A Simplified Analysis on a Pulse Detonation Engine Model

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The performance of pulse detonation engines was analytically estimated by using a simple model. A pulse detonation engine was modeled as a straight tube. One end of the tube was closed and the other was open, and a detonation wave was ignited at the closed end. One cycle of the pulse-detonation-engine operation was divided into three phases: combustion, exhaust, and filling phases. The combustion and exhaust phases were theoretically analyzed with some simplifications, using the Hugoniot relation for the Chapman-Jouguet detonation wave and flow relations for self-similar rarefaction waves. Based on the simplified theoretical analysis, useful formulas for impulse density per one-cycle operation and time-averaged thrust density were derived.

Key Words: Pulse Detonation Engine, Theoretical Analysis, Gas Dynamics

Nomenclature

a : sonic speed of gas
 c_v : specific heat at constant volume per unit mass
 D : propagation velocity of specified surface in x coordinate
 D_{CJ} : Chapman-Jouguet detonation speed of detonable mixture
 f_{cyc} : frequency of cyclic pulse-detonation-engine operation
 I_{cyc} : impulse density acting on thrust wall per one-cycle operation
 L : length of pulse detonation engine
 ℓ_f : characteristic length of detonable-mixture injection
 M_{CJ} : Mach number of Chapman-Jouguet detonation wave ($= D_{CJ}/a_1$)
 p : thermodynamic pressure of gas
 p_{av} : time-averaged thrust density
 $p_w(t)$: pressure acting on thrust wall as a function of time
 q : heat released in chemical reaction per unit mass
 R_u : universal gas constant
 T : thermodynamic temperature of gas
 t : time relative to instant of ignition
 T_{cyc} : period of cyclic pulse-detonation-engine operation
 t_I : time at which x_3 becomes L
 t_{II} : time at which front boundary of rarefaction wave originating from open end reaches closed end
 t_{III} : time at which exhaust of burned gas is completed
 u : flow velocity of gas in x coordinate
 V_f : characteristic speed of detonable-mixture injection
 x : coordinate along axis of pulse detonation engine, where $x = 0$ and $x = L$ correspond to closed and open ends, respectively

γ : specific-heat ratio of gas
 μ : average molecular weight of gas
 ρ : mass density of gas
 τ_f : duration of filling phase

Subscripts

1 : undisturbed detonable mixture
 2 : Chapman-Jouguet surface of detonation wave
 3 : rear boundary of rarefaction wave following detonation wave
 ex : open-end boundary of rarefaction wave originating from open end
 N : von Neumann spike of detonation wave

1. Introduction

A pulse detonation engine (PDE) is expected to be a high-performance next-generation aerospace propulsion engine. In PDEs, fuel is burned as a self-sustaining detonation wave propagating through a tube. The combustion as a detonation wave is close to an isochoric process rather than an isobaric one. Therefore higher heat-cycle efficiency is expected for PDEs than for conventional aerospace propulsion engines, in which combustion is close to an isobaric process.¹⁾

Although research on PDEs has more than a half-century of history, interests in them have been renewed in the past several years. Among the worldwide evolution of PDE researches, a comprehensive review on propulsion applications of detonation waves was recently published.²⁾ A great part of current PDE research seems to be more practical than fundamental, seriously aiming at the commercial use of PDEs.

Because PDEs are of simple structures, performance estimations on them are carried out mostly by methods of computational fluid dynamics (CFD). It is true that performance estimations by CFD methods are more accurate than those by simplified analytical methods, analytical methods

are often more suitable for understanding the performance of PDEs in perspective. Although Bussing and Pappas¹⁾ already showed an analytical treatment for performance estimations on PDEs, their treatment was not adequate enough to elucidate the dependence of PDE performance on governing parameters. In this paper, a simple performance analysis on PDEs is presented. Using the Hugoniot relation for the Chapman-Jouguet (CJ) detonation wave and flow relations for self-similar rarefaction waves, pressure on the thrust wall is estimated as a function of time. Based on the analysis, some useful formulas are derived.

