Multi-Instability Analysis of Swept-Wing Boundary Layers Part 2. Effects of Wall Curvature and Nonparallelism on the e^N Method

By Takashi ATOBE, Nobutake ITOH and Kiyoshi YAMAMOTO

National Aerospace Laboratory, Chofu, Tokyo 182–8522, Japan

(Received December 20th, 2001)

Multi-instability characteristics of the three-dimensional boundary layer around a yawed circular cylinder are investigated by an e^N method involving the effects of wall curvature and nonparallelism. Velocity profiles of the boundary layer are approximated by members of Falkner-Skan-Cooke similarity solutions and the local dispersion relation of each member is determined from the nonparallel eigenvalue problem proposed in part 1 of this study. A complex ray theory is adopted in the integration procedure for N factor, and a numerical estimation of N is made to describe wedge-shaped disturbances originating from a point source. The analysis using these methods shows that the influence of wall curvature and nonparallelism stabilizes and destabilizes the flow, respectively, though their quantitative effects on the N factor depend on kinds of instability, range of frequency, and values of flow parameters. It is also found that the destabilizing effect of nonparallelism increases with the increase of sweep angle.

Key Words: Three-Dimensional Boundary Layer, Laminar-Turbulent Transition, Linear Stability Theory, e^N Method

1. Introduction

One prediction method for laminar-turbulent transition is the e^N method established by Smith and Gamberoni¹⁾ and by Van Ingen²⁾ in 1957, and the most famous numerical code based on this method was SALLY code released by NASA.^{3–6)} Although rapid progress has been made in computer performance since then, it is still difficult to predict the transition location directly by the Navier-Stokes codes or direct numerical simulations. Even now, therefore, one must rely on the e^N method together with some attempts to improve the accuracy.^{7,8)}

The prediction code, in general, consists of three independent parts, corresponding to boundary-layer computation, local stability estimation, and N factor integration; thus each part of the code must be improved individually to get more accurate predictions. The boundary-layer calculation is usually constructed by similarity equations or a Navier-Stokes code. There are many schemes to solve the boundary layer, and this part mainly depends on the resolution of the numerical meshes. Therefore it seems that the accuracy of this part has attained to a relatively high level with the development of the supercomputer. However, the similarity equations are still useful for stability analysis, since the profiles of physical quantities in the boundary layer are sufficiently smooth. The stability equations used in the e^N method have usually been the Orr-Sommerfeld equation derived under the parallel flow assumption; the results are thus relatively inferior for a region with large wall curvature, especially for the leading edge of a wing. To overcome this problem, part 1 of this study proposes new model equations, which include wall curvature, streamline curvature, and non-

parallelism. The new equations are also used for stability analysis in the present paper.

The most significant problem of the e^N method exists in the final part for N factor integration, and many strategies have therefore been proposed by various researchers.^{8–14)} The typical one is referred to as the “envelope method”, which defines the N factor as an envelope curve of all N curves satisfying the dispersion relation concerned. To save labor of the N factor integration, Mack¹²⁾ proposed a simple method on the assumption that the growth direction of disturbance wave is taken to be the direction of the group velocity or the potential flow. These methods are convenient to get an N factor, but they are of low accuracy and of no physical meaning in the calculation. For this crucial problem, Itoh¹⁵⁾ recently proposed a disturbance propagation theory, referred to as the complex ray theory. This theory can uniquely determine a wave number or frequency of the most unstable disturbance by using an extended kinematic wave theory¹⁶⁾ with a realization condition based on physical causality.

In the present study, a precise estimation of the effects of wall curvature and nonparallelism on N factor of the e^N method is made by adopting Falkner-Skan-Cooke similar solutions¹⁷⁾ for boundary-layer calculation, the new model equations proposed in part 1 for stability analysis, and the complex ray theory for N factor integration. The flow field considered here is the three-dimensional boundary layer around a yawed circular cylinder with various swept angles. The flow conditions and stability equations used in the present study are shown in section 2. The complex ray theory for the integration of a local amplification rate is briefly explained in section 3. The effects of wall curvature, nonparallelism, and sweep angle on N factor are numerically shown in section 4. The validity of the results is discussed in section 5.

