Wavelet Multiresolution Analysis Applied to Coherent Structure Eduction of a Turbulent Jet

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Abstract

To evaluate coherent structures in the dimension of time and scale, a definition of wavelet multiresolution autocorrelation based on the discrete wavelet transform is first developed. Then a new identification technique that combines the wavelet multiresolution analysis combined with the wavelet multiresolution autocorrelation analysis is proposed. By analyzing u- and r-components of fluctuating velocity, the coherent structure and its scales can be identified when larger local amplitude fluctuation and stronger autocorrelation appear at the same wavelet level. For a turbulent jet at a downstream distance of x/d=6, the coherent structures with frequency 39 Hz can be deduced about times 0.29, 0.53, 0.6, and 0.67 s. This also represents the passing of eddies through the shear layer and concentration of the energy of the flow at these instants.

1. Introduction

Many different elementary structures in turbulent flows exist. One important structure is called coherent structure, and it is usually defined as a region of the flow over which at least one of the fundamental flow variables exhibits significant correlation with itself or with another variable over a range of space and/or time that is significantly larger than the smallest local scales of the flow.† Coherent structures are known to exist in a turbulent jet and to be responsible for most of the substantial mass, heat, and momentum transport. The coherent structures are condensation of the vorticity field into organized structure, and they concentrate much of the energy of flow. Characterizing its physics through observations and its behavior under varying conditions is still a topic of much continued study.

Until now the conventional statistical methods, such as space-time correlation functions, coherent functions, proper orthogonal decomposition (POD), conditional sampling methods, and visualization techniques, are well-established usual techniques for gaining information regarding the nature of the coherent structure. However, the coherent structure is characterized by an unsteady and localized structure of multiple spatial scales, and the coherent structure in both time and scale spaces has not yet been clarified and explained. Neither the Fourier analysis nor the traditional correlation method gives us sufficient information because of the nonlocal nature of the Fourier analysis.

In recent decades, interest has been growing in wavelet analyses of the turbulent signals, which can combine time-space and frequency-space analyses to produce a potentially more revealing picture of time-frequency localization of turbulent structure. The continuous wavelet transform has been proposed to track coherent structures§-§ in terms of time and scale. They offered the potentials of extracting new information from turbulent flows. The coefficients of continuous wavelet transform are known to extract the characterization of local regularity, but it is unable to reconstruct the original function. In signal processing, it is important to reconstruct the original signal from the wavelet composition and to study the multiresolution signal in the range of various scales.

The discrete wavelet transform allows an orthogonal projection on a minimal number of independent modes and is invertible; in fact it is orthogonal inverse transform. This analysis is known as a multiresolution representation and might be used to compute or model turbulent flow dynamics. Charles§ used the discrete wavelet transform to study local energy spectra and the flux of kinetic energy in the experimental and direct numerical simulation flows. Staszewski et al.§ identifed the turbulent structures of the atmospheric boundary-layer with the discrete wavelet transform. Li et al.§ employed both the continuous and discrete wavelet transforms to evaluate eddy structures of a jet in the dimension of time and scale. Li et al.§,§ also applied the two-dimensional orthogonal wavelets to the turbulent
images and extracted the multiresolution turbulent structures. However, few investigations concerned the extraction of coherent structure in terms of time and scale.

In this paper we fist proposed a definition of wavelet multiresolution autocorrelation based on the discrete wavelet transform. Then the wavelet multiresolution analysis combined with the wavelet multiresolution autocorrelation analysis to produce a new technique that was employed to identify the coherent structures of turbulent jet in the dimension of time and scale.

2. Discrete Wavelet Transform

Here, a brief review of the discrete wavelet transform is introduced from the view of a matrix.

The discrete wavelet transform is a transformation of information from a fine scale to a coarser scale by extracting information that describes the fine scale variability (the detail coefficients or wavelet coefficients) and the coarser scale smoothness (the smooth coefficients or mother-function coefficients) according to

\[ \{S_j\} = [H]\{S_{j+1}\}; \quad \{D_j\} = [G]\{S_{j+1}\} \quad \ldots \ldots (1) \]

where \(S\) represents mother-function coefficients, \(D\) represents wavelet coefficients, \(j\) is the wavelet level, and \(H\) and \(G\) are the convolution matrices based on the wavelet basis function. High values of \(j\) signify finer scales of information. The complete wavelet transform is a process that recursively applied Eq. (1) from the finest to the coarsest wavelet level (scale). This describes a scale by scale extraction of the variability information at each scale. The mother-function coefficients generated at each scale are used for the extraction in the next coarser scale.

The inverse discrete wavelet transform is similarly implemented via a recursive recombination of the smooth and detailed information from the coarsest to the finest wavelet level (scale):

\[ \{S_{j-1}\} = [H]^{T}\{S_j\} + [G]^{T}\{D_j\} \quad \ldots \ldots (2) \]

where \(H^{T}\) and \(G^{T}\) indicate the transposition of \(H\) and \(G\) matrices, respectively.

The matrices \(H\) and \(G\) are created from the coefficients of the basis functions, and represent the convolution of the basis function with the data.

