Error Analysis for CAMUI Type Hybrid Rocket Regression Simulation

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This paper describes the error and uncertainty analysis of the CAMUI hybrid rocket regression simulator. Simulation errors compared to test firings are described and followed by an analysis of the potential uncertainties causing this error. For each uncertainty identified, a sensitivity analysis is then performed with the help of a custom-built simulator to evaluate its impact on the simulator accuracy. It was found that uncertainties in LOX travel time, Reynolds number grouping and model assumptions for the first upstream burning surface have the largest impact on the simulator accuracy and are identified as the main focus points for further research.

Key Words: Hybrid, Rocket, Error, CAMUI, Simulation

1. Introduction

Hybrid rocket engines have several advantages over liquid or solid engines but have traditionally seen limited use due to a low thrust to weight ratio. The Cascading Multi-Stage Impinging-jet (CAMUI) hybrid rocket engine addresses this by using a series of impinging jets and fountain-like flow paths to improve mixing and increase heat transfer to the solid fuel surface. A cut-away view of a typical CAMUI engine is shown in Fig. 1.

Due to the complex CAMUI fuel block geometry, the standard hybrid rocket regression formulas do not sufficiently predict the combustion behaviour of the fuel blocks. For this reason, specific CAMUI fuel regression formulas have been developed which are shown below in Eqs. (1), (2) and (3).1

\[ r_{fu} = a \cdot G_{mu}^{m_{fu}} \cdot \left( \frac{D_p}{D_{P_i}} \right)^{m_{fu}-1}, \]

\[ r_{fp} = a \cdot G_{mp}^{m_{fp}} \cdot \left( \frac{D_p}{D_{P_i}} \right)^{m_{fp}-1}, \]

\[ r_{fd} = a \cdot G_{md}^{m_{fd}} \cdot \left( \frac{H}{H_i} \right)^{m_{fd}} \cdot \left( \frac{D_p}{D_{P_i}} \right)^{m_{fd}-1}. \]

To enable the simulation of engine performance before manufacturing and testing, a fuel regression simulator has been developed based on these equations (Eqs. (1) to (3)). When using this simulator to run simulations of previously performed
engine tests and comparing the results with the measured regressions, the overall simulation errors can be seen. These are shown below in Fig. 2.

Reducing these errors by an order of magnitude is important for the application of the simulator towards future CAMUI engine development. The authors believe that a careful examination of the uncertainties associated with the current model may help identify what areas may be improved upon to realize this objective. The following is a list of uncertainties that have been encountered throughout the development of the CAMUI model and constitute the focus of this analysis:

- LOX travel time
- 2-phase flow
- LOX tank pressurization time
- Flame spreading time
- Local fuel gasification
- Reynolds number grouping
- Regression formula assumptions

The current work is organized as follows:

Section 2 describes the principle of the CAMUI regression simulator followed by the methods used to evaluate each of the uncertainties as well as the sensitivity of the regression simulator to them. Section 3 then presents the resulting uncertainties, their potential error effect on the input parameters of the regression simulator and their effect on the simulator performance. Lastly in section 4 these results are discussed and the main focus points for further research are identified.

2. Methods

2.1. CAMUI simulator

The principle of the regression simulator is shown in Fig. 3.

![Fig. 3. CAMUI simulator concept.](image)

The simulator imports the test data from one series of engine test firings, and then calculates the regression coefficients. They are then used in the regression simulator together with the regression formula to simulate the performance of any other CAMUI type engine. Throughout this analysis, the burning surfaces evaluated are as shown in Fig. 4.

![Fig. 4. CAMUI burn surfaces.](image)

2.2. Engines tests used

The test series used in this work are the TTY-series and the RIE-series. Unless specifically mentioned, the work in this paper is performed with the data from the test firings of the TTY-series. The basic information on the two test series is shown in Table 1.

<table>
<thead>
<tr>
<th>Engine</th>
<th>Thrust (kN)</th>
<th>Engines tested</th>
<th>Fuel blocks</th>
<th>Oxidizer</th>
</tr>
</thead>
<tbody>
<tr>
<td>TTY</td>
<td>10</td>
<td>6</td>
<td>10</td>
<td>HDPE</td>
</tr>
<tr>
<td>RIE</td>
<td>2.5</td>
<td>7</td>
<td>10</td>
<td>HDPE</td>
</tr>
</tbody>
</table>

Specifically the engine TTY04 is used as an example in section 3 unless otherwise noted. The basic information of this engine test is shown in Table 2.

