Nonlinear Optimal Feedback Control for Deployment/Retrieval of a Tethered Satellite^{*1}

By Toshiyuki OHTSUKA^{*2}

Key Words : Guidance and Control, Tethered Satellite, Nonlinear Feedback Control

Abstract

Nonlinear optimal state feedback control is designed for the deployment and retrieval of a tethered satellite in the orbital plane. The mass and flexibility of the tether are neglected, and the controlled system is expressed by a bilinear state equation. To stabilize the system asymptotically, the receding horizon control is applied with the terminal state constrained to be zero. As a practical approach to the problem, the time-dependent penalty function method is employed to achieve the terminal constraint asymptotically, and an algorithm is derived so that an increase of the terminal penalty will cause no numerical difficulty. The effectiveness and robustness of the present approach are confirmed in numerical simulations.

1. Introduction

Deployment and retrieval control of a tethered satellite is still a challenging problem in spite of much past research. Because of inherent nonlinearities in the dynamics of a tethered satellite, control design is not a straightforward task even if the point mass model is employed, neglecting the mass and flexibility of the tether. Little research deals with nonlinear control design without approximating the nonlinearities. The typical control techniques to avoid nonlinear control design are linear control design for a linearized model around an equilibrium state¹⁾ and tracking control along an optimal trajectory that is calculated off line.²⁾ These methods guarantee control performance or stability only in the neighborhood of the equilibrium state or the reference trajectory. On the other hand, control techniques to decrease Lyapunov functions deal with the nonlinearities without approximation and guarantee the closed-loop stability.³⁻⁵⁾ However, it is difficult in general to find a Lyapunov function that not only guarantees the stability, but also achieves a specified control performance. The development of a new control technique is desirable so that the nonlinearities are handled without approximation and a certain criterion of control performance is also adopted into the design.

For the deployment and retrieval of a tethered satellite, this paper employs receding horizon control, which is a control technique to handle nonlinearities without approximation and to optimize control performance under a certain criterion. The receding horizon control is a kind of optimal state feedback control that solves at every time step an optimal control problem of a relatively short horizon. Real-time algorithms have been proposed recently for the receding horizon control of general nonlinear systems.^{6,7)} This paper

considers a receding horizon control problem with the terminal state constrained to be zero and employs the time-dependent penalty function method to solve the problem. The terminal constraint is achieved asymptotically as a time-dependent weight in the terminal penalty goes to infinity. It is known^{8,9)} that the closed-loop stability of the receding horizon control is guaranteed if the terminal state is constrained to be zero. Some results^{10,11)} show stability conditions without terminal constraint. However, the terminal constraint remains the fundamental problem setting to guarantee stability. Furthermore, even if the closed-loop system is stable without the terminal constraint, the rate of convergence may be improved by imposing the terminal constraint. The main difficulty in the penalty function method for the asymptotic terminal constraint is that the weight in the terminal penalty goes to infinity, and the conventional algorithm cannot be executed with this infinite weight. This paper therefore proposes a modified algorithm that can handle an infinite weight without numerical difficulty. Numerical simulations validate the proposed algorithm and show its robustness against a modeling error.

This paper treats the point mass model of a tethered satellite, neglecting the mass and flexibility of the tether. The length and the attitude angle of the tether are usually chosen as generalized coordinates in the literature, resulting in a highly nonlinear state equation including trigonometric functions. In contrast, this paper employs the Cartesian coordinates and transforms the control input; this results in a simple bilinear state equation. The analysis and controller are greatly simplified by employing the bilinear equation, although it remains nonlinear.

2. Bilinear Model for the Tethered Satellite System

The tethered satellite is deployed from a main body (a space shuttle) on a circular orbit with a radius R_G and a constant angular velocity Ω . The mass and deflection of the tether are neglected for simplicity. The coordinate system is taken with the origin set at the center of mass of the main body, as shown in Fig. 1. This paper employs Cartesian coordinates instead of polar coordinates usually employed in the literature. The equation of motion of the tethered satellite is given by