Many different orthonormal wavelet basis functions, such as the Harr, Daubechies, Meyer, Spline, and Coiflets, were often used in the discrete wavelet transform. Different wavelet basis functions will preferentially move, between scales, different characteristics of the target data sets. For example, use of the Harr basis function may emphasize discontinuity in the target data sets, and the Daubechies family may emphasize smoothness of the analyzed data. In this study, a Daubechies wavelet with orders 20 is used as orthonormal wavelet basis function, since a high order wavelet base has good frequency localization that in turn increases the energy compaction.

3. Multiresolution Analysis

Since Mallat first applied wavelet multiresolution analysis to signal processing, researchers have for several years been making widespread use of wavelet multiresolution analysis. The goal of the wavelet multiresolution analysis is to get a representation of a signal written in a parsimonious manner as a sum of its essential components.

It is well known that a signal often includes too much information for real-time vision processing. Multiresolution algorithms process fewer data by selecting the relevant details necessary to perform a particular recognition task. That is, a parsimonious representation of a signal will preserve the interesting features of the original signal, but will express the function in terms of a relatively small set of coefficients, thus overcoming limitations of the continuous wavelet transform that cannot reconstruct the original signal. Coarse to fine searches process first a low-resolution signal and zoom selectively into finer scales information.

In mathematics, the multiresolution analysis consists of a nested set of linear function spaces \(V_j\), with the resolution of functions increasing with \(j\). More precisely, the closed subspaces \(V_j\) satisfy

\[ \cdots \subset V_j \subset V_{j-1} \subset V_{j-2} \cdots \quad \ldots \ldots (3) \]

with

\[ \bigcup_{j \in \mathbb{Z}} V_j = L^2(\mathbb{R}^2) \quad \text{and} \quad \bigcap_{j \in \mathbb{Z}} V_j = \{0\} \quad \ldots \ldots (4) \]

The basis functions for the subspaces \(V_j\) are called scaling functions of the multiresolution analysis. For every \(j \in \mathbb{Z}\), define the wavelet spaces \(W_j\) to be the orthogonal complement of \(V_j\) of \(V_{j-1}\) We have

\[ V_{j-1} = V_j \oplus W_j \quad \ldots \ldots (5) \]

and

\[ W_j \perp W_{j'} \quad \text{if} \quad j \neq j' \quad \ldots \ldots (6) \]

That is, any function in \(V_{j-1}\) can be written as the sum of a unique function in \(V_j\) and a unique function in \(W_j\). In \(L^2(\mathbb{R}^2)\), the orthogonal basis for \(W_j\) is the family of wavelets that is defined. Thus \(L^2(\mathbb{R}^2)\) can be decomposed into mutually orthogonal subspaces and can be written as

\[ L^2(\mathbb{R}^2) = \bigoplus_{j \in \mathbb{Z}} W_j \quad \ldots \ldots (7) \]

The detail regarding the wavelet transform and multiresolution analysis can be found in many references.\(^{11}\)

In this study, the procedure of the wavelet multiresolution analysis can be summarized in two steps:

1. Wavelet coefficients of a signal are computed by using the discrete wavelet transform of Eq. (1).

2. Inverse wavelet transform of Eq. (2) is applied to wavelet coefficients at each wavelet level, and components
of a signal are obtained at each level or scale.

A sum of these essential components of a signal can recover the original signal.

4. Wavelet Multiresolution Autocorrelation Method

In identifying the self-similarity structure of signals and its evolution in time, the autocorrelation analysis is most used. A difficulty with the conventional autocorrelation method, however, is that the autocorrelation function provides only information about the self-similarity behavior in time delay, but no information about correlation behaviors in scale space because of a lack of scale resolution. Recently, Li developed the wavelet autocorrelation analysis based on the continuous wavelet transform. The wavelet autocorrelation analysis provides the unique capability for decomposing the correlation of arbitrary signals over a two-dimensional time delay-scale plane. In analogy with the wavelet autocorrelation, in this paper we first unfold a signal into its two-dimensional time-scale planes by using wavelet multiresolution analysis. We then use signal components to define a new autocorrelation function, called wavelet multiresolution autocorrelation function, at each wavelet level or scale.

To detect the coherent structures that dominate the turbulent flow in terms of energy concentration, the following procedure is proposed.

1. Compute the components of fluctuating velocity at each wavelet level, using wavelet multiresolution analysis.
2. Compute the wavelet multiresolution autocorrelation coefficients at each wavelet level from the components of fluctuating velocity.
3. The energy-concentrating feature, i.e., coherent structure, exists when large amplitude fluctuation and stronger autocorrelation appear at the same wavelet level.