2. Basic Flow and Local Stability Equations

The flow field considered in the present study is shown in Fig. 1. The infinite cylinder with radius r_w is placed obliquely in the uniform flow. The Reynolds number based on the uniform-flow velocity Q_∞ and the chord length $L = 2r_w / \cos \Lambda$ in the streamwise direction is taken to be 0.5 million when the sweep angle Λ is set to 50 degrees:

$$R_Q \equiv \frac{Q_\infty 2r_w / \cos \Lambda}{\nu} = 0.5 \times 10^6, \quad \text{at } \Lambda = 50^\circ, \quad (1)$$

where ν is the kinematic viscosity. We then introduce a three-dimensional coordinate system with x in the chordwise direction, y in the spanwise direction, and z in the normal-to-the-wall direction, respectively. The corresponding velocity components U , V , and $\hat{W}$ of the basic flow for stability analysis have been normalized with the reference velocity U_E for U and V_∞ for V and $\hat{W}$, where the suffices E and ∞ denote the quantities at the edge of the boundary layer and at the uniform flow field, respectively. The velocity profiles U , V , and $\hat{W}$ at each surface location are calculated by an approximate method of boundary-layer calculation,¹⁸⁾ where each velocity distribution is assumed to be expressed with a member of the Falkner-Skan-Cooke similarity family, and a variation of the Falkner-Skan pressure parameter m along the wing surface is determined by integrating the momentum integral equation from the attachment line in the x direction. The chordwise variations of the pressure parameter m , the ratio of the spanwise velocity to the chordwise velocity at the edge of the boundary layer $\gamma \equiv V_E/U_E$, the local Reynolds number defined by $R \equiv Q_E \delta / \nu$, the nondimensional wall curvature κ_w , and the nondimensional streamline curvature $\hat{\kappa}_s$ are given in Fig. 2; all are used in the present calculations. Here the boundary thickness δ is defined by

$$\delta \equiv \sqrt{\nu m / \frac{dU_E}{dx}}, \quad (2)$$

and the nondimensional quantities κ_w and $\hat{\kappa}_s$ defined from the wall curvature $1/r_w$ and the streamline curvature $1/r_s$ are given by

$$\kappa_w = \frac{\delta}{r_w}, \quad \hat{\kappa}_s = \frac{\delta}{r_s} (1 + \gamma^2)^{3/2} / \gamma. \quad (3)$$

It is a great advantage that the similarity solutions in general are smooth enough for stability analysis, though they may be unsuitable for complicated flow fields such as the flow around a tapered wing.

The stability of flow is estimated by growth or decay of the disturbance velocities written in the form

Fig. 1. The flow field and spatial coordinates.

Fig. 2. Results of the boundary-layer calculation.

$$(u_1, v_1, w_1) = (u(\zeta), v(\zeta), w(\zeta)) \exp(i\Theta(X, Y, T)/\varepsilon_0), \quad (4)$$

where $X = x/l_s$, $Y = y/l_s$, and $\zeta = z/\delta$ denote nondimensional coordinates and $T \equiv t/(l_s/Q_\infty)$ is a nondimensional time. Here l_s denotes a surface length πr_w , ε_0 is a small parameter defined by $\varepsilon_0 \equiv \sqrt{\nu l_s / U_\infty} / l_s$, and i denotes the imaginary unit. Then the complex wave numbers α and β in X and Y directions and the complex frequency ω are defined by

$$\frac{\partial \Theta}{\partial X} = \frac{\alpha}{\Delta}, \quad \frac{\partial \Theta}{\partial Y} = \frac{\beta}{\Delta}, \quad \frac{\partial \Theta}{\partial T} = -\frac{\omega}{\Delta^*}, \quad (5)$$

where

$$\Delta(X) \equiv \frac{\delta}{\sqrt{\nu l_s / U_\infty}}, \quad (6)$$

$$\Delta^*(X) \equiv \Delta / [\sqrt{1 + \gamma^2} \hat{U}_E \cos \Lambda]. \quad (7)$$

Δ and Δ^* are functions of X , and $\hat{U}_E$ denotes the potential flow around a circular cylinder given by $\hat{U}_E = 2 \sin(\pi X)$.

