Jumping Mechanism for Asteroid Rover with the Use of Resonance and Electrical Stiffness Switching

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It is not easy for asteroid rover with wheels to move on the surface of asteroids because such an astral body has two special features. One feature is that there is no air and it induces vacuum metalizing of metal slide components. The other feature is that their quite small gravity induces bad controllability of rover with wheels which requires enough frictional force between ground and wheels. Therefore, it is preferable for rover to use jumping mechanism without metal slide component and with low energy consumption. In this paper, a jumping mechanism is proposed and the mechanism uses an energy which is stored by resonance of flexible part. However, simply giving resonance results in low height of jumping. Therefore, electrical stiffness switching system is implemented to realize an effective jumping. Electrical stiffness switching is realized by piezoelectric element and external capacitor which is connected to them. Two method of stiffness switching are introduced. One is stiffness hardening and the other is stiffness softening which solve the problem of stiffness hardening. To validate the proposed mechanisms, numerical analyses are carried out and feasibilities of application for asteroid rover are studied.

Key Words: Rover, Jumping Mechanism, Asteroid, Piezoelectric Element, Hopping Mechanism

Nomenclature

\( L \) : Length of the flexible element
\( h \) : Thickness
\( w \) : Width
\( \rho \) : Mass density
\( l \) : Second moment of area
\( E \) : Young’s modulus
\( A \) : Cross sectional area
\( m \) : Mass
\( k \) : Stiffness of spring
\( y \) : Vertical displacement
\( f_r \) : Reaction force from ground
\( g \) : Gravitational acceleration
\( d_{31} \) : Piezoelectric constant of piezoelectric element for 31 direction
\( \varepsilon_{33} \) : Permittivity of piezoelectric element for 33 direction
\( s_{11} \) : Material compliance (\( =1/E_p \))
\( \xi_i \) : Modal coordinate of i-th mode
\( \phi_i \) : Shape function of i-th mode
\( z \) : Relative displacement with respect to rigid body
\( v \) : Applied voltage to piezoelectric element
\( q \) : Electric charge within circuit
\( f_a \) : Force generated by actuator
\( \Theta \) : Electromechanical coupling coefficient

Subscripts

\( b \) : flexible beam
\( p \) : piezoelectric element
\( t \) : Tip mass
\( r \) : Rigid body
\( f \) : flexible element

1. Introduction

To reveal the origin of the earth and solar system, asteroids have been gathering a lot of attentions and some explorers have been sent to some asteroids in the last couple of decades. NASA has launched NEAR Shoemaker¹ in 1997 and it approached to asteroid “Mathilde” and “Eros” and it sent a wealth of interesting data. NASA also has launched Deep Space ¹² in 1998 and it explored asteroid “Braille” and comet “Borrelly”. Furthermore, ESA , RKA and JAXA also has sent some explorers to such a asteroid. But almost all missions for small astral bodies include explorations with only remote observation and there are a very few cases to survey on their surfaces. One of the cases, Hayabusa developed by JAXA has reached to asteroid “Itokawa” in 2005 and it has sent a great deal of significant data by remote observation. In addition to the data by remote observation, Hayabusa has carried out touch down mission and released a small rover called MINERVA. Such a rover on asteroid is gaining attentions and some of forthcoming missions will execute a release of rovers. However, there are not a lot of prominent achievements by rovers for asteroid and there are some technical difficulties for such a rover, it is not easy for rover to survey on the asteroid’s surface.

The most significant difficulty on asteroid rover comes from smallness of the asteroid’s gravity, e.g. Eros’s gravity on the surface is less than 0.006 [m/s²] and Itokawa has less than 0.1 [mm/s²]. Such a small gravity causes a small frictional force between rover and asteroid’s surface. Therefore, the
conventional locomotion method on astral bodies, i.e. wheel drives, can not work efficiently. In other words, horizontal locomotion by wheel drive yields non-horizontal components of reaction force and then the rover begins to bounce and loses its controllability. In order to solve such a difficulty, some jumping systems are proposed for astral bodies with small gravity. Forces or torques for jumping systems are generally produced by motor drive, e.g. reaction wheel. MINERVA also employs a jumping locomotion which is realized by torque generated by reaction wheel. However, implementation of motor has two problems due to the environment of space explorations and they are as follows:

A) Extremely high vacuum environment induces vacuum metalizing of metal slide component in the system, e.g. bearing and shaft of a rotating system, therefore isolation from vacuum environment have to be implemented. Consequently the volume of the actuator become larger and reliability of the system become lower.

