Effect of Observation by Angle Only Navigation to Plan Non-Cooperative Approach for ADR

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The objective of this paper is to study the effect of observation by Angle Only Navigation to plan non-cooperative Active Debris Removal approach for debris mitigation, and to have a concept of criteria for safe operation and mission assurance.

Key Words: Observation, AON (Angle Only Navigation), Non-Cooperative Approach, ADR (Active Debris Removal)

Nomenclature

a	: semi measure axis
abs	: absolute
ave	: average
BL	: baseline
C	: non-linear observation matrix
CVL	: curvilinear
D	: representative diameter of target
DCR	: dual co-elliptic rendezvous
e	: eccentricity
h	: relative height
i	: inclination
L	: relative length
$LVLH$	: local vertical local horizontal
$p(\Delta v)$	: length or vector of BL by Δv
R	: range
RM	: ranging model
r	: radius
rms	: root mean square
SOR	: stable orbit rendezvous
v	: velocity
x	: state vector
δ	: error
φ	: elevation angle
η	: azimuth angle
ω	: augment of perigee, or target angular size
Ω	: ascending node

Subscripts

0-3	: number of RM
C	: chaser
T	: target

1. Introduction

It is necessary and important to have any visual reference information or criteria of ADR approach for safety and mission assurance, estimating its ranging accuracy and

confirming observability in case of applying AON, and those requirements or conditions for orbital planning about non-cooperative ADR approach are studied, because the ranging accuracy, derived from observed elevation and azimuth angles of a target by a chaser, seems to depend on the position and velocity of each planned relative orbit. Figure 1 shows a typical relative orbit as the background,¹⁻⁴⁾ of this study. This paper discusses (1) the condition of AON which can provide observability, (2) some approximations of ranging formulation, (3) some necessary augmented data from other sensors such as GPS, and (4) the amount of ΔV to keep the observability of relative navigation for ADR approach.

As in general the requirements for ADR approach orbit, the orbital safety against the thruster failure shall require such a planning that avoids any collision between the chaser and Target, and this paper discusses the relationship between the ranging accuracy and observability mentioned above and orbital safety by using some criteria.

Fig. 1. Typical relative approach orbit. Chaser starts from Station keeping (Phase 0), goes through DCR or SOR (Phase 1), Hopping (Phase 2), V-bar approach, Fly Around, and Final Approach to Target (Phase 3).

2. Observation

2.1. Coordinates

Figure 2 shows the coordinates to define measurements.

It is noted that there are two definitions here expressing the Target elevation angle φ . In this paper, the absolute arc length from Chaser to Target, where $y_{T.HILL} = L_T$, and $L = |L_T|$.

Fig. 2. Coordinates. Chaser observes elevation angle ϕ_T and azimuth angle η_T of Target in LVLH frame, and ϕ_{TG} is estimated elevation in CVL frame.

2.2. Optical Target positioning

In this paper, optical Target positioning, that estimates position and velocity by Angle Only Navigation, is defined as feasible, if both of its observability and enough ranging accuracy are satisfied. Figure 3 shows the feasibility mentioned above by an example of triangulation or trigonometry. If the system has enough measurement accuracy of ϕ , the triangulation in Fig. 3 (a) will succeed, and the distance L as range and height h of Target are positioned. However, if both measured ϕ_1 and ϕ_2 are in parallel, or the length of baseline (BL) is too short, the triangulation will fail. In general, it should be recognized that an optical system itself may not have ranging function. Even if it is an optical stereo system, onboard BL shall be unrealistically longer as L is farther.

$$h = \{ \sin \phi_1 \sin \phi_2 / \sin (\phi_1 - \phi_2) \} * BL \quad (1)$$

$$L_T = h / \tan \phi \quad (2)$$

$$L_{T1} = h / \tan \phi_1 \text{ or } L_{T2} = h / \tan \phi_2$$

If it were a ground observation, better BL could be selected by taking better places to avoid such an ill condition of the slender triangle like Fig. 3 (a), but in space this condition is probable and inevitable, and moreover Target and BL are moving.

