Reduced Dimension Analysis on Combustion Oscillation in a Model Rocket Combustor Using a Deep Neural Network

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(Received June 22nd, 2017)

The similarity between POD (Proper Orthogonal Analysis) and NN (Neural Network) is explained and an example of NN to perform reduced dimension analysis on a combustion oscillation problem is presented. The dimension reduction procedure by Snapshot-POD is shown to be expressed by a three layer AE (Auto-Encoder). Based on this, a DAE (Deep Auto-Encoder), consisting of a four layer encoder and four layer decoder is tested. The encoder has layers of 128-32-8-2 neurons and the decoder has the ones of 2-8-32-128 neurons in its layers. The DAE reduces the dimension of the input data into two, which is the number of the encoder output variables. As a reference, a POD that takes the first and the second mode neglecting higher modes are employed to reduce the dimension into two. A URANS simulated time varying temperature, heat release, and pressure distributions of CVRC (Continuously Variable Resonance Combustor) are analyzed by the POD and DAE. As a result, the 2D data from DAE and POD agreed well. It was confirmed that the dimension reduction performance and the resulting amount of information was almost consistent. By analyzing mode maps, the ability to identify the modes for pumping up the oscillation is demonstrated.

Key Words: Rocket Combustor, Combustion Oscillation, Mode Analysis, Deep Neural Network

1. Introduction

Combustion oscillation in rocket combustors is a very complicated phenomenon that include interaction between various phenomena and interacting parameters. One of the main pathway to excite the oscillation is the positive cross-correlation between pressure and heat release fluctuations, which is called as Rayleigh’s criterion.1) The correlation result in the energy transfer from combustion heat release to the existing pressure wave and amplify the amplitude. The correlation is achieve sometimes through many steps. For example, Tanabe et.al2) pointed out that the hard oscillation like N-wave formation occurs when the pressure wave is amplified through re-ignition event near the dump plane in the CVRC (Continuously Variable Resonance Combustor) developed by Purdue University group.3,4) The re-ignition event is initiated by the arrival of the N-wave at the dump plane; and the ignition event yielded heating and expansion of gas to cause additional pressure wave or to add energy to the existing wave. On the other hand, in many cases, the pathway, through which the pressure and heat release fluctuation correlates, is unclear. A general approach to find the pathway is necessary. Among the many method to analyze the oscillation, POD (Proper Orthogonal Decomposition) and DMD (Dynamic Mode Decomposition) is becoming the major tool to find those dominating phenomenon,5,6) separating each independent phenomenon from complicated oscillatory phenomena. They were tested for the analysis of combustion oscillation in rocket combustors and were successful to find the key oscillation mode. For easy analysis, one need to reduce the dimension of the phenomena without losing information of the phenomena. That means a good decomposition technique must reproduce accurate phenomena with minimum number of modes. Although POD and DMD are useful to distinguish a linear oscillation modes, they outputs many modes if non-linearity exists in the oscillation. Difficulty arises when representing such phenomena with minimum number of dimensions. For example, the reproduction of N-wave requires summing up of infinite number of DMD mode, a DMD mode being corresponding to a single frequency wave. Taking a few major frequency fails to accurately representing the non-linear wave. Thus a more powerful method to decompose combustion oscillation field into elemental phenomena is required. The POD/DMD can be done by singular value decomposition and LU decomposition,7,9) both of which are matrix calculus. Considering the fact that the data processing of NN (Neural Network) is similar to matrix calculus, these decomposition method is thought to be expressed by an appropriate NN. This study introduces a new method for the dimension reduction analysis applying DNN (Deep Neural Network) and examine if it can replace the most basic decomposition method, POD.

2. Theoretical Background and Procedure of Dimension Reduction

2.1. Mathematical background of POD, equivalent NN, and DNN

The simplest neural network is shown in Fig. 1. A dataset is put into the input layer and the network outputs the transformed data from the output layer. There is a hidden
layer in between the two layers. The each node is a weighted sum of the value at the nodes in the previous layer. Usually, the input layer accept a set of data that may be a matrix consisting of a number of vectors. In that case, the layer-to-layer transformation can be interpreted as a kind of projection from vector space of dimensions equal to the number of vectors.

\[ V_{n+1} = f_n(W_n V_n + b_n) \]  

where \( W_n \) and \( b_n \) are weight and bias parameter, respectively. The so-called excitation function, \( f_n \), can be arbitrary defined. When \( f_n \) is taken as linear function and all the components of \( b_n \) are zero, the equation is a form of simple decomposition operation written as:

\[ V_{n+1} = W_n V_n \]  

This means that a decomposition operation is equivalent to a single hidden layer of neural network.