B) The energy for the system is limited due to the limitations on mounting space for batteries and solar cells. Hence, there are possibilities that desirable forces or torques can not be produced. Especially ample margin of energy is required for asteroid rover because it is difficult to obtain detail data of asteroids before arrival of the rover at them.

In other area except for space development, a lot of studies for jumping mechanism have been done. For instance, Hougen, et al. implemented a spring foot for jumping in miniature robot jumping mechanism have been done. For instance, Hougen, et al. also employs a jumping locomotion which is realized by torque generated by reaction wheel. However, implementation of motor has two problems due to the environment of space explorations and they are as follows:

Finally the paper is summarized and some future works are denoted in Section 5.

2. Preliminary Study on 2 DOF Spring–Mass System

Fig. 1 shows 2 DOF spring–mass system. An upper body and a lower body are connected with each other through a spring. The lower body is the only component which has contact with ground and an actuator is mounted on the upper body. Note that any style of the actuator is not specified in the analysis. The mathematical expression of the system is represented by

\[
\begin{bmatrix}
   m_l & 0 \\
   0 & m_u
\end{bmatrix}
\begin{bmatrix}
   y_{l}(t) \\
   y_{u}(t)
\end{bmatrix} + \begin{bmatrix}
   k & -k \\
   -k & k
\end{bmatrix}
\begin{bmatrix}
   y_{l}(t) \\
   y_{u}(t)
\end{bmatrix} = \begin{bmatrix}
   -m_l g + f_r \\
   -m_u g + f_r
\end{bmatrix}
\]

where \( f_r \) is given by

\[
f_r = \begin{cases}
   m_l g - k y_u(t) & (t < t_{t_{hopp}}) \\
   0 & (m_l g - k y_u(t) < 0)
\end{cases}
\]

\[
f_r = \begin{cases}
   m_u g - k y_u(t) & (t_{t_{hopp}} < t < t_{hopp} + t_f) \\
   0 & (m_u g - k y_u(t) < 0)
\end{cases}
\]

\[
y_{t_{hopp}} = y_u(t) - m_u g / k \quad \text{for} \quad 0 \leq t < t_{t_{hopp}}.
\]

Eq. (4) indicates that the amplitude of \( y_u(t) \) grows gradually since the exciting force is supposed to be small as stated above. When the amplitude reaches to a certain level, second condition of Eq. (2) is satisfied and the lower body takes off from ground. Therefore, the trajectory of the center of mass after the takeoff is represented by

\[
y_{t_{t_{hopp}}} = \frac{m_l^2}{2g(m_l + m_u)} \left( \frac{k y_{t_{t_{hopp}}}^2}{m_l} + \frac{m_l g^2}{2k} + 2gy_{t_{t_{hopp}}} \right)
\]

\[
y_{t_{t_{hopp}}} = \frac{m_u}{m_l + m_u} \left( \frac{m_u g}{k} + y_{t_{t_{hopp}}} \right)
\]

Since energy is increased gradually by small force, the energy at \( t = t_{t_{hopp}} \) is almost minimum amount required for the takeoff of the lower body. Therefore, higher jumping can not be expected as long as monotonic increase of energy by small force is applied. Furthermore, \( y_{t_{t_{hopp}}} \) is calculated as

\[
y_{t_{t_{hopp}}} = \frac{m_l + 2m_u}{m_u} g
\]

and it indicates that smaller value of \( g \) results in smaller compression of the spring. Then, the system takes off before enough storage of energy and consequently higher jumping...
can not be expected even in smaller gravity circumstance. Therefore, some additional operation and mechanism is required to utilize the energy stored by resonance for jumping.

3. Jumping Mechanism with Stiffness Hardening

3.1. Stiffness hardening before takeoff

Differentiation of Eq. (5) with respect to \( k \) yields

\[
\frac{\partial y_{\text{max}}}{\partial k} > 0 .
\]  

(7)

Eq. (7) indicates that switching to hard spring at minimum local point right before the jumping results in the increase of the reachable height of the center of mass. Note that an implementation of hard spring as original system does not lead to improvement of the jumping height because the depth of compression of the spring does not become deeper before takeoff of the lower body. Therefore, it is important to switch the stiffness right before the jumping.

3.2. Electrical stiffness switching method

Suppose the system as shown in Fig. 2 and the system consists of a flexible beam, a tip mass, a piezoelectric element and an external capacitor. One end of the flexible beam is fixed and the tip mass is attached to the other end of the beam. The piezoelectric element is attached on the beam and the external capacitor is connected to the element.

