against Eq. (3).

Some stars are possibly going out of image frame considering the camera FOV. If the predicted position \((P_{i,x}, P_{i,y})\) satisfies the following inequality,

\[
|P_{i,x}| < \frac{W}{2}
\]

(16)

\[
|P_{i,y}| < \frac{H}{2}
\]

(17)

then this star is assumed to remain in the FOV, otherwise it goes out of the FOV.

3.2. Neighbor star search technique

Since the sensor direction towards the sky changes every moment, all of tracked stars will eventually go outside the image frame. Therefore, it is necessary to recognize star IDs which have newly entered the FOV. However, it takes too much time to search on entire star catalog.

To solve this problem, the star neighborhood catalog is used. It contains the list of at most nine star IDs in the order of closeness in distance against each respective cataloged star as shown in Table. 2. All the neighbor star lists are concatenated side by side. We can obtain the k-th neighbor star closest to the star whose star ID is j by referring to the (9j+k)th star ID to the list. The size of the star neighborhood catalog is 28,296 bytes, which is about 8 % of the size of conventional star catalog whose size is 355,432 bytes and is highly compact. This catalog is appended to the original star catalog.

The method of using the star neighborhood catalog is shown as follows. All adjacent star IDs against already identified ones are found by referring to the star neighborhood catalog like as described in Fig. 7. Next, those detected stars are checked whether they exist within the image frame by calculating predicted star positions from Eq. (14)-(17). Then, the next adjacent stars are also detected from newly detected star IDs and examined in the same way. Repeating this process, all of stars within the FOV in the next time step can be detected one after another, based on even at least one star ID remaining in the FOV in the previous time step. At the same time, predicted future star positions can be calculated. The search is finished when neither new adjacent star is found in the FOV. This method can decrease the frequency of star catalog access compared to the conventional methods.
Table 2. Example of star neighborhood catalog.

<table>
<thead>
<tr>
<th>Reference ID</th>
<th>Neighbor Star ID (In the order of closeness from the left)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1 6 5 4 33 3 2 32 28</td>
</tr>
<tr>
<td>1</td>
<td>0 4 6 5 3 2 33 28 11</td>
</tr>
<tr>
<td>2</td>
<td>13 11 1 4 0 12 15 3 57</td>
</tr>
<tr>
<td>3</td>
<td>4 19 22 20 15 1 16 28 17</td>
</tr>
</tbody>
</table>

3.3. Star matching

The predicted star positions are compared to actual star centroids measured by CCD. In this process, centroids nearest to the predicted stars are investigated. The same centroids should not select in duplication. If the distance between measured star and predicted one is less than the threshold value, then the star ID is assigned to the star centroid. In other word, tracking radius is defined as shown in Fig. 8 against each predicted star position, and the nearest centroid out of all detected ones within this circle which satisfies Eq. (18) can be chosen.

$$\sqrt{(P_x - C_x)^2 + (P_y - C_y)^2} < d$$  (18)

Large tracking radius increase the success rate of tracking stars, but it also increase a risk of misidentification of stars. In this research, tracking radius is defined as 5 pixels. The tracked star IDs are recorded in the memory and used for the next period of attitude determination.

3.4. Attitude calculation by QUEST method

In order to derive more accurate attitude output, QUEST method is applied in star tracking phase. Consideration of all tracked stars for attitude calculation contributes to reduce the attitude error caused by centroiding error which mainly comes from aberration and distortion of the lens. It also ensures robustness against the noise. The computational complexity of this method is a bit large compared to conventional TRIAD method, but it will complete within relatively a less time than searching star catalog.

4. Software Simulation

Software simulation was carried out to verify the effect of the tracking algorithm. It is confirmed whether it can decrease the access frequency of star catalog and perform faster and properly. This simulator is written by C++ language and generates virtual attitudes in 360 seconds in one second intervals according to satellite dynamics law. Acceleration is calculated by the following Euler’s equation of motion and the attitude integration is carried out at a cycle of 50 milliseconds by the 4th-order Runge-Kutta method. The control torques generated by actuators and disturbance torques are neglected.

$$I\ddot{\omega} + \omega \times I\omega = 0$$  (19)

At the same time, star images including star centroid positions are simulated based on star catalog. Centroid position value regularly contains noise caused by aberration and distortion of the lens and limitation of resolution in pixels in an actual hardware. Thus, Gaussian noise was artificially added to each axis of X and Y of centroid positions for reproduction of real situation.

Simulation condition is given by following. The initial angular velocity was 0.2 deg/sec for each axis, the initial quaternion was \{0, 0, 0, 1\} and inertial matrix was given by Eq. (20).

$$I = \begin{bmatrix}
1.7 & -0.3 & -0.3 \\
-0.3 & 1.69 & 0.02 \\
-0.3 & 0.02 & 1.88
\end{bmatrix}$$  (20)

The magnitude of the noise level in 1σ was set from 0 (without noise) to 1.5 pixels in the interval of 0.25 pixels.

In this simulation, two methods were applied respectively; first one is the conventional method (only Pyramid algorithm), and second one is tracking method which was introduced in this paper. For each method, the access frequency of star catalog and the success rate of attitude determination were measured while changing the centroid noise level.