5. Experimental Apparatus

A definition sketch of the plane jet is shown in Fig. 1, where $x$ is the streamwise coordinate and $y$ is the lateral coordinate. The jet is generated by a blower-type wind tunnel having flow-straightening elements, screens, settling length, and a 24:1 contraction leading to a rectangular nozzle of $300 \times 20$ mm (the nozzle width $d$ is 20 mm). The measurements were carried out at a constant Reynolds number (based on exit mean velocity, $U_0$, and nozzle width, $d$) of approximately $Re = 5,000$, which corresponds to an exit velocity of $U_0 = 5$ m/s. The measured exit turbulent intensity on the jet centerline is less than 0.04%. The velocity components of $u$ and $v$ are measured simultaneously by using an X-type hot wire probe in the $xy$ plane. The measurements in this investigation are taken on the centerline and in the shear layer and are shown in Fig. 1. The recording frequency of data is $10$ kHz, and the number of sampled data of each signal was $15,000$.

6. Results and Discussion

When the wavelet multiresolution analysis is used, the $u$-component of fluctuating velocity in the shear layer of $y/d = 0.5$ at $x/d = 6$ is decomposed in a range of wavelet levels 1-8, which correspond to the central frequency range of $\omega = 9.8-1.250$Hz. The components of fluctuating velocity are shown in Fig. 2(a). The original fluctuating velocity is also plotted at the bottom of this figure. This figure exhibits the time behavior of the fluctuating velocity within different scale bands and gives their contribution to the total turbulent energy. The smaller amplitude of velocity fluctuating at level 1 indicates that the turbulent motion with this scale doesn't exist. However, the larger amplitude fluctuating can be clearly observed at wavelet levels 2-5, where it corresponds to the central frequency range of $19.5-156.3$Hz. This indicates that the velocity components at this scale range contain almost all the turbulent energy. Among these levels, the largest amplitude fluctuating occurs at wavelet level 3. At other wavelet levels (from levels 6 to 8) the smaller amplitude of velocity fluctuating appears again, which corresponds to the small-scale turbulent motions.

Figure 2(b) shows the wavelet multiresolution analysis of the $v$-component of fluctuating velocity at the same position. The large amplitude fluctuating can be clearly observed at levels 3-6. This implies that most turbulent energy is concentrated in this scale range. Here levels 3-6 correspond to the central frequency range of $39-312.5$Hz.

The wavelet multiresolution autocorrelation coefficients of the $u$- and $v$-components of fluctuating velocity at each wavelet level are shown in Fig. 3. The conventional autocorrelation coefficients are also plotted at the bottom of this figure. With an increase of the time delay, the conventional autocorrelation coefficient decreases to zero. It is difficult to obtain information on the self-similarity flow structures. The distribution of wavelet multiresolution autocorrelation coefficients shows a map of apparent multiscale self-similarity flow structures. Within the range of a smaller scale, i.e., from wavelet levels 5 to 8, the wavelet multiresolution autocorrelation coefficients become weak as the time delay increases. This appears to show no organized structures at higher frequencies. Although the larger amplitude of autocorrelation coefficients occurs at wavelet levels 1 and 2, no
coherent structure exists within the range of the larger scale because the components of fluctuating velocity exhibit the smaller amplitude in the wavelet multiresolution analysis of Fig. 2. As increasing the wavelet levels, i.e., a decreasing scale, the amplitude of autocorrelation coefficients decreases in the range of wavelet levels 3–5. This implies a disappearance of the organized structures. The periodic large peaks can be clearly observed at wavelet level 3, which corresponds to the central frequency of 39 Hz, and this indicates an apparent coherent structure or dominant structure. Comparing Fig. 3(a) with Fig. 3(b) at wavelet level 3, it can be observed that the distribution of autocorrelation coefficients is similarity and the positions of peaks in Fig. 3(a) correspond to that of Fig. 3(b) at the same time delay.

After determining the scale of coherent structures, it is important to analyze the coherent structures responding to a change in the time. The results of Yule's and our flow visualization experiment have shown that when an eddy passes, the local fluctuating velocity exhibits unusually large positive and negative peaks. It can say that the positive or negative peaks of the component of velocity fluctuation at wavelet level 3 correspond to the passing of the coherent structure or organized eddy motions. It is evident that the larger peaks occur around times 0.29, 0.53, 0.6, and 0.67 s in Fig. 2, which correspond to the larger coherent structures. These peaks also imply the passing of dominant eddies through the shear layer. We note that these features could not have been extracted under the conventional Fourier-based analysis.

7. Conclusion

In this paper the wavelet multiresolution autocorrelation analysis is first developed based on the discrete wavelet transform. Then the wavelet multiresolution analysis is combined with the wavelet multiresolution autocorrelation analysis to produce a new technique that is applied to identification of the coherent structures in the dimension of time and scale. The following results can be summarized.
Fig. 3(a). Wavelet multiresolution autocorrelation coefficients of u-fluctuating velocity at $x/d=6$ and $y/d=0.5$.

(1) Using this new technique, the coherent structure can be identified when the larger amplitude fluctuation and stronger autocorrelation appear at the same wavelet level.

(2) When $u$- and $r$-components of fluctuating velocity of a jet at $x/d=6$ and $y/d=0.5$ are analyzed, the coherent structures with a central frequency of 39 Hz can be deduced around times 0.29, 0.53, 0.6, and 0.67 s. This also represents the passing of eddies through the shear layer and concentration of the flow energy at these instants.

References


