DSMC Simulation of Parallel, Oblique and Normal Free Jet Impingements on a Flat Plate*¹

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Abstract

In the present paper, jet impingements of gas expanded through an orifice into a vacuum were numerically simulated by using the direct simulation Monte Carlo (DSMC) method. Three types of jet impingement, parallel, oblique and normal, were treated. The parallel and oblique impingements were considered as a three-dimensional problem, whereas normal impingement was treated as an axisymmetric problem. The numerical simulations were carried out using the variable soft sphere (VSS) model for nitrogen. For the energy exchange between translational and rotational modes, the Borgnakke-Larsen model was employed together with the temperature-dependent energy exchange probability of Boyd. Simulated results were compared with Legge's experiments and a simple impingement model in the range of the reservoir pressures from 0.1×10^4 to 1.6×10^4 Pa. The present results agreed well with Legge's experimental data of pressure and shear stress distributions on a flat plate.

1. Introduction

Satellites and spacecraft are usually equipped with thrusters for attitude control. Exhaust gas from the thruster expands into the space vacuum and generates a huge jet plume. As a result, the plume may impinge on the satellite body, antennas or solar panels. This impingement causes significant problems of surface degradation, contamination, heat load, disturbance torque and so forth. Hence, it is worthwhile investigating the detailed structure of the plume impingement for the purpose of space engineering.

Rarefied plume impingements have been studied experimentally¹⁻³⁾ and numerically.^{4,5)} In the experimental investigation, the flat-plate impingements of sonic and nozzle jets have been examined by Legge¹⁻³⁾ in a high-vacuum facility. He measured pressure and shear stress on a flat plate for normal, parallel and oblique impingements. Considering the numerical simulation, the direct simulation Monte Carlo (DSMC) method introduced by Bird^{6,7)} has been widely used in calculating rarefied gas flows with the use of computers. Kim & Soga⁴⁾ carried out DSMC simulation of nozzle plume impingement on a perpendicular surface, in which the nozzle exit Mach number was 5. They compared the simulation results with the experimental data of nozzle plume impingement done by Legge for pressure and shear stress distributions on a flat plate, and an agreement between the simulation and experiments was indicated. Recently, Boyd⁵⁾ conducted DSMC simulations of normal, parallel and oblique free-jet impingements of a sonic jet, and compared the results with Legge's

experiments. The simulation results for the normal and oblique impingements agreed with experiments. For the parallel impingement, however, a quantitative comparison with the experimental results was not performed since the reservoir pressure in the simulation is 0.1×10^4 Pa, whereas the reservoir pressure used in Legge's experiment is 0.4×10^4 Pa.

In the present paper, we have performed a DSMC analysis of parallel, oblique and normal jet impingements on a flat plate in a wider range of reservoir pressures (0.1×10^4 – 1.6×10^4 Pa), and made a quantitative comparison of Legge's experimental data and our results for pressure and shear stress distributions on the flat plate. The plume considered here is a sonic jet, different from Kim & Soga's jet with a nozzle exit Mach number of 5. The purpose of the present paper is to validate the capability of the DSMC method to simulate rarefied free-jet impingement.

2. Impingement Models

To calculate an impingement model theoretically, the plume model of Simon⁸⁾ was used. The density of a nozzle plume is described by

$$\frac{\rho(r, \theta)}{\rho^*} = A_p \left(\frac{r^*}{r} \right)^2 \cdot f(\theta), \quad \dots (1)$$

where ρ^* is the density at the orifice exit, A_p the plume constant, r the polar coordinate (radius), r^* the orifice exit radius, $f(\theta) = \cos^{2/(\kappa-1)}[(\pi/2)(\theta/\theta_{\text{lim}})]$, θ_{lim} the maximum turning angle of a stream line at the orifice exit, and κ the adiabatic exponent. The local angle of at-

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Fig. 1. Geometry of impingement.