Fig. 2. Flexible beam with piezoelectric element and external circuit.

The mathematical expression about the system is given by

\[
m \ddot{\xi} + k \xi - \Theta v = 0
\]  

(8)

\[
\Theta \xi_i + C \dot{\xi}_i v = q .
\]  

(9)

Taking the voltage drop at external capacitor into account, Eq. (8) and Eq. (9) yields the following expression as

\[
m \ddot{\xi} + k \xi_i \left( \frac{\Theta^2}{C^2 + C_E} \right) \dot{\xi}_i = 0 .
\]  

(10)

Eq. (10) indicates that Eq. (8) and Eq. (9) can be represented by an equivalent mechanical model and that the stiffness of the mechanical system can be changed by electrical method. Smaller capacitance of the capacitor realizes the larger stiffness of the system. Illustration of Eq. (10) is shown in Fig. 3. Note that the mass and equivalent spring in Fig. 3 has same structure of the upper mass and the spring in Fig. 1. Therefore, it can be expected that the system in Fig. 3 can be used to realize the stiffness switching mentioned in Subsection 3.1.

3.3. Switching of capacitance

As discussed in Subsection 3.2, switching of capacitance is significant in stiffness switching. In this subsection, the meaning of switching of capacitance is shown by the observation of energy change before and after the switching.

Suppose \( C_{E1} \) and \( C_{E2} \) are the capacitance of the external capacitors. Then, the change of the potential energy \( \Delta U_p \) is denoted as

\[
\Delta U_p = \frac{1}{2} \Theta^2 \xi_i^2 \left( \frac{1}{C^2} + \frac{1}{C_E^2} \right) = \frac{1}{2} \frac{q^2}{C^2} + \frac{1}{2} \frac{q^2}{C_E^2} .
\]  

(11)

where \( C_1 = C_S + C_1 \), \( C_2 = C_S + C_2 \) and the relation \( q = \Theta \xi_i \) is applied to the transformation of Eq. (11). Note that Eq. (11) indicates that the capacitance should be changed under the condition that the electric charge within the circuit is kept in same value. For example the circuit shown in Fig. 4 can realize such a switching of capacitance.

Fig. 3. Equivalent mechanical model of the system shown in Fig. 2.

Fig. 4. Example of external capacitor.

Fig. 4 shows that \( C_{E1} \) is inserted between point A and B at first and \( C_{E2} \) with electric charge \( q \) is inserted between point A and B after capacitance switching. Such a switch can be done by electrical method by the use of FET and so on, therefore reliable switching is possible.

3.4. Proposed jumping mechanism by stiffness hardening

Suppose the system as shown in Fig. 5. The system consists of a rigid body, a tip mass and a flexible element. Furthermore, the flexible element consists of a flexible beam and a piezoelectric element and an external capacitor as shown in the right figure of Fig. 5. For the sake of convenience, the component which consists of the flexible element, the tip mass and the actuator is called “excited component” in the later.

To derive the mathematical expression of the system, following assumptions are introduced.

- The flexible element has uniform shape and uniform material property.
- The collision between ground and rigid body is completely inelastic.
- The deformation of the beam does not have influence on the rotational motion of the system and only vertical translational motion is considered for deriving the mathematical model.
- Only bending deformation and the first mode deformation are supposed for flexible beam, i.e. \( z_i(x,t) = \phi_i(x) \xi_i(t) \).
- Internal viscosity of flexible element is omitted.

Then the mathematical expression is given by

\[
M \ddot{X} + K X = F
\]  

where

\[
X = \begin{bmatrix} y_1(t) \\ \xi_1(t) \end{bmatrix}, \quad F = \begin{bmatrix} f_r + f_a - M \dot{g} \phi(t) \\ \phi(L) f_a - M \dot{g} \phi(t) \end{bmatrix} ,
\]
Introducing the absolute coordinate of the tip mass as \( y_i(t) = y_r(t) + z_i(L,t) \) and transformation matrix \( \Omega \), which satisfies \( X = \Omega X \) for \( X = [y_r(t) \ y_i(t)]^T \), Eq. (12) is rewritten as
\[
\begin{bmatrix} M & 0 \\ 0 & M \end{bmatrix} \dot{X} + \begin{bmatrix} K & 0 \\ 0 & K \end{bmatrix} X = \begin{bmatrix} F \\ 0 \end{bmatrix}.
\]
where
\[
M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}, \quad K = \begin{bmatrix} 0 & 0 \\ 0 & K_{22} + \Theta^2 C_p + C_E \end{bmatrix}
\]
and the details of \( M_{11}, M_{12}, M_{21}, M_{22}, K_{22}, \Theta, C_p, C_E \) are shown in the appendix.