Model equations for stability analysis proposed in Part 1 of this study are given by

$$\begin{aligned} & \left[D^2 - \alpha^2 - \beta^2 + iR(\omega - \alpha U - \beta V) - \hat{W}D - \frac{2\hat{\kappa}_s \beta^2 U}{\alpha^2 + \beta^2 - i\alpha \hat{\kappa}_s / R} \right] u \\ & - \left[\frac{i\alpha}{\alpha^2 + \beta^2 - i\alpha \hat{\kappa}_s / R} \left\{ D^2 - \alpha^2 - \beta^2 + iR(\omega - \alpha U - \beta V) - \hat{W}D \right\} D \right. \\ & \quad \left. + \frac{R\beta(\beta U' - \alpha V')}{\alpha^2 + \beta^2 - i\alpha \hat{\kappa}_s / R} \right] w = 0, \end{aligned} \quad (8)$$

$$\begin{aligned}
& \left[\left\{ D^2 - \alpha^2 - \beta^2 + iR(\omega - \alpha U - \beta V) - \hat{W}D - \hat{W}' \right\} \left(D^2 - \alpha^2 - \beta^2 + i\alpha \frac{\hat{\kappa}_s}{R} \right) \right. \\
& \quad \left. + iR(\alpha U'' + \beta V'') + \hat{\kappa}_s(U'D + U'') \right] w \\
& + \left[2\hat{\kappa}_s \left(i\alpha + \frac{\hat{\kappa}_s}{R} \right) (UD + U') - 2\kappa_w \left(\alpha^2 + \beta^2 - i\alpha \frac{\hat{\kappa}_s}{R} \right) U \right] u = 0, \quad (9)
\end{aligned}$$

where D and prime denote differentiation with respect to ζ . Also, the boundary conditions at $\zeta = 0$ are given by

$$u = w = w' = 0, \quad (10)$$

and those at $\zeta = \zeta_E$ are

$$\begin{aligned}
u' - \rho_1 u + K_1(w'' - \rho_1 w') &= 0, \\
w'' - (\rho_2 + \rho_3)w' + \rho_2 \rho_3 w + K_2(u + K_1 w') &= 0, \\
w''' - (\rho_1 + \rho_2 + \rho_3)w'' + (\rho_1 \rho_2 + \rho_2 \rho_3 + \rho_3 \rho_1)w' \\
&\quad - \rho_1 \rho_2 \rho_3 w = 0, \quad (11)
\end{aligned}$$

where ρ_1 , ρ_2 , and ρ_3 are the solutions with negative real parts of the algebraic equations

$$L(\rho_1) - 2\hat{\kappa}_s \beta^2 U_E / S = 0, \quad \rho_2^2 - S = 0, \quad L(\rho_3) - \hat{W}_E' = 0,$$

with the operator $L(\rho) \equiv \rho^2 - \hat{W}_E \rho - \alpha^2 - \beta^2 + iR(\omega - \alpha U_E - \beta V_E)$, the coefficients K_1 , K_2 , and S are given by

$$\begin{aligned}
K_1 &= -\frac{i\alpha}{S}, \quad K_2 = \frac{2U_E \{ \hat{\kappa}_s (i\alpha + \hat{\kappa}_s / R) \rho_1 - \kappa_w S \}}{(\rho_1 + \rho_3 - \hat{W}_E)(\rho_1 + \rho_2)}, \\
S &= \alpha^2 + \beta^2 - i\alpha \frac{\hat{\kappa}_s}{R}.
\end{aligned}$$

These equations include, as a part, the stability equations for the attachment-line flow with no wall curvature and the equations for the Görtler instability of two-dimensional stagnation-point flow along a concavely curved surface. Furthermore, if terms proportional to $1/R$ are ignored except for the viscous terms, the above equations exactly reduce to those for parallel-flow problems.

The homogeneous equations (8) and (9) with homogeneous boundary conditions (10) and (11) pose an eigenvalue problem to determine the complex frequency ω as a function of the complex wave numbers α and β together with the flow parameters of the local Reynolds number R and the sweep angle Λ :

$$\omega = \omega(\alpha, \beta; R, \Lambda). \quad (12)$$

This relation is regarded as a complex dispersion relation connecting the frequency ω with the two wave numbers α and β when the flow conditions are given.