(a) Example of L/δ application.

(b) Conceptual L/δ Scale and error.

Fig. 3. Ranging model and error δ . (a) shows an example of ranging accuracy and observability, by using triangulation model and error δ , and (b) gives some example numbers of L/δ to indicate the accuracy of this model.

The priority is considered higher to have an allowable condition for Target optical positioning (in this case triangulation) than the analysis of baseline accuracy.

Figure 4 shows an example simulated with random parameters of $d\phi$ and dh to have simple criteria of L/δ , which represents ranging accuracy δ by the following Eq. (3), like heuristic Signal-to-Noise ratio of this ranging system.

$$dL_T = dh / \tan \phi - h d\phi / \sin^2 \phi \quad (3)$$

$$\delta_{abs} = |dL_T|, \quad \delta_{rms} = dL_{T,rms}, \quad \delta_{ave} = |\sum_{i=1}^4 dL_{Ti}|$$

Fig. 4. L/δ with random samples. 3 types of L/δ and envelope of 1 ~ 3σ rms applied to evaluate the accuracy of ranging model, with $L = 18\text{km}$, $h = 3\text{km}$, $dh = 27.55\text{m}$ (1σ), $\phi = 9.462\text{deg}$, $d\phi = 0.0098\text{deg}$ (1σ).

In this paper, the ranging accuracy is evaluated by a non-dimensional number L/δ ; if >20 as allowable (as minimum), >30 as nominal, >50 as good, >100 as high. Figure 4 shows that L/δ_{ave} (4 samples of L/δ averaged) has higher accuracy than $L/\delta_{rms} = 100$.

2.3. Ranging model and accuracy

Because probably there is no single accurate ranging model, which can be applied in space, Table 1 shows three kinds of simple Ranging Model (RM), by using the following equations. Each model depends on relative range, with keeping the same observation ϕ and η for AON;

$$L_{T0} = y_{T,Hill} = 2r_C \theta \quad ; \text{ RM0} \quad (4)$$

$$dL_{T0} = 2dr_C \theta + 2r_C d\theta$$

$$L_{T1,2} = h / \tan \phi_{TG} \quad ; \text{ RM1, 2} \quad (5)$$

$$dL_{T1,2} = dh / \tan \phi_{TG} - h d\phi / \sin^2 \phi_{TG}$$

$$L_{T3} = D / \omega \quad ; \text{ RM3} \quad (6)$$

$$dL_{T3} = -L^2 d\omega / D \text{ (assuming } dD = 0 \text{ as } D \text{ is known)}$$

RM0 of Eq. (4) depends on simple curvilinear estimation.

The difference between RM1 and RM2 is if there is any orbit control or not. RM1 based on Fig. 3 expects only static ranging without control. However, RM2 includes any control for ranging. RM3, based on multi pixel of Target object, is not

a kind of AON. However, RM3 is used to confirm switching from single pixel navigation to multi pixel navigation.

Table 1. Ranging Models.

RM	Target	L/δ	Ranging Scheme	Ranging Model
RM0 (Phase 0~1)	Single Pixel	L/δ_0 Circle	- Curve linear. - Observing Target light in Single Pixel	
RM1 (Phase 0~1)	Single Pixel	L/δ_1 Base line (BL)	- Estimated BL - Target OD data or Chaser GPS	
RM2 (Phase 0~2)	Single Pixel	L/δ_2 ΔV	- Estimated BL - ΔV calibrated	
RM3 (Phase 2~3)	Multi Pixel	L/δ_3 Object	- Target object by Multi Pixel	

3. Condition of AON Providing Enough Accuracy

3.1. Static prediction of ranging accuracy

The objective here is to confirm such area that gives high ranging accuracy by using non-dimensional criteria L/δ in Table 1. If this is possible, it is expected to plan a relative approach orbit generally, and to make it easier to have initial and final conditions of each area.