$$m\ddot{\mathbf{r}} + \frac{\mu m}{\|\mathbf{R}_G\|^3} \left(I - \frac{3\mathbf{R}_G\mathbf{R}_G^T}{\|\mathbf{R}_G\|^2} \right) \mathbf{r} + m\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) + 2m\boldsymbol{\omega} \times \dot{\mathbf{r}} = -\frac{\mathbf{r}}{\|\mathbf{r}\|} T_c \quad \dots (1)$$

where m denotes the mass of the tethered satellite, $\mathbf{r} = [x \ y \ z]^T$ the position vector of the tethered satellite with respect to the main body, $\boldsymbol{\omega} = [0 \ -\Omega \ 0]^T$ the angular velocity vector of the frame fixed on the main body, $\mathbf{R}_G = [0 \ 0 \ -R_G]^T$ the position vector of the main body from the center of attraction, μ the gravitational constant, I the identity matrix, and T_c the control tension applied on the tether. Higher order terms of $O(\|\mathbf{r}\|^2/\|\mathbf{R}_G\|^2)$ are neglected to obtain Eq. (1).

The dimensionless state equation of the in-plane motion of the tethered satellite is obtained as a bilinear system as

$$\mathbf{x}' = \mathbf{A}\mathbf{x} + \mathbf{B}xu \quad \dots (2)$$

where

$$\mathbf{x} = [x_1 \ x_2 \ x_3 \ x_4]^T = [X \ Z \ X' \ Z']^T \quad \dots (3)$$

$$u = T_c/m\Omega^2 \|\mathbf{r}\| \quad \dots (4)$$

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 2 \\ 0 & 3 & -2 & 0 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix} \quad \dots (5)$$

$X = x/L$, $Z = z/L$ with L denoting the desirable tether length, and $(\)' = d(\)/d\eta$ with $\eta = \Omega t$ denoting the dimensionless time, respectively. One orbital period corresponds to 2π in the dimensionless time, which hereafter is denoted by t instead of η , which causes no serious confusion and allows a transparent comparison between the algorithm proposed in this paper and the conventional one. The control tension T_c is transformed by Eq. (4) to a new control input u of the same sign. The transformation and its inverse are well-defined anywhere but $\mathbf{r} = 0$. The bilinear state equation is much simpler than the state equation obtained by using polar coordinates. The state equation (2) is locally accessible at any state but the origin, and its linearized equation is

Fig. 1. Tethered satellite system.

controllable around the objective state $[0 \ 1 \ 0 \ 0]^T$.

3. Optimal State Feedback Control

This paper considers the receding horizon control that minimizes the following performance index with a moving horizon:

$$J = \frac{e^{\beta t}}{2} [x(t+T) - x_f]^T S_f [x(t+T) - x_f] + \frac{1}{2} \int_t^{t+T} [(x - x_f)^T Q (x - x_f) + \rho(u - u_f)^2] dt^* \quad \dots (6)$$

where $x_f = [0 \ 1 \ 0 \ 0]^T$ denotes the objective state that is an equilibrium point with the constant control input $u_f = 3$, and ρ , Q , and S_f are weights of appropriate sizes. The terminal penalty includes an exponential function, and the terminal constraint $x(t+T) = x_f$ is achieved asymptotically by the penalty function method if β is positive. As described later, the length of the horizon T is a time-dependent function in general. The optimal control input is determined over the horizon $[t, t+T]$ to minimize the performance index and depends on the current state $x(t)$. Only the optimal control input at time t is used as the actual control input in the receding horizon control; this results in the state feedback control depending on the state $x(t)$. The optimal control input is obtained by solving a two-point boundary-value problem associated with the optimal control problem at each time step.

The algorithm of the receding horizon control must avoid numerical difficulty caused as the terminal penalty goes to infinity. A modified algorithm to avoid this difficulty is presented in the following. Its derivation is straightforward in a similar manner as the conventional algorithms^{6,7)} and is summarized in the appendix. To describe the algorithm, the Hamiltonian H is defined as follows:

$$H = \frac{1}{2} [(x - x_f)^T Q (x - x_f) + \rho(u - u_f)^2] + \lambda^T (\mathbf{A}\mathbf{x} + \mathbf{B}xu) \quad \dots (7)$$

where $\lambda(t)$ denotes the costate vector associated with the state vector $x(t)$. The control input $u(t)$ is determined from the state and costate by

$$u(t) = \arg\{H_u[x(t), \lambda(t), u(t)] = 0\} \quad \dots (8)$$

The costate is updated by the following ordinary differential equation.