3. Description of Wedge-Shaped Disturbances

In this section, the propagation and growth of wavy disturbances originating from a point source under a continuous excitation in the boundary layer are described by the use of

the complex ray theory. The complex wave numbers α and β and the complex frequency ω are related to one another by the compatibility conditions

$$\begin{aligned}
\frac{\partial}{\partial Y} \left(\frac{\alpha}{\Delta} \right) &= \frac{\partial}{\partial X} \left(\frac{\beta}{\Delta} \right), \\
\frac{\partial}{\partial T} \left(\frac{\alpha}{\Delta} \right) &= -\frac{\partial}{\partial X} \left(\frac{\omega}{\Delta^*} \right), \\
\frac{\partial}{\partial T} \left(\frac{\beta}{\Delta} \right) &= -\frac{\partial}{\partial Y} \left(\frac{\omega}{\Delta^*} \right), \quad (13)
\end{aligned}$$

which are obtained by eliminating the phase function Θ from (5). For wedge-shaped disturbances considered here, the imaginary part of ω and the differentiation with respect to T in (13) may be equated to 0, because the pattern of disturbances does not vary over time. These conditions mean that ω/Δ^* must be constant. Using the dispersion relation (12), we can then simplify the compatibility conditions (13) to the form

$$\frac{\partial}{\partial X} \left(\frac{\beta}{\Delta} \right) + \frac{\omega_\beta}{\omega_\alpha} \frac{\partial}{\partial Y} \left(\frac{\beta}{\Delta} \right) = 0. \quad (14)$$

This equation implies that β/Δ is constant on the ray defined by

$$\frac{dY}{dX} = \frac{\omega_\beta}{\omega_\alpha}. \quad (15)$$

For convenience, the dispersion relation and the group velocity $\omega_\beta/\omega_\alpha$ are rewritten in the form

$$\hat{\alpha} \equiv \frac{\alpha}{\sqrt{E_1} \Delta} = \hat{\alpha}(X; \hat{\beta}, \hat{\omega}), \quad C \equiv \frac{\omega_\beta}{\omega_\alpha} = C(X; \hat{\beta}, \hat{\omega}), \quad (16)$$

where $E_1 \equiv \pi/2$, and $\hat{\beta}$ and $\hat{\omega}$ are constant values determined by $\hat{\beta} \equiv \beta/(\sqrt{E_1} \Delta)$ and $\hat{\omega} \equiv \omega/(\sqrt{E_1} \Delta^*)$, respectively. Since the group velocity C takes complex values even if X is chosen as real, the characteristic ray must be extended into the complex space of Y . With C and Y written as $C = C_r + iC_i$ and $Y = Y_r + iY_i$, we may therefore consider that the physical space is realized on the plane $Y_i = 0$. If the source point is chosen as $(X, Y) = (X_0, 0)$ and the observation of disturbances is made at a downstream station $X = X_1$, the complex wave number $\hat{\beta}$ and the real frequency $\hat{\omega}$ must satisfy the relation

$$Y_i = \int_{X_0}^{X_1} C_i(X; \hat{\beta}, \hat{\omega}) dX = 0, \quad (17)$$

and the spanwise location of the observed disturbance component at $X = X_1$ is obtained in the form

$$Y = Y_r = \int_{X_0}^{X_1} C_r(X; \hat{\beta}, \hat{\omega}) dX. \quad (18)$$

The total amplification rate N between X_0 and X_1 is then given by

$$N = -E_1 \sqrt{R_Q} \cos \Lambda \int_{X_0}^{X_1} [\hat{\alpha}_i(X; \hat{\beta}, \hat{\omega}) + \hat{\beta}_i C_r(X; \hat{\beta}, \hat{\omega})] dX. \quad (19)$$

This value corresponds to the usual N factor defined by $N = \log(A_1/A_0)$ when A_0 and A_1 denote the amplitudes of disturbances at $X = X_0$ and X_1 , respectively.

4. Numerical Results and Discussion

The global parameters governing the flow around a yawed circular cylinder are the uniform-flow Reynolds number $R_Q \equiv (Q_\infty d / \cos \Lambda) / \nu$ and the sweep angle Λ . A stability diagram for the cross-flow instability, streamline-curvature instability, and Tollmien-Schlichting instability is given in Fig. 3, which shows that unstable regions for the cross-flow and streamline-curvature instabilities extend from the vicinity of the leading edge to the midchord of the cylinder, but that the unstable region for T-S mode exists on the downstream half of the cylinder. Thin dotted lines in the figure indicate variations of the local Reynolds number for fixed values of the uniform-flow Reynolds number R_Q and for the constant sweep angle $\Lambda = 50^\circ$. Thus the flow is unstable in the region between the two points where the critical curve crosses the local Reynolds number curve for a given R_Q .