tack α and the angle θ are indicated in Fig. 1. Combining Eq. (1) with the definitions of pressure and shear stress coefficients, and the equations for isentropic flow of a perfect gas, we obtain for impingement pressure P and impingement shear stress τ

$$\frac{P}{P_0} \left(\frac{z_N}{r^*} \right)^2 = c_p \sin^2 \alpha \cdot A_p \cdot f(\theta) \cdot \frac{\kappa}{\kappa - 1} \cdot \left(\frac{2}{\kappa + 1} \right)^{1/(\kappa - 1)}, \quad \dots (2)$$

$$\frac{\tau}{P_0} \left(\frac{z_N}{r^*} \right)^2 = c_\tau \sin^2 \alpha \cdot A_p \cdot f(\theta) \cdot \frac{\kappa}{\kappa - 1} \cdot \left(\frac{2}{\kappa + 1} \right)^{1/(\kappa - 1)} \quad \dots (3)$$

where P_0 is the reservoir pressure and z_N the distance between the flat plate and the center of the orifice. For the impingement pressure coefficient c_p , the simple equation of modified Newtonian theory corrected by Chiu et al.,⁹⁾

$$c_p = \frac{\kappa + 3}{\kappa + 1} \sin^2 \alpha \cdot (0.6 + 0.63/\alpha), \quad \dots (4)$$

is employed. Moreover, the impingement pressure coefficient and the shear stress coefficient of the free molecular theory³⁾

$$c_{p,FM} = 2 \sin^2 \alpha + \sqrt{\frac{\kappa - 1}{\kappa}} \pi \sqrt{\frac{T_w}{T_0}} \sin \alpha \quad \dots (5)$$

$$c_{\tau,FM} = 2 \cos \alpha \cdot \sin \alpha, \quad \dots (6)$$

was used in order to make a comparison with the simulation results; where subscript FM indicates the free molecular value, T_w the wall temperature and T_0 the reservoir temperature.

3. Numerical Calculations

A jet impingement considered in the present study is such that gas is expanded through an orifice into a vacuum as a free jet and then interacts with a flat plate. Three types of impingement are treated (i.e., parallel, oblique and normal). The parallel and oblique impingements are considered as a three-dimensional problem. On the other hand, the normal impingement is treated as an axisymmetric problem. Boundary conditions for the axisymmetric and three-dimensional

Fig. 2. Physical plane (upper: axisymmetric calculation, lower: three-dimensional calculation).

calculations are shown in the upper and lower sections of Fig. 2, respectively. In the case of the axisymmetric simulation, Usami's method¹⁰⁾ was employed. He simulated both upstream and downstream of the orifice by using the DSMC method. As a result, good agreement in the calculated and experimental flow fields was obtained. For the three-dimensional simulation, two methods are considered. One is the method for calculating both upstream and downstream of the orifice, similar to the axisymmetric simulation. The other is the method for calculating only downstream of the orifice using the orifice exit conditions obtained by the axisymmetric simulation. There was good agreement between the two calculated results for the oblique impingement. Hence, in the present paper, the latter method was applied to the three-dimensional impingement because the calculated time in the latter method was shorter than that in the former method. The orifice radius r^* is set to 1 mm to compare the present DSMC results with the experiments. Wall conditions considered here are such that molecules impinging on a flat plate suffer diffuse reflection, and the temperature on the flat plate is assumed equal the laboratory temperature (300 K). Molecules across the symmetric plane (xz -plane) suffer specular reflection. Outflow conditions are imposed on the upper, left and right boundaries. In the case of the three-dimensional DSMC analysis, the computational domain consists of 12 subregions for efficient computations, as shown in Fig. 3, with total cells of 2.0×10^5 . The total number of simulated molecules in this domain is 2.0×10^6 .

In this calculation, nitrogen was adopted as a test gas for comparison with Legge's experiments. Collisions of

Table 1. Calculation conditions for DSMC simulation.

Case	Impingement type	Angle β	$P_0 (\times 10^4 \text{ Pa})$	z_N/r^*
1	Parallel impingement	0°	0.4	20
2		0°	1.6	20
3		0°	1.6	40
4	Oblique impingement	45°	0.1	40
5		45°	0.4	40
6		45°	1.6	40
7	Normal impingement	90°	0.1	40
8		90°	0.4	40
9		90°	1.6	40

Fig. 3. Mesh for oblique impingement.

molecules are simulated using the variable soft sphere (VSS) model.¹¹⁾ For the calculation of energy exchange between translational and rotational modes, the Borgnakke-Larsen statistical model¹²⁾ is employed together with the temperature-dependent energy exchange probability of Boyd.¹³⁾

In order to make a comparison with Legge's experiments, the pressure P_i and shear stress τ distributions on the flat plate were calculated according to the following equations:

$$P_i = \rho_i \overline{C_{iz}^2} \quad \dots (7)$$

$$\tau = -\rho_i \overline{C_{ix} C_{iz}} \quad \dots (8)$$

where ρ_i is the density in cell i on the flat plate, and C_{ix} and C_{iz} are the thermal velocities in the x and z directions in cell i , respectively.