$$\frac{d\lambda(t)}{dt} = -H_\lambda^T[x(t), \lambda(t), u(t)] - \Lambda^{*-1}(0, t)r^*(0, t) \quad \dots (9)$$

Furthermore, the matrix $\Lambda^*(0, t)$ and the vector $r^*(0, t)$ in Eq. (9) are obtained by the following calculation along the fictitious time τ .

i) Forward integration on the τ axis:

$$x_\tau^*(\tau, t) = H_x^T(x^*, \lambda^*, u^*); x^*(0, t) = x(t) \quad \dots (10)$$

$$\lambda_\tau^*(\tau, t) = -H_\lambda^T(x^*, \lambda^*, u^*); \lambda^*(0, t) = \lambda(t) \quad \dots (11)$$

$$u^*(\tau, t) = \arg[H_u(x^*, \lambda^*, u^*) = 0] \quad \dots (12)$$

ii) Boundary conditions at $\tau = T$:

$$\Lambda^*(T, t) = e^{-\beta t} S_f^{-1} \quad \dots (13)$$

$$r^*(T, t) = \left[S_f^{-1} A_{2s} S_f - \beta \left(1 + \frac{dT}{dt} \right) \right] x^* - \left(1 + \frac{dT}{dt} \right) f - e^{-\beta t} S_f^{-1} \left[A_{2s} \lambda^* + \left(1 + \frac{dT}{dt} \right) H_x^T \right] \quad \dots (14)$$

iii) Backward integration on the τ axis:

$$\Lambda_\tau^* = A^* \Lambda^* + \Lambda^* A^{*T} + \Lambda^* C^* \Lambda - B^* \quad \dots (15)$$

$$r_\tau^* = (A^* + \Lambda^* C^*) r^* \quad \dots (16)$$

where A_{2s} is a stable matrix given by the designer to specify the rate of convergence of the numerical error in the optimal solution. The matrices A^* , B^* , and C^* are evaluated along the trajectory $[x^*(\tau, t), \lambda^*(\tau, t)]^T$, which is obtained by the forward integration of Eqs. (10)–(12), on the τ axis as

$$A^* = f_x - f_u H_{uu}^{-1} H_{xu}, \quad B^* = f_u H_{uu}^{-1} f_u^T, \quad C^* = H_{xx} - H_{xu} H_{uu}^{-1} H_{ux} \quad \dots (17)$$

where $f = Ax + Bxu$.

In summary, steps i) to iii) are executed at each time, and the costate is updated by Eq. (9) using the results of steps i) to iii). The initial condition $\lambda(0)$ for the differential equation (9) is given by the trivial solution of the two-point boundary-value problem associated with the optimal control problem with the horizon $T=0$ as follows:

$$\lambda(0) = S_f[x(0) - x_f] \quad \dots (18)$$

The solution $\lambda(t)$ is then traced, increasing the horizon T to a specified value T_f smoothly, e.g.,

$$T(t) = T_f(1 - e^{-\alpha t}) \quad (\alpha > 0) \quad \dots (19)$$

It should be noted that the inverse of the terminal weight appears in the boundary conditions (13) and (14) at $\tau = T$. These terms converge to zero as the weight goes to infinity and cause no numerical difficulty. They cannot be set to be zero from the beginning of the algorithm, because Eq. (9) is not executable if $\Lambda^*(T, t) = 0$ with $T = 0$, which is consistent with the inability to satisfy the terminal constraint with $T = 0$. Therefore the time-dependent terminal penalty is necessary to achieve the terminal constraint asymptotically.