2. Model

A PDE is modeled as a straight tube with a constant cross section. One end of the tube is closed and the other is open. For simplicity, gases are treated as polytropic gases.³⁾ And it is also assumed that the viscous effects and thermal conduction are negligible and that flow is one-dimensional. In the analysis below, one cycle of PDE operation is examined.

Initially, the tube is filled with a uniform detonable mixture at rest, as shown in Fig. 1(a). The initial undisturbed mixture is characterized by ρ_1 , p_1 , γ_1 , $u_1 (= 0)$. At time $t = 0$, it is ignited at $x = 0$, and the CJ detonation wave starts to propagate from the closed end to the open one. The initiation process of the CJ detonation wave is among the most important issues for PDEs.⁴⁾ But in this model, this issue is not treated, and it is assumed that the CJ detonation wave starts to propagate from the closed end, namely $x = 0$, at $t = 0$.

By the passing of the CJ detonation wave through the gas, the gas is accelerated in the same direction as the wave propagation because the CJ detonation wave is a compression wave. Since the gas attached to the closed end should be at rest, the gas between the CJ detonation wave and the closed end is decelerated. This deceleration is realized in a self-similar rarefaction wave following the CJ detonation wave. This situation is shown in Fig. 1(b). The front boundary of the rarefaction wave coincides with the CJ surface of the detonation wave. The gas at the CJ surface of the detonation wave is characterized by ρ_2 , p_2 , γ_2 , u_2 . The rear boundary of the rarefaction wave also propagates toward the open end. The gas at the rear boundary of the rarefaction wave is characterized by ρ_3 , p_3 , $\gamma_3 (= \gamma_2)$, $u_3 (= 0)$, which also characterize the whole gas in region $0 \leq x \leq x_3$. In this model, x_3 becomes L at time $t = t_I$, and at this instant, all gas in the tube is characterized by ρ_3 , p_3 , $\gamma_3 (= \gamma_2)$, $u_3 (= 0)$, as shown in Fig. 1(c). The period $0 \leq t \leq t_I$ is named the combustion phase.

At time $t = t_I$, another rarefaction wave is created and starts to propagate from the open end to the closed one. The front boundary of this rarefaction wave propagates at sonic speed a_3 , and reaches the closed end at $t = t_{II}$. Figures 1(d) and 1(e) show this situation. Through this rarefaction wave, the burned gas is exhausted from the open end of the tube. The exhaust of the burned gas lasts until time $t = t_{III}$, as

Fig. 1. Pressure distributions at some instants: (a) $t = 0$, (b) $0 < t < t_I$, (c) $t = t_I$, (d) $t_I < t < t_{II}$, (e) $t = t_{II}$, (f) $t = t_{III}$, (g) $t = T_{cyc}$. Calculation was carried out by using a typical parameter set the same as that used in "numerical example" of the text.

shown in Fig. 1(f). The period $t_I \leq t \leq t_{III}$ is named the exhaust phase. After time $t = t_{III}$, the tube is recharged with a fresh detonable mixture, and the recharging of the tube is completed at time $t = T_{cyc}$, shown in Fig. 1(g). Namely, T_{cyc} is the period of the cyclic PDE operation. The period $t_{III} \leq t \leq T_{cyc}$ is named the filling phase.

3. Analysis

First, the quantities on the CJ surface ρ_2 , p_2 , u_2 , D_2 are examined. When the heat released in the chemical reaction per unit mass q is much larger than the internal thermal energy per unit mass of the unburned gas $c_{v1}T_1$, they are written by the Hugoniot relation as follows,⁵⁾

$$\begin{aligned}\rho_2 &= \frac{\gamma_2 + 1}{\gamma_2} \rho_1, \\ p_2 &= \frac{\gamma_1}{\gamma_2 + 1} M_{CJ}^2 p_1, \\ u_2 &= \frac{1}{\gamma_2} a_2 = \frac{1}{\gamma_2 + 1} D_{CJ}, \\ D_2 &= D_{CJ} = \sqrt{2(\gamma_2^2 - 1)q}.\end{aligned}$$