The POD technique is a decomposition of data matrix into a weighted sum of a set of Eigen functions which are orthogonal with each other. In a more practical use, the snapshot-POD technique uses a time series of spatial distribution of an oscillating parameter as a data matrix, \( V \), and it is decomposed into the three matrices as:

\[ V = T \Sigma C \]

The snapshot POD and dimension reduction process is reproduced by a set of matrix operation shown in Fig. 2. The input data matrix is projected to the second left Eigen function matrix by multiplying weight from the left. If the weight function is the reciprocal of the topos matrix and taking zero bias and linear excitation function, this operation is done by a neural network layer. The dimension reduction is achieved by the second left operation in the figure that extract only the dominant components. This is known as a “pooling layer” in NN. The reduced dimension data is reconstructed by applying the topos matrix again as is shown in the right most operation in the figure. The last step also is equivalent to a NN layer. The necessary condition for the POD dimension reduction is that the RMS error of reconstructed data is minimized. Thus the POD dimension reduction procedure is exactly reproducible by the NN shown in the figure.

The NN shown in Fig. 2 is a kind of Auto-Encoder (AE). In general, AE consists of encoder network and decoder network. It encodes a set of data into another form and decodes to output a data set that is the same as input. The above analysis showed the potential ability of a neural network with two hidden layer to perform POD dimension reduction. When one introduces more layers, one expect more flexible and accurate dimension reduction. Considering the expectation, a deep neural network (DNN), that has more than three layers shown in Fig. 3, is designed and programmed to examine if it can, at least, work as POD dimension reduction of a sample combustion oscillation phenomenon. The DNN forms an auto-encoder. Thus, this is a Deep Auto-Encoder (DAE). The simulated data set of combustion oscillation field are input to an encoder that has fully connected four layers (128-32-8-2 neurons) to output two parameters. Thus this NN projects the input data to 2D space. The output parameters, that are feature functions, are compared with the topos of the most and the second most dominant oscillation modes derived by POD. The decoder part is designed symmetrical (2-8-32-128) to the encoder. The reconstructed data is compared with the input data and

![Fig. 1. A simple feed-forward neural network.](image)

![Fig. 2. A neural network (NN) equivalent to POD dimension reduction and reconstruction.](image)
the weights of all the layers are optimized to minimize RMS error. The optimization is done by a stochastic optimizer, Nadam. To avoid overfitting, dropout layers are inserted in between each layer. The DAE is coded using Keras deep learning library with Theano library as backend. The machine learning process is done using a big-data that is 12,000 patterns of temperature, heat release, and pressure distributions from a model rocket combustor that are produced by unsteady RANS (URANS) simulation. During learning process, the big-data are randomly sorted to flatten the learning process. After completing the learning process and all the weights (i.e. neuron activity) are fixed, the time series of the combustion field data is sorted with time progress and put in the DAE again. This process output the time varying analyzed data. The output 2D data from the encoder becomes a latent variable to extract the time variation of the distribution patterns that also varies with time. Thus the output of this final process becomes just equivalent to the topos matrix of POD.

2.2. Big-data from URANS simulation of CVRC.

The DAE needs big-data to learn the typical distributions of characteristic properties during combustion oscillation. Among many data base, the CVRC data are useful for oscillation analysis, because its geometry is simple having a single coaxial injector and because it covers various typical combustion patterns, being able to test from stable state to unstable to stable again during one firing test. Here, we chose simulated data for CVRC as the big-data. Unsteady RANS calculation is done to simulate the experiment of Tanabe et al. The ANSYS Fluent pressure based solver is employed. Transition SST is employed for viscous model. Time stepping is set to $10^{-6}$ sec, so that the Nyquist frequency is beyond the frequency of combustion oscillation in CVRC that is below 2 kHz. The distributions of temperature, pressure, and heat release rate is captured every 10 time steps from 0.03 sec through 0.15 sec after ignition. The dimensions of the combustor can be found in the dissertation of Yu, and the flow rates, inlet conditions are taken as the experiments. The oxidizer is decomposed hydro-peroxide and the fuel is gas methane. Axis symmetry is assumed for simplicity and for the fact that the reported experimental result has rather ordered flow field. Only the difference is the range and the speed to vary the oxidizer post length during firing, which is set to 1 m/s, to save run time. Figure 4 shows the cross sectional distributions of temperature and pressure (Fig. 4a), oxidizer post length and the resultant pressure fluctuation measured at the recess wall of the injector outlet (Fig. 4b). While the post length is long, combustion is stable. The temperature distribution shows no distinctive flame wrinkling structure and pressure is evenly distributed anywhere. When the time is 0.07 sec and the post length is shortened, a severe oscillation exists. A plume-like structure of the core flow can be seen and the pressure is high near the dump plane. A pressure trace around the moment is magnified in Fig. 4b) that shows the experimentally observed N-wave. The combustion field is unstable passing through the time 0.11 sec and becomes marginal stable at 0.15 sec. This stable to unstable to stable transition agrees with the experimentally observed phenomenon. The URANS simulation qualitatively able to reproduce combustion field in CVRC.