The present study considers the continuously excited wedge-shaped disturbances introduced from a point source, for which the appropriate parameters of the ray theory are the complex wave number $\hat{\beta}$ and the real frequency $\hat{\omega}$, as shown in (16). For each set of these parameters satisfying the realization condition (17), the Y location and the N factor are

Fig. 3. The critical Reynolds numbers R_c with local Reynolds numbers R . Solid line demonstrates R_c originated from the cross-flow instability. Also, the dotted line is of the stream-line curvature, and the chain-dotted line is of the Tollmien-Schlichting instability. The dashed lines are based on each uniform-flow Reynolds number for different mean flow velocity.

obtained by integrations (18) and (19). Since the maximum amplification rate of these disturbances always appears at a zero of $\hat{\beta}_i$, as reported by Itoh,¹⁵⁾ the present study focuses on the case of $\hat{\beta}_i = 0$. This system can then be regarded as a function of only one free parameter, and the real frequency $\hat{\omega}$ is hereinafter taken to be this free parameter. Consequently, global parameters in the present study are the uniform-flow Reynolds number R_Q and the sweep angle Λ , and a local parameter is the real frequency $\hat{\omega}$.

At first we fix the sweep angle at $\Lambda = 50^\circ$ to examine the characteristics of cross-flow disturbances in the flow with the uniform-flow Reynolds number 0.5 million. The introduction point X_0 and the observation location X_1 of disturbances are chosen as $X_0 = 0.1$ and $X_1 = 0.3$, respectively, to put the disturbances in the unstable region during the integration. The effects of wall curvature and nonparallelism on the characteristics of disturbances are investigated by using the approximate equation systems of three different levels: the Orr-Sommerfeld equation based on the parallel-flow approximation, the O-S equation plus the wall curvature terms, and the O-S equation plus all the wall curvature and nonparallel terms in (8) and (9).

4.1. Characteristics of disturbances at a fixed sweep angle

Figure 4 shows the variations of the local amplification rate $-\hat{\alpha}_i$ with X for the case of a real wave number $\hat{\beta} = 0.745$ and a real frequency $\hat{\omega} = 0.08$. The amplification-rate curve obtained from the 'OS' system is much higher than the other two curves, especially for the large X region. If the wall curvature terms are added to the 'OS' system, the amplification-rate curve moves downward, but a further addition of the nonparallel terms raises the curve upward again. These results indicate that the wall curvature stabilizes the flow and the nonparallelism destabilizes it. On the other hand, variations of the local amplification rate $-\hat{\alpha}_i$ with the real frequency $\hat{\omega}$ are given in Fig. 5 in the case of a real wave number $\hat{\beta} = 0.745$ at a fixed location $X = 0.2$. This figure again shows the stabilizing effect of the wall curvature and

Fig. 4. The variations of local amplification rate $\hat{\alpha}_i$ with location X for the case of $\hat{\beta} = 0.745 + 0.0i$ and a frequency $\hat{\omega} = 0.08 + 0.0i$.

the destabilizing one of the nonparallelism. It is also found from this figure that the stabilizing effect of the wall curvature is strong for small $\hat{\omega}$, and the destabilizing effect of the nonparallelism is strong for large $\hat{\omega}$. As a result, the curves of the 'OS' system and the 'OS+wall+nonparallelism' system cross each other at a large $\hat{\omega}$.

In regard to the integrated results using the complex ray theory, which can determine not only the total growth rate of disturbances, but also the wave numbers as a function of the real frequency mentioned before. The variations of wave numbers $\hat{\alpha}_r$ and $\hat{\beta}_r$ and the total growth rate N with the real frequency $\hat{\omega}$ obtained from the three equation systems are shown in Fig. 6. It is found that the wave numbers increase with the increase of $\hat{\omega}$, but the total growth rate N varies in a convex manner with a peak at the real frequency $\hat{\omega} \simeq 0.08$. Furthermore, the N curve from 'OS' system is highest at almost all the region, and the curve from the 'OS+wall' system is smallest. Thus it can be concluded again that the wall curvature stabilizes the flow and the nonparallelism destabilizes it. It is also found that the stabilizing (destabiliz-

Fig. 5. The variations of local amplification rate $\hat{\alpha}_i$ with frequency $\hat{\omega}$ for the case of $\hat{\beta}_i = 0.745 + 0.0i$ at $X = 0.2$.