Figure 5 is an example of integrated navigation chart for ranging with respect to Table 1, with a set of tentative parameters as rms, where all models (RMs) provide $\delta_{n rms}$ by using Eqs. (4), (5), and (6), for whole area related to every relative approach. The area is decomposed first in the basis of $L/\delta_{n rms} > 30$, and composed again as integrated Area connected by squares of blue line as sufficient condition (designed), where all ranging models can be satisfied as $L/\delta_{n rms} > 30$. Now “n” in Fig. 5 is related to relative orbit phase “n” in Fig. 1. Every interface between each Area is shown as blue arrows. The lower interface of height is drawn tentatively as 3km between Area 0 and Area 1, hoping less Δv consumption, but this height is not so sensitive in Area 0. There is no need to confirm the observability in Area 0 by using linear control theory. It is also recognized that SOR orbit has poor ranging accuracy near v-bar above Area 1.

3.2. Phase 0 circle ranging

RM0 is simulated to validate the feasibility of this application by $L/\delta_{0 abs}$. After Fig. 5 is derived, it is easier than before to find a reasonable initial condition of Phase 0. Figure 6 is an example of relative orbit of Phase 0 based on the Chaser LVLH coordinate defined in Fig. 2. Chaser is planned as relative station keeping, but it is gradually approaching Target. The objective is to estimate this relative orbit of Target by the orbital parameters of Chaser as known and AON without relative propagation. The results of RM0 shown in Table 2 give much higher than expected $L/\delta_{0 abs} > 30$. “t” is when $\varphi_{T max}$ is observed, and $\varphi_{T min}$ is measured just before each observation of $\varphi_{T max}$. $L/\delta_{0 abs}$ and $h/\delta_{h abs}$ are in high accuracy, except $h/\delta_{h abs}$ at $t=170,477$ sec (~46hour). This may be affected by the integration or its interval.

Fig. 5. Integrated Navigation Chart (static). $dr_C = h$, $dh_1 = 27.55$ m, $dh_2 = 0.5$ m, $d\phi = d\theta = d\omega = 0.0098$ deg, $D=4$ m

Fig. 6. Phase 0 Simulation to confirm L/δ_0 . Target: $a=7078.136$ km, $e=0.00104637$, $i=98.192$ deg, $\Omega=109.947$, $\omega=90.000$ deg, $M=0.000$ deg, Chaser relative initial condition: $(x_C, y_C, z_C) = (0, -108511.82, -0.02)$ [m], $(v_{x_C}, v_{y_C}, v_{z_C}) = (-2.167, -0.011, 0)$ [m/s] in CVL by special perturbation for two days, with J_{12} and gravity of Sun and moon.

Table 2. Simulation results of L/δ_0 of RM0.

Parameter	Case 1	Case 2
$(\varphi_{T min}, t_{min})$ [deg, s]	(-0.66, 1450)	(-0.77, 167487)
$(\varphi_{T max}, t_{max})$ [deg, s]	(1.48, 4460)	(1.52, 170477)
L_{est} [m]	101443	92762
δ_0 [m]	645	-715
$L/\delta_{0 abs}$ bias	157.2	129.7
h_{est} [m]	726.0	607.0
δ_h [m]	-6.0	18.0
$h/\delta_{h abs}$ bias	113.4	32.4
$L/\delta_{0 rms}$ 1 σ	37.5	41.9
$L/\delta_{0 abs}$ ave	165.2	159.8
$L/\delta_{0 abs}$ min	28.8	21.6

Note: $dh = 27.55$ m (1 σ), $d\phi = 0.0098$ deg (1 σ)

L_{est} is estimated at $\varphi_{T ave} = (\varphi_{T min} + \varphi_{T max}) / 2$ in Table 2, and the average approach speed is predicted as 0.052m/s, whereas the simulation result is 0.044m/s. The effect of air drag is confirmed as 5m (along track) and 0.8m (radial) after 1 revolution, and is not included, where each area /mass ratio is