If the horizon T is positive, some results guarantee the positive definiteness of $\Lambda^*(0, t)$. If $\Lambda^*(T, t) > 0$ and $B^* \geq 0$ hold, then a well-known property¹²⁾ of the Riccati differential equation implies $\Lambda^*(0, t) > 0$. The matrix B^* is positive semidefinite if H_{uu} is positive definite, apparent from Eq. (17). When the terminal constraint is not imposed, $\Lambda^*(T, t) > 0$ holds if S_f is positive definite, as shown in Eq. (13). When the terminal constraint is imposed, $\Lambda^*(0, t)$ must be nonsingular even if $\Lambda^*(T, t)$ converges to zero. Refer to Ref. 13 for a sufficient condition for $\Lambda^*(0, t) > 0$ with $\Lambda^*(T, t) = 0$. That condition is rather complicated and is omitted here. In some cases, $\Lambda^*(0, t)$ may become singular as $\Lambda^*(T, t)$ converges to zero, and consequently the receding horizon control with the terminal constraint may not be applicable. Therefore the terminal constraint restricts the application of the receding horizon control.

4. Simulation Results

4.1. Simulation conditions The preceding algorithm of the receding horizon control is examined in numerical simulations in this section. The Adams method is employed for the numerical integration of the differential equations in the simulations, and the step of the dimensionless time is 0.002. For the integration on the τ axis in the algorithm, the horizon is divided into 20 steps in the case of $T_f = 0.5$ and into 40 steps in the case of $T_f = 1$. Some parameters are fixed throughout the simulations as $\alpha = 0.5$, $S_f = I$, $Q = 10I$, and $A_{2s} = -100I$. For a simulation over the dimensionless time of 15, which is slightly less than two-and-a-half orbital periods, the computational time is about 70 s, which is sufficiently small for real-time implementation in an actual tether system in space. The computational time can be made even smaller with the expense of the accuracy of the optimal solution.

4.2. Deployment control First, the proposed algorithm is examined in the deployment control. The initial state is set as $x(0) = [0 \ 0.1 \ 0 \ 0.01]^T$, and the objective state is $x_f = [0 \ 1 \ 0 \ 0]^T$. The weight for the control input is set as $\rho = 0.1$. Time histories are shown in Fig. 2 for the case of $T_f = 0.5$ and Fig. 3 for the case of $T_f = 1$. The solid lines show the case of $\beta = 0.5$, which results in the exponential growth of the terminal penalty and achieves the terminal constraint $x(t+T) = x_f$ asymptotically. On the other hand, the broken lines show the case of $\beta = 0$, that is, the constant ter-

Fig. 2. Time histories of the closed-loop systems with $T_f = 0.5$ (dashed lines : $\beta = 0$; solid lines : $\beta = 0.5$).

Fig. 3. Time histories of the closed-loop systems with $T_f = 1$ (dashed lines : $\beta = 0$; solid lines : $\beta = 0.5$).

Fig. 4. Root loci of the closed-loop system linearized around the objective state. The terminal time T_f is varied with $\beta=0$ ($\times$: $T_f=0$, $+$: $T_f=0.5$, $*$: $T_f=1$, $\circ$: $T_f \rightarrow \infty$).

Fig. 5. Root loci of the closed-loop system linearized around the objective state. The terminal time T_f is varied with $\beta=0.5$ ($+$: $T_f=0.5$, $*$: $T_f=1$, $\circ$: $T_f \rightarrow \infty$, $\times$: $T_f=0$).

minimal penalty. Each figure shows the four state variables, the dimensionless tension $T_c/mL\Omega^2 = u\|\mathbf{r}\|/L$, and the norm of $\hat{\mathbf{F}}$ defined by

$$\hat{\mathbf{F}}(t) = S_f[x^*(T, t) - x_f] - e^{-\beta t} \lambda^*(T, t) \quad \dots (20)$$

which indicates the error in the optimality condition. A small value of $\|\hat{\mathbf{F}}(t)\|$ implies that the optimal solution is calculated accurately. Although an inequality constraint is not imposed on the tension, the tension is always positive in all time histories. Convergence to the objective state is also satisfactory in all cases. However, if $T_f=0.5$, the asymptotic terminal constraint yields an overshoot in $x_2(Z)$ and the faster responses than in the case without the constraint, as shown in Fig. 2. In contrast, if $T_f=1$, the fast responses are achieved without overshoots, regardless of the asymptotic terminal constraint. The settling time of x_2 is about 7 in the dimensionless time, which corresponds to slightly more than one orbital period. When the horizon is large, the responses do not significantly depend on the asymptotic terminal constraint because the receding horizon control approaches an infinite horizon regulator that is independent of the terminal constraint.