![Fig. 3. A Deep Auto-Encoder (DAE) equivalent to POD dimension reduction and reconstruction.](image)

![Fig. 4. URANS simulated combustion oscillation in CVRC.](image)
3. Result of Reduced Dimension Analysis.

3.1. POD analysis

The proposed DAE is designed to reduce the input dimension into two as is explained with Fig. 3. As a reference, conventional POD is done to check the ability of the DAE to do the same. To save computation time Sparse-PCA function\(^{15}\) assuming two singular value, instead of conventional PCA, is employed for the POD. The 12,000 time steps of temperature, heat release, and pressure distributions in the main chamber (i.e. from the injector recess to the entrance of the nozzle) is analyzed together as DAE case. To adjust the sensitivity to the error minimization in the PCA and the DAE processing, the each distribution is regularized divided by its maximum value among the whole time steps. Thus the data do not retain the physical unit and the discussion will be done only qualitatively, hereafter. Figure 5a) shows the typical combustion field of flame anchoring area near injector outlet and dump plane of CVRC. The relatively cold oxidizer flow exist near the center line; the fuel is mixed with the oxidizer around the boundary of the two flows. Consequently, the mixed gas burns at around the outer boundary of the oxidizer flow. The burnt gas forms recirculation zone stabilized at the dump plane. This hot burnt gas hold the flame anchoring point. There are vortices in the share layer and the flame is highly turbulent. Figure 5b) shows the simulated time-averaged distributions of temperature, heat release, and pressure. The upper boundary of the each figure corresponds to the symmetry axis (center line in the Fig. 5a)) of the cylindrical chamber and the lower boundary is the outer wall of it. From the simulated average field, one sees that the low temperature core flow is seen near the center and hot gas is surrounding it. The heat release distribution approximately corresponds to the location of flame. Since, during oscillatory combustion, the recirculation area is lighted by entraining the oxidizer and compressed by pressure wave, the heat release distribution shows bright area near outer wall in addition to the location of flame schematically shown in Fig. 5a). On average, pressure distribution is uniform.

The result of the POD is shown in Fig. 6. The mode maps are shown for the first dominant mode (POM1) in Fig. 6a) and for the second mode (POM2) in Fig. 6b). The associated chronos is plotted at the bottom of the each figure. The red part is where the deviation of either temperature/heat release/pressure is positive and the blue part negative, when the chronos value is positive. Thus, for example, when the pressure is low compared with average as seen near the dump plane, temperature is low there in POM1. The POM2 show the same trend but in another way. When the pressure near the dump plane is high, temperature is high there. In both modes, when the pressures are high/low (i.e. red/blue), the heat releases are high/low, near dump plane. Thus the general trend of positive cross correlation can be confirmed there. This implies Rayleigh index is positive near the dump plane, too.