To validate the proposed method, numerical analysis is carried out about the parameter shown in Table 1. Note that the parameters about piezoelectric element are reference values about “C-91 piezoelectric ceramic” produced by Fuji Ceramics Corporation.

3.6. Numerical analysis and discussions

The results of numerical analyses are shown in Fig. 7. The upper, middle and lower figure of Fig. 7 indicate the time history of rigid body \( y_r(t) \), the tip mass of the 1st excited component \( y_{11}(t) \) and the other excited component, i.e. \( y_{ii}(t) \) for \( i = 2, 3, \ldots, N \) are same with that of the 1st because all components are excited synchronously. Therefore, only the time history of \( y_{11}(t) \) is shown in Fig. 7. The results given by solid line and

Table 1. Parameters for analysis in the case of stiffness hardening.

<table>
<thead>
<tr>
<th>N</th>
<th>( h_0 = 0.1[mm] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>0.06[mm]</td>
</tr>
<tr>
<td>( w_s = w_p = 0.5[mm] )</td>
<td>( g = 9.8[m/s^2] )</td>
</tr>
<tr>
<td>( m_s = 5.0[g] )</td>
<td>( \rho_n = 2.710^4[kg/m^3] )</td>
</tr>
<tr>
<td>( m_i = 0.2[g] )</td>
<td>( \rho_p = 7.710^4[kg/m^3] )</td>
</tr>
<tr>
<td>( E_s = 7.010^{11}[Pa] )</td>
<td>( C_{gh} = 1.010^{-3}[F] )</td>
</tr>
<tr>
<td>( E_p = 5.910^{11}[Pa] )</td>
<td>( C_{gh} = 1.010^{-3}[F] )</td>
</tr>
<tr>
<td>( \varepsilon_{33} = 5500 )</td>
<td>( d_1 = -33010^{-12}[m/V] )</td>
</tr>
</tbody>
</table>

The results of numerical analyses are shown in Fig. 7. The upper, middle and lower figure of Fig. 7 indicate the time history of rigid body \( y_r(t) \), the tip mass of the 1st excited component \( y_{11}(t) \) and the other excited component, i.e. \( y_{ii}(t) \) for \( i = 2, 3, \ldots, N \) are same with that of the 1st because all components are excited synchronously. Therefore, only the time history of \( y_{11}(t) \) is shown in Fig. 7. The results given by solid line and
is quite small, Eq. (19) can be instantaneous amplitude. Consequently, the force and the compressed displacement . Then the resultant force by the springs are given by

\[ F_{(2)} = -k_2 \sin \omega t \]  \hspace{1cm} (18)

\[ \Delta F = k_2 \sin \omega t \]

Furthermore, suppose that \( t \) is quite small, Eq. (19) can be expressed as

\[ F_{(1)} + F_{(2)} = \Delta k \sin (\omega t + \phi) \]  \hspace{1cm} (20)

As is clear from comparison between Eq. (18) and Eq. (20), Eq. (20) indicate that the softening of the spring of one side functions as the generation of force by equivalent spring with the spring constant \( \Delta k \) and the compressed displacement \( \Delta \omega \). Therefore, even if stiffness changing rate is not enough high, the exciting method denoted in Subsection 4.1 can provide a larger force by the growth of oscillation amplitude. Therefore, the stiffness switching by piezoelectric element which shows small stiffness changing rate can be applied to other functional materials are used for stiffness switching instead of piezoelectric element, e.g. Piezoelectric elastomer.

4. Jumping Mechanism with the Use of Electrical Stiffness Switching

4.1. Excitation method without change of reaction force

As denoted in Subsection 3.6, the jumping mechanism proposed in Section 3 shows disadvantage in smaller gravity. To overcome such a disadvantage, following exciting forces by the actuators of the jumping mechanism shown in Fig. 6 are introduces as

\[ f_{(1)} = \begin{cases} f_0 \cos \omega t & \text{for } i = 1, 3, 5, \ldots, 2n - 1 \\ -f_0 \cos \omega t & \text{for } i = 2, 4, 6, \ldots, 2n \end{cases} \]  \hspace{1cm} (16)

where \( 2n \) is the number of excited component and \( N = 2n \), that is, \( N \) is set to be even number. Eq. (16) indicates that the displacements of half of the excited components are opposite to the displacements of the other half of the excited components. Such an opposite excitation enables the cancellation of the forces generated by deformation of the excited components and leads to no change of reaction force from ground. Therefore, the deformation of the excited components can grow without takeoff of the lower body. Fig. 8 shows the schema of above discussion in the case of \( N = 2 \).