The influences of the terminal constraint and the horizon are investigated on the convergence in the neighborhood of the objective state. The eigenvalues of the closed-loop system linearized around the objective state are plotted in Fig. 4 for $\beta=0$ and in Fig. 5 for $\beta=0.5$, with the horizon varied. The limits of the eigenvalues as $T_f \rightarrow \infty$ are identical in both figures. When T_f is small, the eigenvalues are far from the imaginary axis in Fig. 5, showing the case with the terminal constraint, and they are close to the imaginary axis in Fig. 4, showing the case without the terminal constraint. It can also be concluded from these figures that the terminal constraint with a small horizon yields fast responses, which is consistent with the preceding simulation results. It should

Fig. 6. Trajectories of the closed-loop system with $T_f=0.5$ and $\beta=0$ (solid line: $x(0)=[0 \ 0.1 \ 0 \ 0.01]^T$; dashed line: $x(0)=[0.3 \ 0.1 \ 0 \ 0]^T$; dashed-dotted line: $x(0)=[-0.3 \ 0.1 \ 0 \ 0]^T$; dotted line: $x(0)=[0 \ 0.1 \ 0.2 \ 0]^T$).

be noted that the fast responses with a small horizon do not necessarily imply a satisfactory performance. In general, a larger horizon is preferable, as shown in Figs. 2 and 3. However, if the horizon is large, the numerical error in the algorithm may grow, and the algorithm may fail to trace the optimal solution. Therefore the terminal constraint is useful to obtain fast responses with a small horizon that makes the algorithm easy to compute.

The closed-loop trajectories are shown in Fig. 6 with $T_f=0.5$, $\beta=0$, and the initial condition varied. The asymptotic terminal constraint is not imposed because satisfactory trajectories are achieved with $\beta=0$ without overshoots in $x_2(Z)$. Since inequality constraints are not imposed on the

Fig. 7. Trajectories of the closed-loop system by the linearized feedback with $T_f=0.5$ and $\beta=0$ (solid line: $x(0)=[0 \ 0.1 \ 0 \ 0.01]^T$; dashed line: $x(0)=[0.3 \ 0.1 \ 0 \ 0]^T$; dashed-dotted line: $x(0)=[-0.3 \ 0.1 \ 0 \ 0]^T$; dotted line: $x(0)=[0 \ 0.1 \ 0.2 \ 0]^T$).

motion of the tethered satellite, it moves over the main body in two cases in Fig. 6. If the initial horizontal velocity is so small that pendulous motion of the tethered satellite is induced when the tether length is fixed, the motion over the main body can be avoided if the tether length is fixed until the tethered satellite attains an appropriate initial condition for control. In all cases, the convergence to the objective state is satisfactory and control tension is always positive, though the time histories are omitted. To clarify the effectiveness of the nonlinear state feedback control, the closed-loop trajectories are shown in Fig. 7 for linear state feedback control that is obtained by linearizing the nonlinear receding horizon control around the objective state. The trajectories are unstable in two cases and are oscillatory even in stable cases, which shows the necessity of the nonlinear feedback control. It is concluded that the nonlinear receding horizon control achieves desirable control performance in the wide range of the state space.

To investigate the robustness of the nonlinear receding horizon control, the closed-loop responses with a modeling error are shown in Fig. 8. The bilinear state equation Eq. (2) does not include any model parameters, and the modeling error affects only the transformation of Eq. (4) between the control input and the tension. It is also observed from Eq. (4) that the error in the mass m of the tethered satellite is equivalent with the error in the angular velocity Ω of the orbit. The mass of the tethered satellite is varied in this simulation. Figure 8 shows cases with the modeling error and cases without it for both asymptotically constrained and unconstrained problems. In cases with the modeling error, the actual mass is 20% larger than in the model. The actual mass is set larger than the model because the tether mass may not be negligible. It is apparent from Fig. 8 that the steady-state deviation is smaller if the terminal state is as-

Fig. 8. Trajectories of the closed-loop systems with $T_f=0.5$ (solid line: $\beta=0$ without error; dashed line: $\beta=0.5$ without error; dashed-dotted line: $\beta=0$ with error; dotted line: $\beta=0.5$ with error).

ymptotically constrained ($\beta=0.5$) in spite of the modeling error. That is because the feedback gain around the objective state is larger and cancels the modeling error in the constrained case. To be exact, the steady-state deviation is not zero even in the constrained case, because the state is constrained to be the objective state at the time $t+T$, not the current time t . Therefore the steady-state deviation can grow as the horizon increases even in the constrained case. In all cases the tension is always positive, though the time histories are omitted.