3.2. Deep auto-encoder analysis

The two time varying output of the encoder of the DAE is shown in Fig. 7. The 12,000 patterns are expressed by the two parameters. The two functions are similar to the two chronos of the POM1 and POM2. The function shown in the lower raw of the Fig. 7 seems to express the amplitude of oscillation and the upper one shows some measure of stability.
To assess the difference of information contained in the POD and DAE output, the pair of the output from the two method is plotted on the 2D phase space spanned by the two function that is shown in Fig. 8. For easy comparison, the phase plot are realigned to a new orthogonal space with which the correlation between two function is none. The scale of the each function is regularized so that the standard deviation is unity. The upper small diagram shows the original phase plot and the lower large diagram shows the aligned plot for comparison. The shape of the two realigned phase plot of POD and DAE are nearly identical with each other, though the two dimension reduction method are different. This verifies that the DAE can be a substitutional method to POD in reducing the dimension of the oscillating combustion field. The plotted line starts from the left on the horizontal axis as is indicated by the color variation from green to blue. By time goes on, the plot starts going up and down that shows the start of weak combustion oscillation. After oscillation is amplified and the amplitude seen in the vertical direction reaches a certain level, the line starts moving to the right. This left-to-right transition corresponding to the oscillation mode switching. As is known from the previous work,\cite{ref2} N-wave and re-ignition in the recirculation zone occurs and the flame anchoring point is shifted. Thus the two method is thought to separate the combustion oscillation phenomena according to the oscillation component and flame shape, each of which has timescale of acoustic resonance and that of the motion of the oxidizer post length, respectively. This implies that the both of the two dimension reduction methods has potential to distinguish between the two different combustion modes.

The DAE do not output mode-map like POD does. The two input variable of the decoder of the DAE is transformed to 128 variables through the 2-8-32-128 neural network. The two dimensional information is up-sampled to 128 dimensions by going through the network. The 128 neuron activation patterns of the last layer of the decoder corresponds to the mode map, because the output from the decoder is the internal product of the neuron pattern and the generated 128 variables from the preceding layer. In this interpretation, the chronos matrix in POD corresponds to the time varying 128 variables in DAE and the mode map does to the neuron activation pattern. In other words, the neuron activation patterns are the typical elements of combustion field appeared during the sequence. Figure 9 shows four arbitrary sampled neurons among the 128 neurons at the last layer of the decoder, which shows typical patterns of combustion field in terms of coupling between the three fields. Compared with the mode map of POD, smaller structure is captured in these figures. This indicates the future potential to capture non-linear N-wave that has many high spatial frequency components.
plane. The pattern 1 has higher pressure amplitude. As seen from the middle figure of the case of pattern 1, heat release is increased near dump plane. This confirms the mode with which pressure and heat release fluctuation has positive correlation. This mode can contribute to pump up the combustion oscillation. For the pattern 2, in spite of the high pressure near dump plane, the heat release there is not influenced. This mode cannot amplify the oscillation. Probably, the less pressure rise could not light the recirculation zone that could happen for the pattern 1. Thus one can separate the two similar but slightly different phenomenon. From the pattern 3, when pressure is uniform everywhere in the combustion chamber, the heat release fluctuation is negligible. The pattern 4 case shows the case of opposite phase compared with the pattern 1. This also is a mode that can pump up the combustion oscillation. These are only a sample of the 128 patterns of neuron activations. Detailed look into the each of the neuron tells us more than the two mode map from POD. From the DAE processing, even though the information is degenerated to only two dimensional space, one can extract information equivalent to 128 dimensions. Thus, the present DAE has the same performance with POD in terms of dimension reduction; but it can separate the phenomenon into more number of elementary phenomena and helps to understand the key physical process.

4. Conclusion

By dimension analysis on the URANS simulated combustion oscillation data of a model rocket combustor, CVRC, and by introducing 8 layer Deep Auto-Encoder that is a kind of deep neural network, the followings are found.

1. Deep auto-encoder is able to perform the same dimension reduction analysis as compared with conventional proper orthogonal decomposition. The high-dimensional data set of the simulated temperature, heat release, and pressure fluctuation field in the chamber is successfully reduced to two dimensional data. The information available from the reduced dimension data was almost identical regardless of the reduction method.

2. The Deep auto-encoder enables more detailed mode separation compared with POD. For the dimension reduction by POD, the number of modes is equal to the reduced dimension. Deep auto-encoder retains number of modes equal to the final layer of the decoder part of the auto-encoder. The more information can be extracted from the increased number of modes.

3. Based on the analysis, simple investigation on the pressure-heat release correlation could be done to find modes that pumps up the combustion oscillation.

Considering the flexibility of the network design of the deep neural network, a more variation of decomposition and reduced dimension analysis is expected. The ability to contain high spatial frequency components in a mode suggest the potential ability to capture the N-wave separately from the standing wave components. Future work is desirable.

Acknowledgment

The author thank Prof. William E. Anderson of Purdue University for fruitful discussion to develop the present URANS simulation.

References