Fig. 6. The wave numbers $\hat{\alpha}_r$ and $\hat{\beta}_r$ and the total growth rate N of the disturbances with $\hat{\beta}_i = 0$ plotted against the real frequency $\hat{\omega}$.

Table 1. Values of wave number $\hat{\alpha}_r$ and $\hat{\beta}_r$ and total amplification rate N for each case at frequency $\hat{\omega} = 0.08 + 0.0i$.

	$\hat{\alpha}_r$	$\hat{\beta}_r$	N
OS	-0.363	0.775	3.448
OS+wall	-0.350	0.753	1.923
OS+wall+nonpara	-0.335	0.745	3.009

Table 2. Correspondence Λ with R_Q at fixed mean flow velocity Q_∞ .

Λ (degree)	35	40	50	55	60	70
$R_Q \times 10^{-6}$	0.39	0.42	0.50	0.56	0.64	0.94

ing) effect of the wall curvature (nonparallelism) is relatively strong for small (large) $\hat{\omega}$ in the same manner as Fig. 5.

It should be emphasized that there is little difference of the variations in $\hat{\alpha}_r$ and $\hat{\beta}_r$ among the three systems; that is, the wave-number vectors of the disturbances are not much affected by the wall curvature or the nonparallelism, even though the N curve is considerably affected. The values of wave number and N factor for each system in $\hat{\omega}_r = 0.08$ are listed in Table 1. It is also interesting that each N curve has a peak at a nonzero value of $\hat{\omega}$, in contradiction to many experimental results^{19,20)} where almost all C-F type disturbances are observed as stationary vortices. This implies that the growth of disturbances is an end result caused by many factors, such as the surface roughness, the receptivity, and so on.

4.2. Effects of sweep angle

To examine the dependence of N on sweep angle, we consider a situation that an infinitely long cylinder of a constant diameter d is placed in the field of a uniform-flow velocity Q_∞ with various values of the sweep angle Λ . Then the uniform-flow Reynolds number R_Q defined in (1) varies with Λ as shown in Table 2. Disturbances considered here are again the wedge-shaped ones originating from a point source under a continuous excitation. With the assumption of $\hat{\beta}_i = 0$, as in the previous section, the total amplification rate N is determined as a function of the real frequency $\hat{\omega}$ and the sweep angle Λ . For the present purpose of examining the effects of Λ , we prefer to fix the frequency $\hat{\omega}$ at a specific value that yields the maximum N value in the case of the 'OS' system, because this $\hat{\omega}$ gives nearly the maximum values of N for the other two systems, as shown in Fig. 6.

The results of the examination are shown in Fig. 7, where the N factor in each equation system increases with the increase of Λ , because the uniform-flow Reynolds number also increases together with the sweep angle in this study. Furthermore, it is found that the wall curvature stabilizes the flow and the nonparallelism destabilizes the flow, as before; that is, the 'OS' system gives the most unstable eigenvalues, and the 'OS+wall' system yields the most stable ones, though the destabilizing effect of the nonparallelism is relatively strong at large sweep angles.

Fig. 7. The variations of total amplification rate N with sweep angle Λ at a real frequency $\hat{\omega}$ corresponding to a maximum N factor.

5. Conclusions

A modified e^N method using the nonparallel stability equations proposed in part 1 was applied to the three-dimensional boundary layer around a yawed circular cylinder to investigate the effects of wall curvature and nonparallelism on the N factor. In this method, the velocity profiles of the basic flow are obtained from the approximate boundary-layer calculation based on the momentum integral equation in the form locally approximated by members of the Falkner-Skan-Cooke similarity family. The integration for the total amplification rate N is carried out subject to the complex ray theory to describe wedge-shaped disturbances originating from a point source under a continuous excitation in the boundary layer. Computations have been made for three equation systems with different levels of approximation, that is, the Orr-Sommerfeld equation based on the parallel-flow approximation, the O-S equation plus the wall curvature terms, and the O-S equation plus all the wall curvature and nonparallel terms.