For the sake of convenience, the excitation procedure in this Subsection is called “opposite oscillation” in the later.

4.2. Application of stiffness softening

Suppose the same system as shown in Fig. 8. When the excited force is enough small, the displacements of the 1st and 2nd excited component for short time range are given by

\[ y_{(1)}(t) = \Lambda \sin \omega t \]
\[ y_{(2)}(t) = -\Lambda \sin \omega t \]

where \( \Lambda \) is instantaneous amplitude. Consequently, the force on the rigid body given by the spring is represented as

\[ F_{(1)} = k_1 \sin \omega t \]
\[ F_{(2)} = -k_2 \sin \omega t \]

where \( y_{(1)} \) and \( y_{(2)} \) are negligible compared to \( y_{(1)} \) and \( y_{(2)} \). Suppose that the rigidity of the spring in the 2nd excited component decreases in small amount \( \Delta k \) and consequently their natural frequency also decreases in small amount \( \Delta \omega \). Then the resultant force by the springs are given as

\[ F_{(1)} + F_{(2)} = k_1 \sin \omega t - (k + \Delta k) \sin (\omega t - \Delta \omega) t \]

\[ = \Lambda^2 \sin (\omega t + \phi) \]  \hspace{1cm} (19)

Furthermore, suppose that \( \Delta \omega \) is negligible compared to \( \omega t \) and \( \omega t + \phi \). Therefore, the stiffness switching by piezoelectric element which shows small stiffness changing rate can be applied to the practical jumping mechanism.
the force lead to the jumping of the system. To validate the proposed jumping mechanism with proposed procedure, numerical analyses are carried out and the parameters for the analyses are shown in Table 2.

Table 2. Parameters for analysis in the case of stiffness softening.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>10</td>
</tr>
<tr>
<td>$h_0$</td>
<td>$h_{pc} = 0.5[m]$</td>
</tr>
<tr>
<td>$L$</td>
<td>$0.10[m]$</td>
</tr>
<tr>
<td>$m_1$</td>
<td>$m_r = 5.0[g]$</td>
</tr>
<tr>
<td>$w_1$</td>
<td>$w_r = 5.0[mm]$</td>
</tr>
<tr>
<td>$g$</td>
<td>$9.8[m/s^2]$</td>
</tr>
<tr>
<td>$C_{EI}$</td>
<td>$1.0 \times 10^{-3}[F]$</td>
</tr>
<tr>
<td>$C_{EL}$</td>
<td>$1.0 \times 10^{-6}[F]$</td>
</tr>
<tr>
<td>$E_h$</td>
<td>$7.0 \times 10^{10}[Pa]$</td>
</tr>
<tr>
<td>$E_e$</td>
<td>$5.9 \times 10^{10}[Pa]$</td>
</tr>
</tbody>
</table>

Figures in Fig. 9 indicate same ones with those in Fig. 7. Analyses are shown in Table 2.

Optimization of the parameters to improve the hopping height of the proposed jumping mechanism.

Construction of an objective valuation parameter in order to evaluate the amount of jumping height for development of practical application, e.g.:

- *Jumping height (Rover mass) × (Rover height) × (Gravity acceleration)*
- Horizontal locomotion method for practical application.
- Application of other functional materials instead of piezoelectric element.
- More strict theoretical analyses without approximation for the jumping mechanism with stiffness softening.

6. Appendix

\[ M_{12} = M_{G1} = m_r + m_r + m_f, \quad M_{12} = \frac{m_f}{L} \int_0^L \phi dx + m_r \phi(L), \]
\[ M_{22} = m_r \phi(L) + m_f \phi(L), \]
\[ M_{F2} = M_{G2} = m_r \phi(L) + m_f \phi(L), \]
\[ K_{22} = \left( \frac{E_r L_0 + \frac{L}{2} \int_0^L \frac{1}{2} \frac{\phi^2}{C} dx}{\frac{1}{2} \int_0^L \frac{\phi^2}{C} dx} \right) \psi, \]
\[ \Theta = \int_0^L \frac{1}{2} \int_0^L \frac{\phi^2}{C} dx dz, \]
\[ C_r = \int_0^L \int_0^L \frac{\psi^2}{C} dx dz \]

References

10. Specification sheet of material characteristics about the lead zirconate titanate materials, Fuji Ceramics Corporation.