Since thickness of the CJ detonation wave is generally negligible compared with tube length, the position of the CJ surface is written by

$$x_2 = D_2 t = D_{CJ} t.$$

On the von Neumann spike, ρ_N and p_N are written as follows,³⁾

$$\begin{aligned}\rho_N &= \frac{\gamma_1 + 1}{\gamma_1 - 1} \rho_1, \\ p_N &= \frac{2\gamma_1}{\gamma_1 + 1} M_{CJ}^2 p_1.\end{aligned}$$

Second, the rarefaction wave following the CJ detonation wave is examined. This wave is a self-similar rarefaction wave whose front boundary coincides with the CJ surface. Therefore the state of the gas at the front boundary is given by ρ_2 , p_2 , γ_2 , u_2 , D_2 . The state of the flow inside the rarefaction wave is written by the relation for the self-similar rarefaction wave as follows,⁶⁾

$$\begin{aligned}\rho &= \left(\frac{1}{\gamma_2} + \frac{\gamma_2 - 1}{\gamma_2} \frac{x}{x_2} \right)^{\frac{2}{\gamma_2 - 1}} \rho_2, \\ p &= \left(\frac{1}{\gamma_2} + \frac{\gamma_2 - 1}{\gamma_2} \frac{x}{x_2} \right)^{\frac{2\gamma_2}{\gamma_2 - 1}} p_2, \\ u &= u_2 - \frac{2}{\gamma_2 + 1} \frac{x_2 - x}{t} = -a + \frac{x}{t} \left(\leq \frac{1}{\gamma_2} a \right), \\ a &= a_2 - \frac{\gamma_2 - 1}{\gamma_2 + 1} \frac{x_2 - x}{t}.\end{aligned}$$

At the rear boundary of the rarefaction wave, the flow velocity is zero:

$$u_3 = 0.$$

Therefore the state of the gas at the rear boundary of the rarefaction wave is given as follows,

$$\begin{aligned}x_3 &= \frac{1}{2} x_2, \\ D_3 &= a_3 = \frac{1}{2} D_{CJ}, \\ \rho_3 &= 2 \left(\frac{\gamma_2 + 1}{2\gamma_2} \right)^{\frac{\gamma_2 + 1}{\gamma_2 - 1}} \rho_1, \\ p_3 &= \frac{\gamma_1}{2\gamma_2} \left(\frac{\gamma_2 + 1}{2\gamma_2} \right)^{\frac{\gamma_2 + 1}{\gamma_2 - 1}} M_{CJ}^2 p_1.\end{aligned}$$

Although these solutions are not exactly valid after the time when the detonation wave exits the open end of the tube, an estimation

$$t_I = \frac{L}{D_3} = \frac{2L}{D_{CJ}}$$

is adopted for simplicity. And at time $t = t_I$, all gas in the tube is considered to be characterized by ρ_3 , p_3 , $\gamma_3 (= \gamma_2)$, $u_3 (= 0)$. In the combustion phase $0 \leq t \leq t_I$, the pressure acting on the thrust wall $p_w(t)$ is given by p_3 .

Third, the exhaust phase is examined. At time $t = t_I$, another rarefaction wave is created and starts to propagate through the tube from the open end to the closed one. The front boundary of the rarefaction wave propagates with sonic speed a_3 in the $-x$ direction. Therefore the time required for the propagation of the rarefaction wave from the open end to the closed one is estimated by

$$t_{II} - t_I = \frac{L}{a_3} = \frac{2L}{D_{CJ}}.$$

Therefore, t_{II} is given by

$$t_{II} = \frac{2L}{D_{CJ}} + t_I = \frac{4L}{D_{CJ}}.$$

The state of the gas inside the rarefaction wave is written by the relation for the self-similar rarefaction wave as follows,⁶⁾