0.013 for Target and 0.045 for Chaser respectively. Table 2 shows a result of bias and one sample for random error defined in the note, $L/\delta_{0\text{abs}}$ is 157.2 (bias), 37.5 (1 σ), 165.2 (mean), 28.8 (minimum) for 30 measurements in the first column. It is expected that the moving average of observed angles for φ is effective, and $L/\delta_{0\text{abs}}$ min can also exceed 30, if 4 observed angles are averaged. Because Figure 5 suggests that better $L/\delta_{0\text{abs}}$ will be acquired in farther case than Table 2, another simulation is tried, by only changing L (L_{est}) = 110 ~ 125km, and the results are so high as $L/\delta_{0\text{abs}}$ = 311.2, 154.7 and $h/\delta_{h\text{abs}}$ = 51.6, 40.7 respectively, as $t=1450$ and 4460s in Table 2. Here, the estimated average approach speed is 0.037m/s, whereas the simulated true speed is 0.035m/s. For $\delta a = -0.0207\text{km}$, the estimated results at each moment is $\delta a = -0.0177, -0.0200\text{km}$. In the case of Table 2, Chaser may be in eclipse (shadow) when $\varphi_{T\text{max}}$ is observed, and so it is necessary to design the measurement comes in sunlight, or to interpolate the history of elevation angle by using only sunlit data to estimate $\varphi_{TG\text{max}}$ using proper model.^{4,5)}

3.3. Phase 1 triangle ranging with baseline (BL)

Figure 7 shows an example of DCR1 in LVLH coordinate, where Chaser goes 3608.59m below Target in CVL coordinate if seen in Fig. 1. It is understood by using Fig. 2 that the trajectory in Fig. 7 goes down in LVLH coordinate, as Chaser approaches Target. The trajectory is composed of some slope due to the effect of LVLH coordinate with small sinusoidal orbit. Because RM1 depends on CVL coordinate, the elevation shall be transformed from φ_T in LVLH to φ_{TG} in CVL coordinate as follows;

$$\begin{aligned}\varphi_{TG} &= \text{atan} \{ (2r_T \theta (\varphi_T + \theta) / (2r_C \theta)) \} \quad (7) \\ &\sim (r_T / r_C) (\varphi_T + \theta) \\ &\sim \varphi_T + \theta\end{aligned}$$

Fig. 7. Phase 1 Simulation to confirm L/δ_l . Target: $a = 7078.136\text{km}$, $e = 0.00104637$, $i = 98.192\text{deg}$, $\Omega = 109.947$, $\omega = 90.000\text{deg}$, $M = 0.000\text{deg}$, Chaser relative initial condition: $(x_C, y_C, z_C) = (-3608.59, -99940.46, 0.02)$ [m], $(v_{x_C}, v_{y_C}, v_{z_C}) = (0.0, 6.0, 0)$ [m/s] in CVL by special perturbation for five hours, with J_{12} and gravity of Sun and moon.

Table 3 shows the results of $L/\delta_{l\text{abs}}$ in case of Fig. 7, where φ_T is larger, and true model means the simulated $\varphi_T + \theta$ in Eq. (7), and error model may include such an allowable set of errors that keeps $L/\delta_{l\text{abs}} > 30$. Especially the initial L/δ_0 is assumed as >100 by the result of Table 2, and the allowable

error of h_T in LVLH frame is assumed as $0.071 = \sqrt{2} * 50\text{m}$, which means that RM1 is observable in theory,¹⁾ but may not be practically observable,⁶⁾ as this model without these support data. If the amplitude of sinusoidal part ($\sim 1\text{km}$) in Fig. 7 is estimated accurately by only GPS of Chaser, it can be used as a BL as shown in Fig. 3 to make RM1 accurate and observable in a special condition like DCR, where the apsis of both Chaser and Target are closely in a line.

Table 3. Simulation results of L/δ_l of RM1.