4.3. Retrieval control The proposed algorithm is also applied to the retrieval control. The initial state is set as $x(0)=[0 \ 10 \ 0 \ 0]^T$, and the objective state is $x_1=[0 \ 1 \ 0 \ 0]^T$. That is, an appropriate scaling is assumed so that the same objective state is used in both deployment and retrieval. The weight for the control input is increased to $\rho=1$ to realize moderate control action that does not cause wrapping of the tether round the main body. Other parameters are identical with the simulations for deployment control.

Time histories are shown in Fig. 9 for deployment control with and without the asymptotic terminal constraint. A small overshoot is observed in x_2 when the terminal state is constrained asymptotically; this is similar to the simulation result of deployment control. Time histories are smoother when the terminal state is not constrained asymptotically.

Closed-loop trajectories of retrieval control are shown in Fig. 10 from various initial conditions. The tethered satellite is retrieved successfully in all trajectories. In contrast, as shown in Fig. 11, closed-loop trajectories are oscillatory in the case of the linear state feedback that is obtained by linearizing the nonlinear receding horizon control around the objective state. It is confirmed that the nonlinear receding horizon control achieves satisfactory performance in a wide range of the state space for the retrieval control as well as for the deployment control.

Fig. 9. Time histories of the closed-loop systems with $T_f=0.5$ (dashed lines : $\beta=0$; solid lines : $\beta=0.5$).

Fig. 10. Trajectories of the closed-loop systems with $T_f=0.5$ and $\beta=0$ (solid line: $x(0)=[0 \ 10 \ 0 \ 0]^T$; dashed line: $x(0)=[2 \ 10 \ 0 \ 0]^T$; dashed-dotted line: $x(0)=[-2 \ 10 \ 0 \ 0]^T$; dotted line: $x(0)=[0 \ 10 \ -5 \ 0]^T$).

Fig. 11. Trajectories of the closed-loop system by the linearized feedback with $T_f=0.5$ and $\beta=0$ (solid line: $x(0)=[0 \ 10 \ 0 \ 0]^T$; dashed line: $x(0)=[2 \ 10 \ 0 \ 0]^T$; dashed-dotted line: $x(0)=[-2 \ 10 \ 0 \ 0]^T$; dotted line: $x(0)=[0 \ 10 \ -5 \ 0]^T$).

Finally, the robustness of the retrieval control is investigated by decreasing the actual mass of the tethered satellite to 20% smaller than the design model. The actual mass is decreased because an overestimate of the mass may cause excessive control tension and consequent wrapping of the tether around the main body. The closed-loop trajectories

with and without the error are compared in Fig. 12, where trajectories with and without the asymptotic terminal constraint are also compared. When the terminal state is not constrained asymptotically and the modeling error is imposed, the tether wraps around the main body. In contrast, when the terminal state is constrained asymptotically, the

Fig. 12. Trajectories of the closed-loop systems with $T_f=0.5$ (solid line: $\beta=0$ without error; dashed line: $\beta=0.5$ without error; dashed-dotted line: $\beta=0$ with error; dotted line: $\beta=0.5$ with error).

trajectory around the objective state is not affected seriously by the modeling error. Therefore the receding horizon control with the asymptotic terminal constraint is more robust against the modeling error in the retrieval control as well as in the deployment control.

