Robust Adaptive Variable Structure Control of a Flexible Spacecraft Containing Input Nonlinearity/Dead-Zone

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1. Introduction

Fine attitude control and stabilization are key technologies required for modern flexible spacecraft, whose missions, such as stereoscopic mapping, require high pointing accuracy and stabilization. However, in the realistic environment, knowledge about system parameters such as inertia matrix and modal frequencies are usually not known, and various disturbances also occur. Therefore, disturbance rejection control strategies that are also robust to parametric uncertainty are of great interest in spacecraft applications.

Variable structure control (VSC) is an efficient control technique that is applicable to a wide class of nonlinear systems subject to modeling uncertainty and external disturbances.^{1,2)} In those works,^{1,2)} VSC is limited to systems with full-state feedback. In practical application, however, full-state measurement might be neither possible nor feasible. The problem of designing output feedback controllers via VSC to stabilize multi-variable plants has been investigated by Wang³⁾ and Yallapragada.⁴⁾ In addition, the inputs of a system are usually restricted by its physical structure and energy consumption, which also gives the system inputs such nonlinear characteristics as saturation, dead-zone, etc. The existence of input nonlinearity is a source of degradation or even worse, instability in system performance. Hsu et al.⁵⁾ have proposed VSC schemes for uncertain dynamics systems with dead-zone nonlinearity for solving regulation problems. These control schemes can possess insensitivity to matching uncertainties and disturbances. However, information on the upper boundaries of uncertainty is required, which is generally not available or too expensive to assess. Adaptive approaches may offer a simple and effective tool to solve this problem.^{1,6)}

The contribution of this paper is the design of a nonlinear controller to achieve attitude maneuvering of a three-axis stabilized flexible spacecraft under parametric uncertainty, external disturbances and control input nonlinearity/dead-zone. The proposed controller ensures the global reaching condition of the sliding mode of an uncertain system in the presence of input nonlinearity/dead-zone without the limitation of knowing the boundaries of uncertainty and the perturbations in advance. Numerical simulations show that precise attitude control and vibration suppression can

be accomplished using the derived controller for both asymptotically and exponentially stable design cases.

2. System Statement

A general description of nonlinear systems containing nonlinearity/dead-zone in the input function is given in the form of

$$\begin{aligned}\dot{x}(t) &= (A + \Delta A)x(t) + B\Phi(u) + f(t) \\ y &= Cx,\end{aligned}\quad (1)$$

where $x(t) \in R^n$ is a vector of state variables, $u \in R^m$ is a vector of control inputs of the system, $A \in R^{n \times n}$ is the state matrix of the normal part of the uncertain dynamic system, $B \in R^{n \times m}$ is the input matrix of the uncertain dynamic system, $f(t) \in R^m$ stands for external disturbances of the system, ΔA is the bounded uncertainty matrix of A , $\Phi(u) \in R^m$ is a vector of nonlinear input functions, and $\Phi(0) = 0$.

Throughout the remainder of this paper, the following assumptions are made:

Assumption 1: For the nominal part of the uncertain dynamic system with nonlinearities indicated in Eq. (1), the triplet (A, B, C) of the nominal system is controllable and observable.

Assumption 2: The uncertainty matrix and the disturbance $(\Delta A, f)$ meet the following matching condition

$$\Delta A = BH, \quad \|H\| \leq \beta_1 \quad (2a)$$

$$f(t) = Bd, \quad \|d\| < \beta_2, \quad (2b)$$

where H and d are appropriate dimensions, and β_1 and β_2 the unknown bounds of uncertainty and disturbance, respectively.