Parameter	Source	True Model	Error Model	Input Error
φ_T [deg]	Obs by Cam	-2.70	-2.69	0.010
r_C [km]	Obs by GPS	7070.8	7070.83	0.030
L [km]	Sim	68.401	69.085	0.684
h_T (LVLH) [km]		-3.221	-3.150	0.071
θ [deg]		0.277	0.280	-
θ [rad]		0.005	0.005	-
r_T [km]		7067.910	7068.018	-
h_T (Hill) [km]	Est in Hill	-2.890	-2.812	-
L [km]		68.298	66.822	-
$L/\delta_{l\text{abs}}$		666.4	30.5	-

3.4. Phase 2 triangle ranging with Δv

In Fig. 3, both RM1 and RM2 are applicable in similar area.

It is studied that an orbital maneuver can provide observability.^{7,8)} Figure 8 shows two examples of maneuver, and $p(\Delta v)$ called as “Probing” to provide observability. “ $p(\Delta v)$ ” is a kind of BL, and it is the difference of these two models that RM2 only gives observability with maneuvering.

Fig. 8. Examples of observable situation with Δv . a) and b) give observability with maneuver.

3.4.1. Δv calibration

As an example, perigee or apogee up maneuver can be a Probing, that is, $p(\Delta v)$ for navigation. Table 4 shows a simple relationship between control and navigation, where δh is 7m if $h > 280\text{m}$, and δh will be $< 7\text{m}$ if $h < 280\text{m}$ after calibrated.

By this Δv calibration shown in Fig. 9 a), better $\delta p(\Delta v)$ is expected as following condition;

- 1) $\delta\Delta v_{GPS}$ and δv_{GPS} are applied if $h > h_{CP}$ (Calibration Point)
- 2) $\delta\Delta v_{THR}$ and δh_{THR} are applied, if $h < h_{CP}$,
where $\delta h_{THR} = \delta h v_{GPS} * \delta\Delta v_{THR} / \delta\Delta v_{GPS}$

Table 4. Orbital control, Probing, and navigation error.

Orbital control		$P(\Delta v)$	Nav. Error	
Δt	Δv	Δh	$\delta \Delta h$ (δP)	Note
[s]	[m/s]	[m]	[m]	
10	0.075	280	7	Δv calibration
5	0.0375	140	3.5	
1	0.0075	28	0.7	Δv calibration
0.5	0.00375	14	0.35	GPS accuracy
0.1	0.00075	2.8	0.07	
0.05	0.000375	1.4	0.035	

Condition: GPS data measured 4 times, accuracy $14\text{m}/\sqrt{4 \text{ samples}} = 7\text{m}$ for perigee/apogee up maneuver.

Chaser mass: 400kg, Thruster: 3N, $\Delta h/\delta\Delta h = 40$ ($= 280/7$).

For $\Delta t < 1\text{s}$, Δv model will be calibrated by firing for 1s, or an accelerometer may be necessary for Chaser.

Fig. 9. Δv calibration. After a height transfer (perigee/ apogee up or down) of h_{GPS} measured as $p(\Delta v)$ by Δv_{GPS} , $\delta\Delta v_{GPS}$ and δh_{GPS} are estimated at calibration point and $\delta\Delta v_{THR}$ is predicted.

3.4.2. Condition of AON providing observability

Let's discuss DCR in Fig. 8 b) simply as follows;

$$\alpha_1 = \tan \varphi_n = \frac{h_0}{L_n + v_n n t} \quad (8)$$

$$\alpha'_n = \tan \varphi_n = \frac{h_0 - p(\Delta v)}{L_n + v_n n t} \quad (9)$$

for $n = 2$ and 3 , that is, second and third observation. By using Eqs. (8) and (9), the following matrix Eq. (10) is composed to understand the situation, when RM2 is applied. Here it is so approximated that v_n are constant simply.

$$\begin{bmatrix} \alpha_1 & \alpha_1 t & -1 \\ \alpha'_2 & 2\alpha'_2 t & -1 \\ \alpha'_3 & 3\alpha'_3 t & -1 \end{bmatrix} \begin{bmatrix} L_0 \\ v_0 \\ h_0 \end{bmatrix} = \begin{bmatrix} 0 \\ -p(\Delta v) \\ -p(\Delta v) \end{bmatrix} \quad (10)$$