$$\begin{aligned}\rho &= \left(1 + \frac{\gamma_2 - 1}{D_{CJ}} \frac{L - x}{t - t_I} \right)^{\frac{2}{\gamma_2 - 1}} \rho_{ex}, \\ p &= \left(1 + \frac{\gamma_2 - 1}{D_{CJ}} \frac{L - x}{t - t_I} \right)^{\frac{2\gamma_2}{\gamma_2 - 1}} p_{ex}, \\ u &= u_{ex} - \frac{2}{\gamma_2 + 1} \frac{L - x}{t - t_I} = a - \frac{L - x}{t - t_I} (\leq a), \\ a &= a_{ex} + \frac{\gamma_2 - 1}{\gamma_2 + 1} \frac{L - x}{t - t_I},\end{aligned}$$

where

$$\begin{aligned}\rho_{ex} &= \frac{\gamma_2 + 1}{\gamma_2^{\frac{\gamma_2 + 1}{\gamma_2 - 1}}} \rho_1, \\ p_{ex} &= \frac{\gamma_1}{\gamma_2^{\frac{2\gamma_2}{\gamma_2 - 1}} (\gamma_2 + 1)} M_{CJ}^2 p_1, \\ u_{ex} &= a_{ex} = \frac{1}{\gamma_2 + 1} D_{CJ}\end{aligned}$$

stand for the state of the gas at the open end during $t_I \leq t \leq t_{II}$. Using these solutions, the mass exhaust rate per unit area is given by

$$\rho_{ex} u_{ex} = \frac{2^{\frac{2}{\gamma_2-1}}}{(\gamma_2+1)^{\frac{\gamma_2+1}{\gamma_2-1}}} \rho_3 D_{CJ} = \left(\frac{2}{\gamma_2+1} \right)^{\frac{\gamma_2+1}{\gamma_2-1}} \rho_3 a_3.$$

At time $t = t_I$ when the exhausting rarefaction wave starts to propagate, the total mass of the gas per unit area in the tube is $\rho_3 L$. When $\rho_{ex} u_{ex}$ is considered to be the characteristic mass exhaust rate per unit area, the period of the exhaust phase is estimated as

$$t_{III} - t_I = \frac{\rho_3 L}{\rho_{ex} u_{ex}} = \frac{(\gamma_2+1)^{\frac{\gamma_2+1}{\gamma_2-1}} L}{2^{\frac{2}{\gamma_2-1}} D_{CJ}} = \left(\frac{\gamma_2+1}{2} \right)^{\frac{\gamma_2+1}{\gamma_2-1}} \frac{L}{a_3}.$$

During $t_I \leq t \leq t_{II}$ in this phase, the front boundary of the exhausting rarefaction wave does not reach the closed end; therefore the pressure acting on the thrust wall $p_w(t)$ remains p_3 . On the assumption of temporally linear decay of the thrust during $t_{II} \leq t \leq t_{III}$, $p_w(t)$ is written as

$$p_w(t; t_{II} \leq t \leq t_{III}) = p_3 \left(1 - \frac{t - t_{II}}{t_{III} - t_{II}} \right),$$

where

$$t_{III} = \left[1 + \left(\frac{\gamma_2+1}{2} \right)^{\frac{\gamma_2+1}{\gamma_2-1}} \right] \frac{2L}{D_{CJ}}.$$

Here the filling phase and some important quantities are discussed. The period of the cyclic PDE operation T_{cyc} is written as

$$\begin{aligned} T_{cyc} = t_{III} + \tau_f &= \left[1 + \left(\frac{\gamma_2+1}{2} \right)^{\frac{\gamma_2+1}{\gamma_2-1}} \right] \frac{2L}{D_{CJ}} + \tau_f \\ &> \left[1 + \left(\frac{\gamma_2+1}{2} \right)^{\frac{\gamma_2+1}{\gamma_2-1}} \right] \frac{2L}{D_{CJ}}, \end{aligned}$$

where $\tau_f > 0$ was used. And thus the frequency of the cyclic PDE operation f_{cyc} is written as

$$\begin{aligned} f_{cyc} = \frac{1}{T_{cyc}} &= \frac{1}{\left[1 + \left(\frac{\gamma_2+1}{2} \right)^{\frac{\gamma_2+1}{\gamma_2-1}} \right] \frac{2L}{D_{CJ}} + \tau_f} \\ &< \frac{1}{\left[1 + \left(\frac{\gamma_2+1}{2} \right)^{\frac{\gamma_2+1}{\gamma_2-1}} \right] \frac{2L}{D_{CJ}}}. \end{aligned}$$