Assumption 3: The nonlinear input function $\Phi_i(u_i)$ applied to the system is mathematically defined as follows:

$$\Phi_i(u_i) = \begin{cases} \bar{\Phi}_{i+}(u_i - u_{i+}) & u_i > u_{i+} \\ 0 & -u_{i-} \leq u_i \leq u_{i+}, \\ \bar{\Phi}_{i-}(u_i + u_{i-}) & u_i < -u_{i-} \end{cases} \quad (3)$$

where $\bar{\Phi}_{i+} > 0$ and $\bar{\Phi}_{i-} > 0$ are nonlinear functions of u_i , and $u_{i+} > 0$ and $u_{i-} > 0$ are constants. In addition, for the i th input u_i , the nonlinear input function $\Phi_i(u_i)$ is assumed to satisfy the following inequalities:

$$(u_i - u_{i+})\Phi_i(u_i) \geq \alpha_{i+}(u_i - u_{i+})^2 \quad \text{for } u_i > u_{i+} \quad (4a)$$

and

$$(u_i + u_{i-})\Phi_i(u_i) \geq \alpha_{i-}(u_i + u_{i-})^2 \quad \text{for } u_i < -u_{i-}, \quad (4b)$$

where $\alpha_{i+} \leq \bar{\Phi}_{i+}$ and $\alpha_{i-} \leq \bar{\Phi}_{i-}$ for all u_i , and all are constants.

3. Adaptive Variable Structure Output Feedback Control Design

A linear switching surface is defined as

$$S(t) = Gy = GCx(t), \quad (5)$$

where $G = [g_1, g_2, \dots, g_m]^T \in R^{m \times n}$ is a specified constant matrix such that $\det(GCB) \neq 0$.

As a consequence, asymptotically and exponentially stable design methods are analyzed for the development of the adaptive variable structure output feedback control law without knowing the upper boundaries of uncertainty and disturbance in advance.

3.1. Asymptotically stable design

Without loss of generality, suppose that the dead-zone function $\Phi_i(u_i)$ is anti-symmetric with respect to origin (i.e., $\alpha_{i+} = \alpha_{i-} = \alpha_i$, $u_{i+} = u_{i-} = u_{i0}$).

Defining the singular value decomposition of GC as $GC = U\Sigma V_1^T$ with $U \in R^{m \times m}$, $\Sigma \in R^{m \times m}$ and $V_1 \in R^{n \times m}$, and letting $\tilde{S} = [\tilde{s}_1, \tilde{s}_2, \dots, \tilde{s}_m]^T = B^T C^T G^T S$ and $\alpha = \min\{\alpha_i\}$, the adaptive variable structure output feedback control can then be

$$u_i = \begin{cases} -\frac{\tilde{s}_i}{\|\tilde{S}\|} \phi(y, t) - u_{i0} & \text{if } \tilde{s}_i > 0 \\ 0 & \text{if } \tilde{s}_i = 0 \\ -\frac{\tilde{s}_i}{\|\tilde{S}\|} \phi(y, t) + u_{i0} & \text{if } \tilde{s}_i < 0 \end{cases}, \quad (6)$$

where

$$\phi(y, t) = \mu \vartheta(y, t), \quad \mu > \frac{1}{\alpha}$$

and

$$\vartheta(y, t) = \frac{-S^T GCBNy + \Gamma \|S\|}{\|B^T C^T G^T S\|} + k.$$

Theorem 1: Consider the uncertain nonlinear system Eq. (1) containing input nonlinearity and a dead-zone which are subject to assumptions 1–3. For the control law in Eq. (6), use

$$N = -(\gamma I + GCAV_1 \Sigma^{-1} U^T)G, \quad (7)$$

where $\gamma > \hat{\beta}_1 \|\Sigma^{-1}\|$ is a scalar, and k is any positive constant. If Γ is chosen to satisfy $\Gamma > \|GCA\|\Omega + \hat{\beta}_2$, $\vartheta(y, t) \geq 0$, and $\Omega > 0$, where $\hat{\beta}_i (i = 1, 2)$ is the estimate of $\beta_i (i = 1, 2)$, and can be obtained from the adaptive law

$$\dot{\hat{\beta}}_1 = \frac{1}{p_1} \|S^T GCB\| \|\Sigma^{-1}\| \|S\| \quad (8a)$$

$$\dot{\hat{\beta}}_2 = \frac{1}{p_2} \|S^T GCB\| \quad (8b)$$

with the adaptive gain $p_i > 0$, then the reaching condition $S^T \dot{S} < 0$ is satisfied for $\{x : \|x_k\| \leq \Omega\}$.