Equation (10) is to be written as Eq. (11).

$$\mathbf{C}(t)\mathbf{x}_0 = \mathbf{p}(\Delta v) \quad (11)$$

$\mathbf{x}_0$ is solved as Eq. (12), and $\delta\mathbf{x}_0$ is derived as Eq. (13) and (14), by differentiating (11), if $\mathbf{p}(\Delta v)$ is not 0, which means Δv performed. If $\mathbf{p}(\Delta v)$ is 0, $\mathbf{C}(t)$ is singular.

$$\mathbf{x}_0 = \mathbf{C}(t)^{-1}\mathbf{p}(\Delta v) \quad (12)$$

$$\delta\mathbf{x}_0 = \mathbf{C}(t)^{-1} \left\{ \delta\mathbf{p} - \left(\frac{\partial\mathbf{C}(t)}{\partial\varphi} d\varphi + \frac{\partial\mathbf{C}(t)}{\partial t} dt \right) \mathbf{x}_0 \right\} \quad (13)$$

where

$$\frac{\partial\mathbf{C}(t)}{\partial\varphi} d\varphi + \frac{\partial\mathbf{C}(t)}{\partial t} dt \quad (14)$$

$$= \begin{bmatrix} \frac{1}{\cos^2\varphi_1} & \frac{t}{\cos^2\varphi_1} & 0 \\ \frac{1}{\cos^2\varphi_2} & \frac{2t}{\cos^2\varphi_2} & 0 \\ \frac{1}{\cos^2\varphi_3} & \frac{3t}{\cos^2\varphi_3} & 0 \end{bmatrix} d\varphi + \begin{bmatrix} 0 & \tan\varphi_1 & 0 \\ 0 & 2\tan\varphi_2 & 0 \\ 0 & 3\tan\varphi_3 & 0 \end{bmatrix} dt$$

Here, $d\varphi$ is assumed as a sensor bias. It is interesting that Eq. (11) seems to be a linear control model at a glance, but it is non-linear, and it is visualized that some interaction between control and navigation is expected. If Δv has been calibrated, $\delta\mathbf{p}$ may be approximated as 0. In case of Eq. (8) and (9), $\mathbf{p}(\Delta v)$ is vertical, but the same equation as Eq. (11) can be shown with similar components in its matrix for a horizontal case. As approaching to Target, if Δv decreases so small, that is, if its hopping height decreases so short, Eq. (12) could fail finally, and RM2 also has its approaching limit or hopping height necessary, and it will be succeeded by RM3.

3.5. Phase 3 object ranging

Phase 3 is a terminal condition of AON to switch object ranging RM3. During continuous thrusting, longer than one second, the condition to estimate $\delta p(\Delta v)$ expected as 1) above Fig. 9. However, impulsive thrusting, shorter than one second, further Δv calibration will be necessary for the final approach to estimate $\delta p(\Delta v)$ expected as 2) above Fig. 9.

Here, the objective is to find where the later calibration can be planned before impulsive thrusting done. When $\Delta t = 1\text{s}$, $P(\Delta v) = 28\text{m}$ in Table 4, and if the along track movement of Chaser is twice $P(\Delta v)$, assuming relative station keeping, which has the same shape of hopping ellipsoid, and assuming $L/\delta_{2\text{ abs}} = 30$, then $L = 28*2*30 = 1680\text{m}$ will be planned.

Figure 10 depicts switching from L/δ_2 to L/δ_3 , where the Target image is given by camera as vicinity range, and it is recognized that $L/\delta_{2\text{ abs}}$ is much higher than $L/\delta_{3\text{ abs}}$. Thus, the final Δv calibration point will be a relative station keeping orbit. To have a planned intersection of L/δ_2 and L/δ_3 like Fig. 10, hopping approach with periodical Δv will be suitable to switch from high $L/\delta_{2\text{ abs}}$ to $L/\delta_{3\text{ abs}}$ waited for increasing enough value, as an example, 80 by $\text{FOV} = 10\text{deg}$, 1024×1024 pixels, in Fig. 10. At this moment, Δt is so designed as larger than 1s. As shown in Table 1, $L/\delta_{3\text{ abs}}$ is for multi pixel ranging,

and it is interesting that L/δ_{abs} is nearly proportional to the number of pixels, which means 80 pixels seen for D of Target by any camera.