<table>
<thead>
<tr>
<th>Engine</th>
<th>Average Burn Thrust (kN)</th>
<th>Burn time (s)</th>
<th>Fuel blocks</th>
<th>Fuel Oxidizer</th>
</tr>
</thead>
<tbody>
<tr>
<td>TTY04</td>
<td>6</td>
<td>5.3</td>
<td>10</td>
<td>HDPE LOX</td>
</tr>
</tbody>
</table>

For all analysis presented here, only the first (upstream) 8 fuel blocks are taken into account as the last 2 fuel blocks in the CAMUI engine act as fuel mixer/post-combustion chamber, and are not expected to exhibit the improved mixing and heat transfer that the upstream blocks do. They are therefore not evaluated in this research.

2.3. Uncertainty analysis

The objective of this section is to carefully review the assumptions and other factors that lead to errors in the standard regression model that often get overlooked. The details will be specific to the test-setup used in this study, but the concept of the error analysis and many of the types of physical phenomenon identified that lead to these errors are believed to be common among hybrid rocket researchers.

2.3.1. LOX travel time

The oxidizer flow rate for each of the CAMUI test firings is determined by measuring the pressure upstream and downstream of a venturi in the feedline. By making the assumption that the oxidizer is a saturated liquid, the flow rate may be calculated from the pressure drop over the venturi and its orifice diameter. Figure 5 shows the test set-up and the position of this measurement point.

The LOX flow is started by pressurizing the tank, pushing the LOX through the feedline to the engine. The time it takes for the LOX to travel from the tank (feedline top) to the engine (injector) in Fig. 5 is calculated as shown in Eq. (4):

\[
\text{Time} = \frac{\text{Distance}}{\text{Flow rate}}
\]
The current work is organized as follows:

- Regression formula assumptions
- Local fuel gasification
- 2-phase flow
- LOX tank pressurization time
- Flame spreading time
- LOX tank pressurization time

2.3.1. LOX travel time

The LOX flow is started by pressurizing the tank, pushing the LOX through the feedline to the engine. The time it takes for the tank pressure measurement to reach a steady state, and is read from the pressure graphs manually.

2.3.2. 2-phase flow

The LOX partially evaporates while travelling down the feedline, causing the flow to become a liquid-vapor mixture until the feedline has been cooled sufficiently. As the mass flow calculation assumes a liquid state, when and only when the 2-phase flow has completely transitioned to a liquid flow at the measurement point does the measurement of the flow give reliable results. The theoretical time it takes for this is calculated as in Ref. 2) with the addition of the latent heat of vaporization of the LOX. This is shown in Eq. (4).

\[ t_{2ph} = \frac{C_t \cdot (T_{lo} - T_d) \cdot \rho_l \cdot A_{in} \cdot L_t}{C_{LOX} \cdot (T_{lo} - 90) \cdot m_\text{Lox} + H_\text{Lox} \cdot m_\text{o} \cdot f}. \]  

Here it is assumed that the start of the rise of the downstream LOX pressure measurement coincides with the oxygen flow becoming fully liquid. The empirical value \( f \) represents the fraction of the LOX that is vaporized and is found by fitting the calculated values to this point in the measured pressure graphs of the engine firing. \( t_{2ph} \) is neither directly part of a test measurement nor the simulator. Instead its effect on burn time \( t_b \) and LOX flow rate \( m_\text{Lox} \) error are estimated.

2.3.3. LOX tank pressurization time

With the current test set-up, the pressurization of the LOX tank is relatively slow. The tank pressurization time is defined as the time it takes for the tank pressure measurement to reach steady state, and is read from the pressure graphs manually.

2.3.4. Flame spreading time

During the start-up of the firing, the flame spreads from the ignition point, at the upstream end face of the first fuel block, downstream to the other blocks. This in theory would mean that upstream surfaces burn longer than downstream surfaces. It is here assumed that the main mechanism for the flame spreading is the travel of hot gasses from the ignition point down through the engine. Based on this, the flame spreading time is calculated by Eq. (6):

\[ t_f = \frac{V_{mp} \cdot \rho_{G\text{Lox}}}{m_o}. \]  

\( t_f \) is neither directly part of a test measurement nor the simulator. Rather its effect on burn time \( t_b \) error is estimated.

2.3.5. Local fuel gasification

The \( O/F \) ratio and the mass flux are calculated using the oxygen and gasified fuel from the burning surface upstream of the gas flow. An example of such calculations for the upstream end face are shown in Eqs. (7) and (8):

\[ O/F_u = \frac{\dot{m}_{\text{Lox}}}{\dot{m}_{fp}} \]  

\[ G_{pu} = \frac{\dot{m}_{fp} + \dot{m}_{\text{Lox}}}{A_p}. \]  

The current regression model does not consider the locally gasified fuel when calculating \( O/F \) ratio and propellant mass flux \( G_p \). The inclusion of local fuel gasification is illustrated for the upstream end face example in Fig. 6.