Table 1 shows the conditions employed in the present calculations. The range of z_N/r^* is from 20 to 40, that is, z_N from 20 to 40 mm, and the range of reservoir pressure P_0 is 0.1×10^4 to 1.6×10^4 Pa (i.e., corresponding Knudsen number is 3.26×10^{-3} to 2.04×10^{-4}). These conditions were taken from Legge's experiments.

4. Results and Discussion

4.1. Parallel impingement Figure 4 shows number density contours on the xz -plane (upper figure) and flat

Fig. 4. Number density contours for case 1.

Fig. 5. Mach number contours and stream lines for case 1.

plate (lower figure) for case 1. The number density increases near the flat plate, and the maximum of n/n_0 on the flat plate is found at $x/z_N = 0.825$. Figure 5 illustrates Mach number contours and stream lines of the xz -plane (upper figure) and flat plate (lower figure) for case 1. It appears that the Mach number decreases gradually towards the flat plate. As shown in the lower figure, a stagnation point on the flat plate is found at $x/z_N = 0.430$, which is different from the position of maximum density, $x/z_N = 0.825$.

The DSMC results of the pressure distribution on the flat plate are compared with Legge's experiments in Fig. 6. The upper figure illustrates the comparison for two different reservoir pressures (cases 1 and 2). As shown in this figure, there is good agreement between simulation and experiments at $x/z_N > 0.8$, whereas the DSMC results are higher than the experimental data at $x/z_N < 0.8$. This presumably means that the number of inflow molecules near the edge of the orifice exit in the simulation is

Fig. 6. Pressure distributions on a flat plate (upper: comparison for two different reservoir pressures, lower: comparison for two different orifice positions).

larger than that in the experiment; thus, the spread of the plume in the DSMC result is larger than that in the experiment, and the impingement pressure increases at $x/z_N < 0.8$. The DSMC results are also compared with the impingement model (Eq. (2)) in Fig. 6. The impingement model agrees well with the present results at $x/z_N > 1.4$. The lower figure demonstrates the comparison of results for two different orifice positions (cases 2 and 3). Good agreement between the DSMC results and Legge's experiments is shown at $x/z_N > 0.8$. The impingement model also agrees with the present simulation at $x/z_N > 1.4$.

Figure 7 illustrates the comparison of DSMC results for shear stress τ on the flat plate with Legge's experiments. The upper figure shows the comparison for two different reservoir pressure (cases 1 and 2). Since case 1 represents more rarefied flow conditions than case 2, as a matter of course, the result for case 1 is closer to the free molecular theory (Eq. (5)). Both of the present results also agree well with Legge's experiments at

Fig. 7. Shear stress distributions on a flat plate (upper: comparison for two different reservoir pressures, lower: comparison for two different orifice positions).

$x/z_N > 0.8$. The lower figure shows the comparison for two different orifice positions (cases 2 and 3). The present results agree well with Legge's experiments at $x/z_N > 0.8$.

4.2. Oblique impingement Concerning the angle between the axis of the orifice and the flat plate for the oblique impingement, 45° is selected to make a comparison with Legge's experiment. In order to investigate the effect of a reservoir pressure on the impingement flowfield, computations were conducted for three different reservoir pressures (cases 4, 5 and 6). Number density contours for cases 4 and 5 are shown in Figs. 8 and 9, respectively. Figures 10 and 11 illustrate Mach number contours and stream lines for cases 4 and 5, respectively. As the reservoir pressure is increased, the local maximum of the Mach number becomes large: 6.2 for case 4, and 8.1 for case 5. The stagnation point for case 4 is found at $x/z_N = 0.209$, whereas that found for case 5 is $x/z_N = 0.237$.

Comparisons of the DSMC results (cases 4, 5 and 6) with Legge's experiments and the impingement model

Fig. 8. Number density contours for case 4.

Fig. 10. Mach number contours and stream lines for case 4.