Since the thrust density as a function of time in this simplified model is summarized as

$$p_w(t) = \begin{cases} p_3 & (0 \leq t \leq t_{II}) \\ p_3 \left(1 - \frac{t - t_{II}}{t_{III} - t_{II}} \right) & (t_{II} \leq t \leq t_{III}) \\ 0 & (t_{III} \leq t \leq T_{cyc}) \end{cases},$$

Fig. 2. Pressure acting on the thrust wall as a function of time. Calculation was carried out by using a typical parameter set the same as that used in “numerical example” of the text.

which is shown in Fig. 2, the impulse density acting on the thrust wall per one-cycle operation I_{cyc} is written as follows,

$$\begin{aligned} I_{cyc} &= \int_0^{T_{cyc}} p_w(t) dt \\ &= \int_0^{t_{II}} p_3 dt + \int_{t_{II}}^{t_{III}} p_3 \left(1 - \frac{t - t_{II}}{t_{III} - t_{II}} \right) dt \\ &= \frac{1}{2\gamma_2} \left(\frac{\gamma_2+1}{2\gamma_2} \right)^{\frac{\gamma_2+1}{\gamma_2-1}} \left[3 + \left(\frac{\gamma_2+1}{2} \right)^{\frac{\gamma_2+1}{\gamma_2-1}} \right] \rho_1 D_{CJ} L. \end{aligned}$$

In this solution, $a_1^2 = \gamma_1 p_1 / \rho_1$ was used. Qualitatively, the pressure acting on the thrust wall is proportional to $(D_{CJ}/a_1)^2 p_1$, and the duration for which this wall is pushed by the thermodynamic gas pressure is proportional to L/D_{CJ} ; therefore dependency of the impulse is

$$I_{cyc} \propto \left(\frac{D_{CJ}}{a_1} \right)^2 p_1 \frac{L}{D_{CJ}} \propto \frac{D_{CJ}^2}{p_1} p_1 \frac{L}{D_{CJ}} \propto \rho_1 D_{CJ} L.$$

When the equation of state for an ideal gas,

$$p_1 = \rho_2 \frac{R_u}{\mu_1} T_1,$$

is used, I_{cyc} is written as follows,

$$I_{cyc} = \frac{1}{2\gamma_2} \left(\frac{\gamma_2+1}{2\gamma_2} \right)^{\frac{\gamma_2+1}{\gamma_2-1}} \left[3 + \left(\frac{\gamma_2+1}{2} \right)^{\frac{\gamma_2+1}{\gamma_2-1}} \right] \frac{p_1 \mu_1}{R_u T_1} D_{CJ} L.$$

Furthermore, the time-averaged thrust density p_{av} is written as follows,

$$\begin{aligned}
p_{\text{av}} &= \frac{I_{\text{cyc}}}{T_{\text{cyc}}} \\
&= \frac{\frac{1}{2\gamma_2} \left(\frac{\gamma_2 + 1}{2\gamma_2} \right)^{\frac{\gamma_2+1}{\gamma_2-1}} \left[3 + \left(\frac{\gamma_2 + 1}{2} \right)^{\frac{\gamma_2+1}{\gamma_2-1}} \right] \rho_1 D_{\text{CJ}} L}{\left[1 + \left(\frac{\gamma_2 + 1}{2} \right)^{\frac{\gamma_2+1}{\gamma_2-1}} \right] \frac{2L}{D_{\text{CJ}}} + \tau_f} \\
&< \frac{1}{4\gamma_2} \left(\frac{\gamma_2 + 1}{2\gamma_2} \right)^{\frac{\gamma_2+1}{\gamma_2-1}} \frac{3 + \left(\frac{\gamma_2 + 1}{2} \right)^{\frac{\gamma_2+1}{\gamma_2-1}}}{1 + \left(\frac{\gamma_2 + 1}{2} \right)^{\frac{\gamma_2+1}{\gamma_2-1}}} \rho_1 D_{\text{CJ}}^2.
\end{aligned}$$