Adding the local fuel gasification, Eqs. (7) and (8) become Eqs. (9) and (10):

\[ O/F_u = \frac{\dot{m}_{\text{Lox}}}{\dot{m}_{fp} + \dot{mf}_f}. \]  

\[ G_{pu} = \frac{\dot{m}_{fp} + \dot{mf}_f + \dot{m}_{\text{Lox}}}{A_p}. \]  

Note that as the value of \( mf_u \) itself is a function of Eq. (10), this is only possible because the simulator runs as an iterative time process calculating all parameters for a very short time step and then readjusting for the next time step. This allows \( mf_u \) to be taken from the previous time step. The calculation for the other surfaces are done in the same way with the relevant variables.

2.3.6. Reynolds number grouping

The current regression model as shown in Eqs. (1) to (3) assumes constant \( m \) exponents which is derived as an average best fit for a given engine for each of the three burn surfaces. Ref. 3) shows that the \( m \) exponent is strongly dependent on \( Re \) with \( m \) values ranging from 0.5 to 0.8 for \( Re \) values ranging from 10^4 to 10^5. For the TTY-series of test firings, the range of \( Re \) for each engine is shown below in Table 3.

<table>
<thead>
<tr>
<th>Engine (TTY)</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Max Re [10^4]</td>
<td>5</td>
<td>10</td>
<td>4</td>
<td>6</td>
<td>10</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>Min Re [10^5]</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

In this case, as it is known from Ref. 3) that smaller engines typically have lower \( Re \) values, a second engine series is
also investigated. The RIE-engine is essentially the TTY-engine scaled down to 20% of the thrust level. The $Re$ values for the RIE-series are shown below in Table 4.

<table>
<thead>
<tr>
<th>Engine (RIE)</th>
<th>01</th>
<th>02</th>
<th>03</th>
<th>04</th>
<th>05</th>
<th>06</th>
<th>07</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max $Re [10^6]$</td>
<td>20</td>
<td>20</td>
<td>30</td>
<td>20</td>
<td>20</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Min $Re [10^6]$</td>
<td>7</td>
<td>6</td>
<td>1</td>
<td>9</td>
<td>8</td>
<td>6</td>
<td>5</td>
</tr>
</tbody>
</table>

These values are compared to the $Re$ values and the resulting $m$ values from Ref. 3) to estimate the potential error in $m$ values.

### 2.3.7. Regression formula assumptions

Figure 7 shows the measured vs simulated regression for each burning surface of the TTY04 engine test giving an idea of the error spread across the fuel blocks and burning surfaces.

![Regression simulator error distribution (TTY04).](image)

In the example in Fig. 7 block 1 is closest to the LOX injector and block 8 is the CAMUI block closest to the nozzle. Similar comparisons of other tests in the TTY-series show similar results. Overall there is a tendency for the simulator to underestimate the regression. Furthermore, it can be seen that the error is considerably larger for the upstream blocks than the downstream blocks. The current regression model for the upstream end face assumes, as shown in Eq. (1), that the only effect in regression is convective heat transfer. As the largest error is on the upstream end face of block 1, it is assumed that the main cause of this error is a faulty model for the regression of this surface. This error propagates down through to the other burning surface calculations through the upstream propellant mass flux term $G_p$. To evaluate this, the regression simulation is run with the regression of the upstream end face of block 1 forced to fit the measured regression.

### 2.4. Regression simulator sensitivity analysis

To analyze the sensitivity of the regression simulator to the various uncertainties the CAMUI simulator is run a number of times with input error values, ranging from ±1%, to ±30% as relevant, added to the error source being evaluated. The resulting simulated regression values are compared to evaluate the weight of the given error source. For the regression formula assumption, this is not necessary as the uncertainty and the effect are the same, namely the total regression over the full burn time.

### 3. Results

First, the calculated potential errors are presented for each uncertainty, followed by the effect these have on the simulator performance.

#### 3.1. Potential simulator input errors

##### 3.1.1. LOX travel time

Calculating the LOX travel time from Eq. (4) for TTY04 gives Eq. (11):

$$t_t = \frac{3.8 \times 10^{-4} \cdot (1.5 + 0.8) \cdot 1142}{1} = 1 \text{s.}$$

When considering each term separately, the following physical meanings can be discerned:

- Tank to measurement point = 0.6 s.
- Measurement point to engine = 0.4 s.

With test burn times between 2 s and 10 s this gives a potential LOX flow error, before the flow travels, from the tank to the measurement point, of ≈ 5 – 25%. Similarly it gives a potential burn time error, before the flow reaches from the tank to the engine, of ≈ 10 – 50%.

##### 3.1.2. 2-phase flow

The TTY04 LOX flow measurement pressure graphs are shown in Fig. 8.