Fig. 9. Number density contours for case 5.

Fig. 11. Mach number contours and stream lines for case 5.

are illustrated in Fig. 12 for pressure on the flat plate. All of the DSMC results are seen to be almost similar. In Legge's experiments, background pressure existed in the facility, and the value of the normalized background pressure was 0.07 in case 4. Away from the orifice, therefore, the normalized impingement pressure ought to approach this value in the experiments. On the other hand, the DSMC downstream pressure condition is not a finite pressure condition but an outflow condition, and the normalized impingement pressure becomes smaller than 0.07 at $x/z_N > 2.6$. As a result, a discrepancy exists between the present results and experiments downstream. However, it is seen that the impingement model agrees well with the DSMC results at $x/z_N > 1.5$. As the reservoir pressure is decreased, simulation and experimental results more closely approach the free molecular theory at $x/z_N < 1.5$. It is indicated that the impingement model underpredicts the simulation and experiments at $x/z_N < 1.5$. Figure 13 illustrates shear stress distributions for cases 4, 5 and 6. All of the DSMC results also agree well with Legge's experiments. It is seen that the present results come close to the free molecular theory as the reservoir pressure P_0 is decreased.

4.3. Normal impingement Number density contours

Fig. 12. Pressure distributions on a flat plate ($\beta = 45^\circ$).

Fig. 13. Shear stress distributions on a flat plate ($\beta = 45^\circ$).

Fig. 14. Number density contours (upper: case 7, middle: case 8, lower: case 9).

for cases 7, 8 and 9 are shown in Fig. 14. It is seen that the gradient of the number density near the flat plate becomes steep as the reservoir pressure is increased. Mach number contours for cases 7, 8 and 9 are illustrated in Fig. 15. The maximum Mach number is 5.8 for case 7, whereas the maximum is 7.8 for case 8 and 9.5 for case 9.

A comparison of the DSMC results with Legge's experiments and the impingement model for pressure distribution on the flat plate is shown in Fig. 16. Fairly good agreement between the DSMC results and experiments is found near $x/z_N = 0$, whereas the discrepancy

Fig. 15. Mach number contours (upper: case 7, middle: case 8, lower: case 9).

Fig. 16. Pressure distributions on a flat plate.

between the two results becomes large for large values of x/z_N . This is presumably because Legge's experimental condition is non-zero background pressure, whereas the DSMC downstream conditions are outflow conditions, similar to oblique impingement ($\beta = 45^\circ$). The impingement model shows features similar to the simulation results. As the flow is rarefied, however, both Legge's experimental data and DSMC results become slightly higher than the impingement model near $x/z_N = 0$. A comparison of the DSMC results and Legge's experiments in terms of shear stress distribution on the flat plate is shown in Fig. 17. All of the DSMC results show good agreement with Legge's experiments.

Fig. 17. Shear stress distributions on a flat plate.

5. Conclusions

In the present study, the DSMC method was employed for the analysis of parallel, oblique and normal jet impingement on a flat plate. The simulations were conducted for different reservoir pressures, and the change of the flowfield caused by different reservoir pressures was successfully calculated. It was shown that the position of the stagnation point is different according to the reservoir pressure and angle β .

The simulation results were also compared with Legge's experiments in terms of pressure and shear stress distributions on the flat plate. In the case of the pressure distribution, the DSMC results agree well with Legge's experimental data at $\beta=0, 45$ and 90° , although there were discrepancies between the experimental and simulation results at two points. One is seen at $x/z_N < 0.8$ for parallel impingement, where the DSMC results are slightly larger than the experimental data. It is likely that there is a difference in the spread of the plume between simulation and experimental results. The other is seen in the downstream, where the experimental results are higher than the DSMC results. In the experiment, the impingement pressure does not become smaller than the background pressure. However, in space, the impingement pressure become smaller further from the orifice because of the absence of background pressure. There-

fore, it is indicated that DSMC simulations reproduces the impingement pressure for the space vacuum. The simulation results for shear stress distributions are different according to reservoir pressure, and become closer to the free molecular theory as reservoir pressure decreases. Good agreement between the DSMC results and Legge's experiments was indicated. It may be mentioned, therefore, that DSMC calculation of rarefied jet impingement can predict impingement pressure and shear stress.

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