When the equation of state for an ideal gas is used, it is written as follows,

$$\begin{aligned}
p_{\text{av}} &= \frac{\frac{1}{2\gamma_2} \left(\frac{\gamma_2 + 1}{2\gamma_2} \right)^{\frac{\gamma_2+1}{\gamma_2-1}} \left[3 + \left(\frac{\gamma_2 + 1}{2} \right)^{\frac{\gamma_2+1}{\gamma_2-1}} \right] \frac{p_1 \mu_1}{R_u T_1} D_{\text{CJ}} L}{\left[1 + \left(\frac{\gamma_2 + 1}{2} \right)^{\frac{\gamma_2+1}{\gamma_2-1}} \right] \frac{2L}{D_{\text{CJ}}} + \tau_f} \\
&< \frac{1}{4\gamma_2} \left(\frac{\gamma_2 + 1}{2\gamma_2} \right)^{\frac{\gamma_2+1}{\gamma_2-1}} \frac{3 + \left(\frac{\gamma_2 + 1}{2} \right)^{\frac{\gamma_2+1}{\gamma_2-1}}}{1 + \left(\frac{\gamma_2 + 1}{2} \right)^{\frac{\gamma_2+1}{\gamma_2-1}}} \frac{p_1 \mu_1}{R_u T_1} D_{\text{CJ}}^2.
\end{aligned}$$

The shorter the duration of filling phase τ_f , the larger the time-averaged thrust density p_{av} . This is to be expected because the filling phase is a dead phase of a PDE operation cycle. The duration of filling phase τ_f can be estimated by ℓ_f/V_f . Generally in PDEs, tube diameter is much shorter than tube length. Therefore the detonable-mixture injection in the radial direction rather than the axial direction may be effective in shortening the duration of the filling phase.

4. Numerical Example

We show a numerical example by using a typical parameter set. First we use $\gamma_1 = \gamma_2 = 7/5$. In this case, following formulas are obtained.

$$\begin{aligned}
\rho_2 &= \frac{12}{7} \rho_1 \\
p_2 &\approx \left(\frac{M_{\text{CJ}}}{1.3} \right)^2 p_1 \\
u_2 &= \frac{5}{7} a_2 = \frac{5}{12} D_{\text{CJ}} \\
\rho_N &= 6\rho_1
\end{aligned}$$

$$\begin{aligned}
p_N &\approx \left(\frac{M_{\text{CJ}}}{0.93} \right)^2 p_1 \\
\rho_3 &\approx 0.79 \rho_1 \\
p_3 &\approx \left(\frac{M_{\text{CJ}}}{2.2} \right)^2 p_1 \\
u_3 &= 0 \\
a_3 &= D_3 = \frac{1}{2} D_{\text{CJ}} \\
\rho_{\text{ex}} &\approx 0.32 \rho_1 \\
p_{\text{ex}} &\approx \left(\frac{M_{\text{CJ}}}{4.3} \right)^2 p_1 \\
u_{\text{ex}} &= a_{\text{ex}} = \frac{5}{12} D_{\text{CJ}} \\
\rho_{\text{ex}} u_{\text{ex}} &\approx 0.17 \rho_3 D_{\text{CJ}} \approx 0.33 \rho_3 a_3 \\
t_I &= 2 \frac{L}{D_{\text{CJ}}} \\
t_{II} &= 4 \frac{L}{D_{\text{CJ}}} \\
t_{III} &\approx 8.0 \frac{L}{D_{\text{CJ}}} \\
t_{III} - t_I &\approx 6.0 \frac{L}{D_{\text{CJ}}} \approx 3.0 \frac{L}{a_3} \\
T_{\text{cyc}} &\approx 8.0 \frac{L}{D_{\text{CJ}}} + \tau_f > 8.0 \frac{L}{D_{\text{CJ}}} \\
f_{\text{cyc}} &\approx \frac{1}{8.0 \frac{L}{D_{\text{CJ}}} + \tau_f} < \frac{1}{8.0 \frac{L}{D_{\text{CJ}}}} \\
I_{\text{cyc}} &\approx 0.85 \rho_1 D_{\text{CJ}} L \\
p_{\text{av}} &\approx \frac{0.85 \rho_1 D_{\text{CJ}} L}{8.0 \frac{L}{D_{\text{CJ}}} + \tau_f} < 0.11 \rho_1 D_{\text{CJ}}^2.
\end{aligned}$$