Before proving Theorem 1, the following lemma has to be introduced.

Lemma: For all the input nonlinearities $\Phi(u)$ that satisfy Assumption 3, if the controller is selected as Eq. (6), then

$$\tilde{S}^T \Phi(u) \leq -\alpha \|\tilde{S}\| \phi(y, t). \quad (9)$$

Proof of Theorem 1: From Eqs. (4) and (6), we can obtain

$$\tilde{s}_i \Phi_i(u_i) \leq -\alpha \frac{\tilde{s}_i^2}{\|\tilde{S}\|} \phi(y, t), \quad (10)$$

Therefore, the following inequality is held,

$$\tilde{S}^T \Phi(u) \leq \sum_{i=1}^m -\alpha \frac{\tilde{s}_i^2}{\|\tilde{S}\|} \phi(y, t) = -\alpha \|\tilde{S}\| \phi(y, t) \quad (11)$$

Proof of Theorem 1: Consider the following Lyapunov function candidate

$$V = \frac{1}{2} [\|S\|^2 + p_1 \bar{\beta}_1^2 + p_2 \bar{\beta}_2^2] \quad p_i > 0, \quad (12)$$

where $\bar{\beta}_i = \hat{\beta}_i - \beta_i$ denotes the adaptation error.

Taking the derivative of V with respect to time t ,

$$\begin{aligned} \dot{V}(t) &= S^T \dot{S} + p_1 \bar{\beta}_1 \dot{\bar{\beta}}_1 + p_2 \bar{\beta}_2 \dot{\bar{\beta}}_2 \\ &= S^T GC[Ax + B\Phi(u) + B(Hx + d)] \\ &\quad + p_1 \bar{\beta}_1 \dot{\bar{\beta}}_1 + p_2 \bar{\beta}_2 \dot{\bar{\beta}}_2. \end{aligned} \quad (13)$$

By Eq. (11) in Lemma, we have

$$-\alpha \|\tilde{S}\| \phi(x, t) \leq S^T GCBNy - \Gamma \|S\|. \quad (14)$$

If x is decomposed as $x = x_k + x_p$, where $x_k \in \mathcal{N}(GC)$ and $x_p \in \mathcal{N}^\perp(GC)$, substituting Eq. (14) into Eq. (13), and using Eq. (7) with assumption $GCB = I$, the upper boundary for the expression in Eq. (13) is

$$\begin{aligned} \dot{V} &\leq S^T GCAx_k - \gamma S^T S - \Gamma \|S\| - k \|S\| + S^T d \\ &\quad + S^T H V_1 \Sigma^{-1} U^T S + p_1 \bar{\beta}_1 \dot{\bar{\beta}}_1 + p_2 \bar{\beta}_2 \dot{\bar{\beta}}_2 \\ &\leq \|S\| \|GCA\| \Omega - \gamma S^T S + \beta_1 \|\Sigma^{-1}\| S^T S \\ &\quad - \Gamma \|S\| - k \|S\| + \beta_2 \|S\| + p_1 \bar{\beta}_1 \dot{\bar{\beta}}_1 + p_2 \bar{\beta}_2 \dot{\bar{\beta}}_2 \\ &< -k \|S\| + (\beta_1 - \hat{\beta}_1) \|\Sigma^{-1}\| S^T S + (\beta_2 - \hat{\beta}_2) \\ &\quad \times \|S\| + p_1 \bar{\beta}_1 \dot{\bar{\beta}}_1 + p_2 \bar{\beta}_2 \dot{\bar{\beta}}_2 \\ &= -k \|S\| - \bar{\beta}_1 \|\Sigma^{-1}\| S^T S - \bar{\beta}_2 \|S\| \\ &\quad + p_1 \bar{\beta}_1 \dot{\bar{\beta}}_1 + p_2 \bar{\beta}_2 \dot{\bar{\beta}}_2. \end{aligned} \quad (15)$$