Simulation results are described in Figs. 9 and 10. While there was no centroid noise, the number of access to star catalog was reduced to 1/16 using tracking algorithm. By adding centroid noise, the access frequency increased exponentially and success rate of attitude calculation decreased according to the magnitude of noise level in the conventional method. On the other hand, the performance was almost the same as in the case of no centroid noise applying the tracking method. Therefore, it is confirmed that the tracking algorithm is effective for both reducing access frequency of star catalog and improving robustness against centroid noise.
5. Establishment of Ground Test Environment for Star Trackers

In order to evaluate the performance of tracking technique in an actual hardware, ground evaluation environment was established. New evaluation circuit board of star tracker, star simulator and satellite dynamics simulator software developed in SRL are introduced in this chapter.

5.1. Star tracker evaluation board

The circuit board for evaluation of star tracker is designed and developed. A commercial evaluation board equipped with Xilinx’s Virtex-5 LX FPGA is the base of this board. The FPGA has a soft-core processor which is driven at 50 MHz. The function is basically the same as the previous version of star trackers mounted on DIWATA-1, but tracking algorithm is newly implemented and it is able to switch to between conventional mode and tracking mode by the external command. This board has a power supply circuit, interfaces with PC and CCD sensor, and an external SRAM as storage of star catalog information and captured star images. For ground verification, a pseudo pulse per second (PPS) signal is generated in FPGA logic and it is sent to processor instead of external PPS signal input from a GPS receiver. The board also implements logic for measuring the processing time of attitude calculation with accuracy of 1 microsecond. The configuration of the board is described in Fig. 11. Also, PC software for control of the board in GUI style was developed to send commands, receive and decode telemetry data, and save calculated attitude and other status.

5.2. Star simulator

The star simulator consists of a monitor that displays virtual stellar sky, a star tracker camera fixed on a 3-axis movable stage and a PC for monitor control. All the equipment is installed in the darkroom. The setup is showed in Fig. 12. The monitor displays up to the certain magnitude of stars in any direction. The star patterns are generated on control PC based on either attitude by manual input or that received from the satellite dynamics simulator introduced later. The operation of actual star tracker hardware can be thoroughly verified by capturing star images towards the monitor which simulates the continuous change of the direction of stellar sky according to the attitude dynamics calculation.

5.3. Satellite dynamics and space environment simulator

This simulation software performs as a part of a genetic satellite development environment called MEViμS for conducting ground evaluation of attitude determination and control system for micro-satellites in SRL. It can simulate orbit and attitude dynamics as well as space environment such as geomagnetic field and planet orbit, operation of sensors and actuators, power consumption and temperature information of each instrument. This system can connect to the control PC of the star simulator in the local network and send simulated attitude information in the form of quaternion continuously in 20 Hz. The star simulator PC generates the corresponding star images of received attitude, and the monitor is updated at 50 milliseconds intervals.

5.4. Integration of ground evaluation environment

It is possible to conduct a ground evaluation for standalone star tracker with the test configuration demonstrated in Fig. 13. Furthermore, the satellite simulator has a special interface box to connect to the real satellite computer. This integrated system can also be used for thorough verification of attitude control system using all real hardware in hardware-in-the-loop simulation.

6. Ground Evaluation of Star Trackers with Tracking Algorithm

The performance of standalone star tracker implemented tracking algorithm was evaluated in the integrated ground test.
environment. The test conditions are configured as follows. The orbit and attitude of satellite were reproduced using the satellite dynamics simulator. The initial orbit information is given by the two line element (TLE) information of DIWATA-1. The initial attitude information was given manually; the initial angular velocity was 0.2 deg/sec for each axis, the initial quaternion was \( \{0, 0, 0, 1\} \) in satellite coordinate system. In this test, the processing time during the process of both star identification and attitude calculation was measured in accuracy of 1 microsecond while switching the methods used for attitude determination. The success frequency of attitude determination was also calculated.

The measurement result of processing time is shown in Fig. 14. It was drastically reduced from 213 milliseconds per step in conventional mode to 3.03 milliseconds in tracking mode. The processing speed became about 70 times faster than before. In addition, success rate of attitude determination was improved from 63.6 % to 93.8 % in average as shown in Fig. 15. Hence, it is confirmed of high validity of tracking algorithm in real hardware environment.

![Fig. 14. Processing time for attitude calculation per step.](image)

![Fig. 15. Success rate of attitude determination on ground evaluation.](image)

**7. Conclusion**

Fast tracking algorithm is introduced in order to improve the success rate of attitude determination for star trackers. The access frequency of star catalog can be decreased by using previous attitude and pre-identified star information as well as referring to star neighborhood catalog. Attitude quaternion can be calculated by QUEST method using all tracked stars. The ground evaluation environment is integrated including star simulator and satellite dynamics simulator. Then, the performance of standalone star tracker is evaluated. The result illustrates that the tracking algorithm works effectively for fast processing of attitude detection and it is valid to obtain continuous and stable attitude output at 1 Hz interval in a real hardware configuration. Currently, improved star tracker with tracking algorithm which is planning to mount on the next micro-satellite is under development. Pre-launch evaluation will be conducted many times over using the ground evaluation system to guarantee the reliable operation on orbit. Furthermore, this test environment will utilize for evaluating full operations of attitude determination and control system with the real star tracker and the satellite computer in hardware-in-the-loop simulation.

**Acknowledgements**

The authors would like to thank Meisei Electric Co., Ltd. for technical support of this research, and the Philippines’ micro-satellite development team for giving the opportunity of demonstration on orbit.

**References**