Furthermore, we use $D_{\text{CJ}} = 2000 \text{ m/s}$, $a_1 = 400 \text{ m/s}$, $p_1 = 10^5 \text{ Pa}$, and $L = 2 \text{ m}$ as a typical parameter set, which corresponds to $\rho_1 = \gamma_1 p_1 / a_1^2 = 0.875 \text{ kg/m}^3$. In this case, the following values are obtained.

$$\begin{aligned}
\rho_2 &= 1.5 \text{ kg/m}^3 \\
p_2 &\approx 15 p_1 \approx 15 \times 10^5 \text{ Pa} \\
u_2 &\approx 830 \text{ m/s} \\
a_2 &\approx 1200 \text{ m/s} \\
\rho_N &= 5.25 \text{ kg/m}^3 \\
p_N &\approx 29 p_1 \approx 29 \times 10^5 \text{ Pa} \\
\rho_3 &\approx 0.69 \text{ kg/m}^3 \\
p_3 &\approx 5.0 p_1 \approx 5.0 \times 10^5 \text{ Pa}
\end{aligned}$$

$$u_3 = 0$$

$$a_3 = D_3 = 1000 \text{ m/s}$$

$$\rho_{\text{ex}} \approx 0.28 \text{ kg/m}^3$$

$$p_{\text{ex}} \approx 1.4 p_1 \approx 1.4 \times 10^5 \text{ Pa}$$

$$u_{\text{ex}} = a_{\text{ex}} \approx 830 \text{ m/s}$$

$$\rho_{\text{ex}} u_{\text{ex}} \approx 230 \text{ kg/(m}^2 \cdot \text{s)}$$

$$t_I = 2.0 \text{ ms}$$

$$t_{II} = 4.0 \text{ ms}$$

$$t_{III} \approx 8.0 \text{ ms}$$

$$T_{\text{cyc}} \approx 8.0 [\text{ms}] + \tau_f > 8.0 \text{ ms}$$

$$f_{\text{cyc}} \approx \frac{1}{8.0 [\text{ms}] + \tau_f} < 130 \text{ Hz}$$

$$I_{\text{cyc}} \approx 3000 (\text{N} \cdot \text{s})/\text{m}^2$$

$$p_{\text{av}} \approx \frac{30}{8.0 + \tau_f [\text{ms}]} \times 10^5 \text{ Pa} < 3.7 \times 10^5 \text{ Pa}.$$

If p_{ex} is less than pressure outside the tube, the treatment of the exhaust phase in this paper is invalid. Initial pressure p_1 is generally not less than pressure outside the tube, and p_{ex} was estimated to be $p_{\text{ex}} \approx 1.4 p_1$ in this example; therefore the above estimation of the exhaust phase is valid.

5. Conclusions

We analytically obtained simple formulas for the performance of pulse detonation engines (PDEs). A PDE

was modeled as a straight tube. One end was closed and the other was open; a detonation wave was ignited at the closed end. We calculated pressure acting on the closed end, with some simplifications, using the Hugoniot relation for the Chapman-Jouguet (CJ) detonation wave and flow relations for self-similar rarefaction waves. The most important formula obtained in this paper is $p_{\text{av}} \approx (0.85 \rho_1 D_{\text{CJ}} L) / [(8.0 L / D_{\text{CJ}}) + \tau_f] < 0.11 \rho_1 D_{\text{CJ}}^2$ for the specific-heat ratio of 7/5, where p_{av} is the time-averaged thrust density; ρ_1 and D_{CJ} are, respectively, the mass density and CJ detonation speed of the detonable mixture; L is the length of the tube; and τ_f is the time required to fill the tube with a fresh detonable mixture.

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