Since β_i is assumed to be constant, we have $\dot{\beta}_i = 0$ and the expression $\dot{\bar{\beta}}_i = \dot{\hat{\beta}}_i$ holds. Inserting $\dot{\bar{\beta}}_i = \dot{\hat{\beta}}_i$ with Eq. (8) into the right-hand side of the inequality Eq. (15) yields

$$\dot{V} \leq -k \|S\|. \quad (16)$$

Since $k > 0$ is chosen, $\dot{V} < 0$. Using Barbalat's Lemma,⁷⁾ one concludes that $S(t) \rightarrow 0$ as t tends to be infinity.

3.2. Exponentially stable design

If the control law Eq. (7) is modified as

$$\bar{u}_i = \begin{cases} -\frac{\tilde{s}_i}{\|\tilde{S}\|} \phi_\alpha(y, t) - u_{i0} & \text{if } \tilde{s}_i > 0 \\ 0 & \text{if } \tilde{s}_i = 0 \\ -\frac{\tilde{s}_i}{\|\tilde{S}\|} \phi_\alpha(y, t) + u_{i0} & \text{if } \tilde{s}_i < 0 \end{cases}, \quad (17)$$

where

$$\phi_\alpha(y, t) = \phi(y, t) + \left(\frac{q}{2\alpha} \right) ((S^T S) / \|\tilde{S}\|),$$

$q > 0$ with the adaptive law in Eq. (8).

Theorem 2: Consider an uncertain nonlinear dynamic system Eq. (1), and suppose assumptions 1–3 are satisfied. The variable structure output feedback control law is chosen as Eq. (17) with the adaptive law in Eq. (8). Then the system state globally and exponentially converges to the sliding mode.

Proof: Consider the same Lyapunov function candidate $V(t)$ as given in Eq. (12). Using Eq. (17) and the same manipulations as Eqs. (14–17), its derivative is given

$$\dot{V} < -\frac{1}{2} q \|S\|^2 = -qV \leq 0. \quad (18)$$

Therefore, the system trajectories globally and exponentially converge to the sliding mode.

4. Application to a Flexible Spacecraft

4.1. Mathematical model

The dynamics of spacecraft with flexible appendages can be obtained from a Lagrangian approach.⁸⁾ With the assumption of small Euler angle rotations, the dynamic model of Di Gennaro⁸⁾ can be approximated as

$$\begin{bmatrix} (J_T - J_R) & \delta^T \\ \delta & I \end{bmatrix} \ddot{Z} + \begin{bmatrix} 0 & 0 \\ 0 & D \end{bmatrix} \dot{Z} + \begin{bmatrix} 0 & 0 \\ 0 & K \end{bmatrix} Z = \begin{bmatrix} I \\ 0 \end{bmatrix} (\Sigma(\varphi, \theta, \psi, t) - u_s) \quad (19)$$

where $Z = [\varphi \ \theta \ \psi \ \eta^T]^T$, φ, θ and ψ are roll, pitch and yaw attitude angles, respectively, and $\Sigma(\varphi, \theta, \psi, t)$ can be obtained by equating similar terms in Eq. (19) and the corresponding spacecraft model given in Di Gennaro.⁸⁾

Defining the state variable $x_s = [Z \ \dot{Z}]^T \in R^{2(N+3)}$, the state-space spacecraft model describing the dynamics behavior of the flexible spacecraft can be described by

$$\begin{aligned} \dot{x}_s(t) &= (A_s + \Delta A_s)x_s(t) + B_s \Phi_s(u_s) + f_s(t) \\ y_s &= C_s x_s. \end{aligned} \quad (20)$$

Here, the nonlinearities in the input $\Phi_s(u_s)$ are given as saturation and dead-zone whose saturated and dead-band values are 10 Nm and 0.5 Nm, respectively.