Fig. 10. Switching from L/δ_2 to L/δ_3 . D of Target is 4m, FOV's are examples, and hopping approach is assumed here.

3.6. Amount of Δv to keep observability of AON

Through Phase 0 to Phase 2, the amount of Δv in plane is due to roughly a) transferring height h to v -bar (6.5m/s) and b) total length of hopping (0.5m/s). For Phase 3, the rest of Δv is estimated as 3m/s including c) station keeping and d) other corrections.

3.7. Navigation chart

Figure 5 and Table 5 summarize a proposed Navigation chart (static and dynamic). Table 5 considers robust design against the variation of parameter related to not only sensors but also thrusters and orbit, by applying quality engineering based on design of experiments, and it will contribute as a guideline for relative orbit planning. Figure 5 will be updated on-line for safety prediction, if new dh or $d\phi$ estimated, while mission operation. These results are cooperative with safe approach design, and no contradiction is confirmed so far.

Table 5. Navigation chart (dynamic).

RM	Target	L/δ	Simulation condition	Approach Limit	Observability	Ranging Accuracy $L/\delta_{rms} > 30$
RM0 (Phase 0~1)	Single Pixel	L/δ_0 Circle	$d\theta = 0.098 \text{ deg}$, $h \sim 0 \text{ km}$, $dh = N/A$	$L > 75 \text{ km}$ due to $d\theta$	- Observable - Higher L/δ_0 by longer L , - dh is not sensitive.	
RM1 (Phase 0~1)	Single Pixel	L/δ_1 Base line	$d\phi = 0.098 \text{ deg}$, $dh \sim 27.55 \text{ m}$	$L > 8 \sim 16 \text{ km}$ $ h > 1.2 \text{ km}$ due to dh	- Observable with augmented data, e.g. OD and GPS.	
RM2 (Phase 0~2)		L/δ_2 ΔV calib.	$d\phi = 0.098 \text{ deg}$, $dh = 0.5 \text{ m}$	$L > 100 \text{ m}$, $ h > 30 \text{ m}$ due to dh	- Observable with ΔV - Approach limit improved by ΔV calib.	
RM3 (Phase 2~3)	Multi Pixel	L/δ_3 Object	$d\omega = 0.098 \text{ deg}$, $dh = N/A$	$L < 600 \text{ m}$ due to $d\omega$	- Observable	

Approach Limit Estimated with criteria $L/\delta_{rms} > 30$ (1σ)

Phase 0 is in a preferable condition for optical navigation (RM0). The approach limit of RM0 by pure AON is analyzed assuming 1σ in Table 5, and the limit can be nearer Target if samples for averaging are increased. The approach limit of

RM1 itself is estimated as 8~16km, but can approach up to 3km or nearer, if supported by some data or RM2, which will contribute in a range of 100m~8km in Phase 2. RM2 seems sensitive. To keep robustness of RM2, the operational range should be suitably short. In Phase 3, RM3 is so designed to take over navigation from AON.

4. Conclusion

In this paper, the effect of observation by AON in Low Earth Orbit to plan non-cooperative approach has been studied, and the following items are summarized;

- Navigation chart proposed.
- with
- Visual map of ranging accuracy for optical navigation.
- Unified and non-dimensional simple criterion as L/δ for ranging accuracy, expected to be useful also for other optical navigation.
- Observability studied as follows;
- Difference of observability confirmed between true model with no relative propagation and algorithm using Hill equation as a relative propagation model.
- Observability of true model checked by simulation, and compared with references.^{1,6)}
- Suitable area or orbit phase visualized for keeping accurate ranging.
- Other suitable condition provided by sensor and operation.

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