5. Conclusions

This paper has designed the nonlinear receding horizon control, which is a kind of nonlinear optimal state feedback control, for a point mass model of a tethered satellite. The motion of the tethered satellite has been described with a bilinear state equation by employing Cartesian coordinates instead of polar coordinates. The exponential terminal penalty has been introduced in the performance index so that the terminal constraint can be taken into account in the control design and is achieved asymptotically as the time-dependent penalty function method. The receding horizon control algorithm has been modified to avoid numerical difficulty even if the terminal penalty goes to infinity in the penalty function method. As a result of the numerical simulations, it has been shown that the receding horizon control with the asymptotic terminal constraint is more robust against the modeling error in comparison with the receding horizon control without the asymptotic terminal constraint. It has also been shown that the asymptotic terminal constraint causes an overshoot in the time histories when the horizon is small. When the horizon is large, the responses do not significantly depend on the terminal constraint. It is contrast from linear feedback control or tracking control that the single nonlinear feedback controller achieves a satisfactory performance for various conditions, as shown in the simulations. This nonlinear feedback control as presented in this paper is useful if the operational condition cannot be limited a priori and the controller must handle

various situations. Necessity of the terminal constraint depends on the possible length of the horizon and the requirement for the robustness.

Acknowledgment

This work was partially supported by the Research Fund of the Kurata Memorial Science and Technology Promotion Foundation and a Grant-in-Aid for Scientific Research Result from Japan Society for the Promotion of Science (No. 11750178).

References

- 1) Bainum, P. M. and Kumar, V. K.: Optimal Control of the Shuttle-Tethered-Subsatellite System, *Acta Astronautica*, **7**, 12 (1980), pp. 1333–1348.
- 2) Fujii, H. A. and Anazawa, S.: Deployment/Retrieval Control of Tethered Subsatellite Through an Optimal Path, *J. Guid. Control Dyn.*, **17**, 6 (1994), pp. 1292–1298.
- 3) Fujii, H. and Ishijima, S.: Mission Function Control for Deployment and Retrieval of a Subsatellite, *J. Guid. Control Dyn.*, **12**, 2 (1989), pp. 243–247.
- 4) Vadali, S. R.: Feedback Tether Deployment and Retrieval, *J. Guid. Control Dyn.*, **14**, 2 (1991), pp. 469–470.
- 5) Fujii, H., Uchiyama, K. and Kokubun, K.: Mission Function Control of Tethered Subsatellite Deployment/Retrieval: In-Plane and Out-of-Plane Motion, *J. Guid. Control Dyn.*, **14**, 2 (1991), pp. 471–473.
- 6) Ohtsuka, T. and Fujii, H. A.: Real-Time Optimization Algorithm for Nonlinear Receding-Horizon Control, *Automatica*, **33**, 6 (1997), pp. 1147–1154.
- 7) Ohtsuka, T.: Time-Variant Receding-Horizon Control of Nonlinear Systems, *J. Guid. Control Dyn.*, **21**, 1 (1998), pp. 174–176.
- 8) Chen, C. C. and Shaw, L.: On Receding Horizon Feedback Control, *Automatica*, **18**, 3 (1982), pp. 349–352.
- 9) Mayne, D. Q. and Michalska, H.: Receding Horizon Control of Nonlinear Systems, *IEEE Trans. Automatic Control*, **35**, 7 (1990), pp. 814–824.
- 10) Parisini, T. and Zoppoli, R.: A Receding-Horizon Regulator for Nonlinear Systems and a Neural Approximation, *Automatica*, **31**, 10 (1995), pp. 1443–1451.
- 11) Michalska, H.: Receding Horizon Stabilizing Control without Terminal Constraint on the State, *Proceedings of the 35th IEEE Conference of Decision and Control* (Kobe, Japan), Inst. of Electrical and Electronics Engineers, New York, 1996, pp. 2608–2613.
- 12) Kodama, S. and Suda, N.: *Matrix Theory for System Control*, Society for Instrument and Control Engineers, Tokyo, Japan, 1978, Sec. 13.2 (in Japanese).
- 13) Kwon, W. H. and Pearson, A. E.: A Modified Quadratic Cost Problem and Feedback Stabilization of a Linear System, *IEEE Trans. Automatic Control*, **AC-22**, 5 (1977), pp. 838–842.

