Stochastic Dependency of Neural Impulse Sequences

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The statistical dependent properties of the spontaneous impulse sequence recorded from the mesencephalic reticular formation (MRF) and the lateral geniculate nucleus (LGN) in cats were clarified with the following procedures. 1) the value of each interspike interval was normalized by the mean value of the intervals. 2) the stochastic dependency of the normalized sequence was calculated. It is suggested that Markov property exists in the spontaneous impulse sequences recorded from MRF single neurons and not from LGN single neurons.

In previous papers, Shannon's entropy was used as one of statistics: a measure for representing the stochastic dependency of neural impulse sequences, where the entropy and the conditional entropy were calculated from the interval histogram and from the joint interval histogram, respectively. In this report this measure is improved for the purpose of practical use.

The entropy of the interval histogram, $H_0(τ)$, is defined as

$$H_0(τ) = \sum_{j=1}^{n} p(j;τ) \log_2 p(j;τ)$$

where $p(j;τ)$ is a probability of the j-th class in the histogram with a bin width of $τ$. The conditional entropy corresponds to the transition matrix between adjacent events. As in the case of $H_0(τ)$, the conditional entropy, $H_1(τ)$, is defined as a function of $τ$.

$$H_1(τ) = -\sum_{i=1}^{n} \sum_{j=1}^{n} p(i,j;τ) \log_2 p_i(j;τ)$$

where $p(i,j;τ)$ is the joint probability between the occurrences of the i-th and the j-th interval classes, and $p_i(j;τ)$ is the conditional probability of the occurrence of the j-th interval class under the condition of occurrence of the i-th interval class. From the above equations, entropy, $S(τ)$, is defined as follows.

$$S(τ) = H_0(τ) - H_1(τ)$$

$S(τ)$ is zero if and only if two adjacent interspike intervals are statistically independent. $S(τ)$ is normalized by $H_0(τ)$, since $H_0(τ)$ is different in value from neuron to neuron. Therefore we could compare different neural impulse sequences. Hence, a statistic

$$D(τ) = S(τ)/H_0(τ)$$

is developed. $H_1(τ)$ of the shuffled sequence is expected to be equal in value to $H_0(τ)$. However, this actually does not so with insufficient number of samples, because $H_1(τ)$ of the shuffled sequence has the value of the covariance matrix of the distribution of the sample points in the transition matrix. Therefore the stochastic dependency, $d(τ)$, is developed by the following equations.

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is defined. Thus $d(\tau)$ is applicable even to the analysis of finite samples of the impulse sequence. It is shown that the value of $d(\tau)$ approaches to that of $D$, defined in the previous papers,\textsuperscript{1-3} for sufficient large number of samples of the impulse sequence. Here we are confronted with a problem that $d(\tau)$ of an original impulse sequence differs in value from the sequence where each interspike interval of the original sequence is multiplied by the same scalar quantity. Therefore we normalized the value of each interspike interval by the mean value of the intervals, and used for the computation of $d(\tau)$.

\begin{align*}
  d(\tau) &= D_{\text{original}}(\tau) - D_{\text{shuffled}}(\tau) \\
  D_{\text{original}}(\tau) &= \frac{H_0(\tau) - H_1(\tau)}{H_0(\tau)} \\
  D_{\text{shuffled}}(\tau) &= \frac{H_0(\tau) - H_1(\tau)}{H_0(\tau)}
\end{align*}

Hence

\[
  d(\tau) = \frac{H_1(\text{shuffled} \, \tau) - H_1(\text{original} \, \tau)}{H_0(\tau)}
\]

is defined. Thus $d(\tau)$ is applicable even to the analysis of finite samples of the impulse sequence. It is shown that the value of $d(\tau)$ approaches to that of $D$, defined in the previous papers,\textsuperscript{1-3} for sufficient large number of samples of the impulse sequence. Here we are confronted with a problem that $d(\tau)$ of an original impulse sequence differs in value from the sequence where each interspike interval of the original sequence is multiplied by the same scalar quantity. Therefore we normalized the value of each interspike interval by the mean value of the intervals, and used for the computation of $d(\tau)$.

Fig. 1. Plots of the values of $d(\tau)$ computed from the normalized interspike intervals. The number of sample intervals is 4,000.

Some examples of the results with the consecutive interspike intervals of spontaneous discharges recorded from the single neurons of the mesencephalic reticular formation (MRF) and the lateral geniculate nucleus (LGN) in cats are given in Fig. 1, indicating that Markov property exists in the spontaneous impulse activity recorded from MRF neurons and not from LGN neurons.

References