![TTY04 LOX line pressure graphs.](image)

When including the 0.6 s travel time from tank to measurement point this gives a $t_{2ph}$ of 0.2 s for this section of the feedline (light blue in Fig. 5) resulting in a $f$ value of 0.3. For the full tank to injector feedline (light blue and purple in Fig. 5) the TTY04 2-phase time is then given by Eq. (12):

$$t_{2ph} = \frac{500 \cdot (283 - 150) \cdot 7930 \cdot 8.4 \cdot 10^{-5} \cdot 1.5}{920 \cdot (150 - 90) \cdot 1 + 214 \cdot 10^3 \cdot 1.3} = 0.6 \text{s.}$$

When considering each term separately, the following physical meanings can be discerned:

- Tank to measurement point = 0.2 s.
- Measurement point to engine = 0.4 s.

With test burn times between 2 s and 10 s this gives a potential LOX flow error, before the flow stabilizes between the tank and the measurement point, of ≈ 1 – 5%. Similarly it gives a potential burn time error, before the flow stabilizes between the tank and the engine, of ≈ 2 – 10%.

##### 3.1.3. LOX tank pressurization time

Figure 8 shows the TTY04 tank pressurization time. This time is defined as the time from start of pressure rise to the time 90% of maximum pressure is reached. In this case the pressurization time is around 2 s. For the rest of the TTY firings it varies from 1-2 s.
3.1.4. Flame spreading time

Using Eq. (6), the TTY04 flame spreading time becomes Eq. (13):

\[
\tau_f = \frac{0.0107 \cdot 10}{1} = 0.1 \text{s.}
\]  

(13)

With test burn times between 2 s and 10 s this gives a potential burn time error of \(\approx 1 - 5\%\).

3.1.5. Local fuel gasification

There are 28 separately calculated burn surfaces in the TTY-engines and overall average \(O/F\) ratio of around 2. This gives an approximation of \(\approx 1 - 2\%\) potential error for the propellant mass flux and \(\approx 2 - 50\%\) potential error in the \(O/F\) ratio caused by the local fuel gasification. Importantly, the larger uncertainties are found at the most upstream burn surfaces.

3.1.6. Reynolds number grouping

Compared with values reported in Ref. 3), the \(Re\) values from the RIE test series show a shift of \(m\) exponent from 0.8 to 0.6, giving a potential \(m\) exponent error of up to \(\approx 30\%\).

3.1.7. Regression formula assumptions

With the upstream end face of block 1 forced to fit the measurement, the simulation performance for TTY04 is as shown in Fig. 9.

Fig. 9. Corrected regression simulator error distribution (TTY04).

When compared to Fig. 7, the errors of the downstream burn surfaces are reduced considerably from up to 43% error to no larger than 21% with average error values (normalized) going from 19% to 9%.

3.2. Regression simulator sensitivity

The sensitivity of the simulator to the identified potential errors, namely LOX flow, burn time, propellant mass flux, \(O/F\) ratio and \(m\) exponent, are shown in Figs. 10 to 14.

4. Discussion

The analysed uncertainties and their potential effect on the regression simulator are summarized below in Table 5.

The results shown in Table 5 indicate that the largest potential simulator errors are found in:

- LOX travel time
- local fuel gasification
- Reynolds number grouping
- model assumptions for the first upstream end face

The largest simulator error from the LOX travel time is by the uncertainty it creates in the burn time. As the LOX flow rate and the burn times are changed independently for each engine firing, this causes the large range in input errors seen in Table 5. This issue could be minimized by upgrading the test set-up in a way to reduce the travel time as close to zero as possible. This could be done, for example, by changing it to allow pre-cooling and pre-filling of the complete feedline.

The largest simulator error from the local fuel gasification is by the effect it has on the \(O/F\) ratio. As the additional fuel mass flow from the locally gasified fuel is approximately the same for each surface and the \(O/F\) ratio drops by orders of magnitude from first to last burning surface, the input error varies considerably as seen in Table 5, with the largest error coming from the
Further effects therefore need to be taken into consideration. As an example, a possible additional effect considered in the case of the upstream end face of the first block could be allowing for direct physical contact between the oxidizer and the fuel surface causing a direct chemical reaction. This is currently part of the ongoing research on the CAMUI engine.

Finally regarding the tank pressurization time, this is taken into account in the simulator and as such does not in itself affect the simulator performance. It does though cause a start-up transient that, though correctly measured, may be undesirable. In this case, it is therefore suggested that the test set-up be improved to minimize or avoid this effect.

5. Conclusion

A CAMUI engine simulator was implemented for hybrid engine performance design purposes. Real engine test fire data was compared with simulated results, and overall regression simulator errors of up to a factor 2 were found.

The identified possible causes for this were discussed and their potential effect on the regression simulator performance were presented. This led to the identification of 3 main points to be the focus of further development of the CAMUI engine. The LOX travel time, through an upgrade to the test set-up, while the Reynolds number grouping and model assumptions are to be the focus points of further theoretical research.

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References