Appendix : Derivation of the Algorithm

First, we define

$$\varphi[x(t+T), t] = \frac{e^{\beta t}}{2} [x(t+T) - x_f]^T S_f [x(t+T) - x_f] \quad \dots (A1)$$

$$F = \varphi_x^T [x^*(T, t), t] - \lambda^*(T, t) \quad \dots (A2)$$

The well-known condition for the optimal control is given by

$$x_\tau^*(\tau, t) = H_\lambda^T \quad \dots (A3)$$

$$\lambda_{\tau}^*(\tau, t) = -H_x^T \quad \dots (A4)$$

$$H_u = 0 \quad \dots (A5)$$

$$x^*(0, t) = x(t), \quad \lambda^*(T, t) = \varphi_x^T[x^*(T, t), t] \quad \dots (A6)$$

We would like to find $\lambda(t) = \lambda^*(0, t)$ such that $F=0$ and Eqs. (A3)–(A5) are satisfied with $x^*(0, t) = x(t)$ given. Since an analytical solution is not available for $F=0$ in general, we derive the differential equation of $\lambda(t)$ so that $F=0$ is stabilized, that is, $dF/dt = A_{2s}F$ with A_{2s} a stable matrix. On the other hand, from the definition Eq. (A2) of F , at $\tau=T$,

$$\begin{aligned} \frac{dF}{dt} &= \varphi_{xx} \left(x_t^* + x_{\tau}^* \frac{dT}{dt} \right) + \varphi_{xt} - \left(\lambda_t^* + \lambda_{\tau}^* \frac{dT}{dt} \right) \\ &= \varphi_{xx}(x_t^* - x_{\tau}^*) - (\lambda_t^* - \lambda_{\tau}^*) + \left(1 + \frac{dT}{dt} \right) (H_x^T + \varphi_{xx}f) + \varphi_{xt} \end{aligned} \quad \dots (A7)$$

where note that $dx^*(T, t)/dt = x_t^*(T, t) + x_{\tau}^*(T, t)dT/dt$, $x_{\tau}^* = f$ and $\lambda_{\tau}^* = -H_x^T$.

We must find $d\lambda(t)/dt$ such that the right-hand side of Eq. (A7) is identical to $A_{2s}F$, i.e.,

$$\varphi_{xx}(x_t^* - x_{\tau}^*) - (\lambda_t^* - \lambda_{\tau}^*) + \left(1 + \frac{dT}{dt} \right) (H_x^T + \varphi_{xx}f) + \varphi_{xt} = A_{2s}F \quad \dots (A8)$$

A condition for $d\lambda(t)/dt = \lambda_t^*(0, t)$ is derived from Eq. (A8)

that gives a condition at $\tau=T$. It is straightforward to show

$$\frac{\partial}{\partial t} \begin{bmatrix} x_t^* - x_{\tau}^* \\ \lambda_t^* - \lambda_{\tau}^* \end{bmatrix} = \begin{bmatrix} A^* & -B^* \\ -C^* & -A^{*T} \end{bmatrix} \begin{bmatrix} x_t^* - x_{\tau}^* \\ \lambda_t^* - \lambda_{\tau}^* \end{bmatrix} \quad \dots (A9)$$

where the matrices A^* , B^* , and C^* are given by Eq. (17). We assume the following relation between the costate and the state on the τ axis.

$$x_t^* - x_{\tau}^* = \Lambda^*(\tau, t)(\lambda_t^* - \lambda_{\tau}^*) + r^*(\tau, t) \quad \dots (A10)$$

It should be noted that in comparison with the conventional algorithm,^{6,7)} the proposed algorithm exchanges the right-hand side and the left-hand side in the relation between the costate and the state on the τ axis.

By comparing Eq. (A8) with Eq. (A10), we obtain the boundary conditions for $\Lambda^*(\tau, t)$ and $r^*(\tau, t)$ as

$$\Lambda^*(T, t) = \varphi_{xx}^{-1} \quad \dots (A11)$$

$$r^*(T, t) = \varphi_{xx}^{-1} [A_{2s}F - \left(1 + \frac{dT}{dt} \right) (H_x^T + \varphi_{xx}f) - \varphi_{xt}] \quad \dots (A12)$$

The differential equations for $\Lambda^*(\tau, t)$ and $r^*(\tau, t)$ are obtained as Eqs. (15) and (16) by substituting Eq. (A10) to Eq. (A9) and setting the coefficients of $x_t^* - x_{\tau}^*$ and $\lambda_t^* - \lambda_{\tau}^*$ at zero. Finally, $d\lambda(t)/dt = \lambda_t^*(0, t)$ is obtained as Eq. (9) by evaluating Eq. (A10) at $\tau=0$.