Numerical results obtained for a fixed sweep angle $\Lambda = 50^\circ$ show that the convex wall curvature has a stabilizing effect on unstable cross-flow disturbances for almost all the frequencies and that the nonparallelism gives a destabilizing effect, though the wave-number vector of disturbances is not affected by these two effects. Furthermore, the stabilizing effect of the wall curvature is relatively weak and the destabilizing effect of the nonparallelism is strong at large frequencies. An investigation of the dependence of these effects on the sweep angle has shown that the stabilizing and destabilizing effects of wall curvature and nonparallelism are weakened and strengthened at large sweep angles, respectively. Consequently, the system, including the wall curvature, only

always gives more stable results than the other two systems do.

The present study has clarified the effects of the convex wall curvature and nonparallelism on the growth of cross-flow disturbances by the use of the model equation system proposed in part 1. It may be expected that this system will be applied to practical problems like wing design and will contribute to a more precise prediction of a laminar-turbulent transition of three-dimensional boundary layers.

References

- 1) Smith, A. M. O. and Gamberoni, N.: Transition, Pressure Gradient and Stability Theory, Douglas Aircraft Co. Rept. ES 26388, El Segundo, California, 1956.
- 2) Van Ingen, J. L.: A Suggested Semi-empirical Method for the Calculation of the Boundary Layer Transition Region, Univ. of Techn., Dept. of Aero. Eng. Rept. UTH-74, Delft, 1956.
- 3) Strowski, A. and Orszag, S.: Mass Flow Requirements for LFC Wing Design, AIAA Paper 77-1222, 1977.
- 4) Kaups, K. and Cebeci, T.: Compressible Laminar Boundary Layers with Suction on Swept and Tapered Wings, *J. Aircraft*, **14** (1977), pp. 661–667.
- 5) Cebeci, T. and Bradshaw, P.: *Momentum Transfer in Boundary Layers*, Hemisphere, New York, 1977, pp. 213–234.
- 6) Cebeci, T. and Stewartson, K.: On Stability and Transition in Three-Dimensional Flows, *AIAA J.*, **18** (1980), pp. 398–405.
- 7) Arnal, D.: Boundary Layer Transition: Predictions Based on Linear Theory, AGARD Rep., 793, 1994, pp. 2-1–2-63.
- 8) Reed, H. L. and Saric, W. S.: Linear Stability Theory Applied to Boundary Layers, *Annu. Rev. Fluid Mech.*, **28** (1996), pp. 389–428.
- 9) Gaster, M.: On the generation of Spatially Growing Waves in a Boundary Layer, *J. Fluid Mech.*, **22** (1965), pp. 433–441.
- 10) Gaster, M.: The Development of Three-dimensional Wave Packet in a Boundary Layer, *J. Fluid Mech.*, **32** (1968), pp. 173–184.
- 11) Gaster, M.: The Development of a Two-dimensional Wavepacket in a Growing Boundary Layer, *Proc. R. Soc. Lond. A*, **384** (1982), pp. 317–332.
- 12) Mack, L. M.: Transition Prediction and Linear Stability Theory, AGARD Conf. Proc. No. 224, 1977, pp. 1-1–1-22.
- 13) Mack, L. M.: Boundary-layer Linear Stability Theory, AGARD Rep., 709, 1984, pp. 3-1–3-81.
- 14) Herbert, T.: Parabolized Stability Equations, AGARD Rep., 793, 1994, pp. 4-1–4-34.
- 15) Itoh, N.: Development of Wedge-shaped Disturbances Originating from a Point Source in a Three-dimensional Boundary Layer, *Fluid Dyn. Res.*, **18** (1996), pp. 337–354.
- 16) Schlichting, H.: *Boundary Layer Theory*, McGraw-Hill, New York, 1968, Chap. 10.
- 17) Rosenhead, J. P.: *Laminar Boundary Layer*, Oxford University Press, Oxford, 1963, Chap. 5.
- 18) Itoh, N.: Effects of Wall and Streamline Curvatures on Instability of 3-D Boundary Layers, *Laminar-Turbulent Transition*, ed. by R. Kobayashi, Springer, Berlin, 1995, pp. 323–330.
- 19) Saric, W. S.: Physical Description of Boundary-layer Transition: Experimental Evidence, AGARD Rep., No. 793, 1994, pp. 1-1–1-51.
- 20) Bippes, H.: Basic Experiments on Transition in Three-dimensional Boundary Layers Dominated by Crossflow Instability, *Prog. Aerospace Sci.*, **35** (1999), pp. 363–412.