4.2. Simulation results

In this section, a numerical application of the proposed control schemes to the attitude control of a flexible spacecraft is presented using MATLAB/SIMULINK software. The nominal moments of inertia parameters, coupling matri-

Fig. 1. Response of disturbance torques.

ces and the natural frequencies are taken from Di Gennaro.⁸⁾

Suppose the initial values of the three attitude angles are $\varphi(0) = 6^\circ$, $\theta(0) = 4^\circ$, and $\psi(0) = -4^\circ$. It is required to control each of the attitude angles approaching zero from their initial value within $t_f = 150$ s, and each of the vibration mode coordinates satisfies

$$\eta_i = \dot{\eta}_i = \eta_i(t_f) = \dot{\eta}_i(t_f) = 0,$$

$i = 1, 2, 3, 4$. $f_s(t)$ is usually imprecise and satisfies Assumption 2. Assume that d is a random disturbance torque with mean “0” and standard deviation “1”, whose maximum absolute $d_{\max}$ has been fixed to 0.25 Nm, as shown in Fig. 1.

In order to reduce chattering in an actual implementation, the discontinuous control given as Eq. (6) is often replaced with a smoothed control of the form:

$$u_i = \begin{cases} -\frac{\tilde{s}_i}{\|\tilde{S}\| + \varepsilon} \phi(y, t) - u_{i0} & \text{if } \tilde{s}_i > 0 \\ 0 & \text{if } \tilde{s}_i = 0 \\ -\frac{\tilde{s}_i}{\|\tilde{S}\| + \varepsilon} \phi(y, t) + u_{i0} & \text{if } \tilde{s}_i < 0 \end{cases} \quad (21)$$

where $\varepsilon > 0$ is small.

For the controller in Eq. (21), four design parameters (Ω , γ , Γ and ε) are involved. A considerable number of simulations have been done for determining parameters Ω , γ , Γ and ε , while here only one case of those numerical studies is given for the space limitation: $\Omega = 10$, $\gamma = 10$, $\Gamma = 5$, $k = 0.1$ and $\varepsilon = 0.01$. In addition, the following initial values are arbitrarily chosen; that is $\hat{\beta}_1 = \hat{\beta}_2 = 0$.

With the designed controller in Eq. (21), the requirements for the inner torque of each flywheel are illustrated in Fig. 2(a). From this figure, we can see that the inner torque of each flywheel approaches “0” at the time of 50 s, and the maximum magnitudes of the torque do not exceed the saturated value, 10 Nm, but with chattering slightly. The responses of roll, pitch and yaw angles corresponding to the initial attitude values are shown in

(a) Inner-torque of flywheels

(b) Attitude angle and rate

(c) Coordinate of 1st-4th vibration modes

Fig. 2. The case for asymptotically stable design method.

(a) Inner-torque of flywheels

(b) Attitude angle and rate

(c) Coordinate of 1st-4th vibration modes

Fig. 3. The case for exponentially stable design method.

Fig. 2(b). We can see that each of the three attitude angles response almost approaches “0” at the time of 150 s, and the steady error of each angle is less than 0.002° . The bottom plot of Fig. 2(b) shows the responses of angular velocities of the flexible spacecraft. It is noted that an acceptable attitude rate response was also achieved with a steady error

less than $0.002^\circ/\text{s}$. However, with control input dead-zone/nonlinearities and external perturbations, fast and precise attitude control was achieved for the current design system. Meanwhile, responses of the first four modes are illustrated in Fig. 2(c). The maximum induced vibration co-

ordinate of the 1st mode is effectively suppressed below $0.1 \text{ kg}^{1/2}\text{m}$. It is noted that all of the vibration modal coordinates and vibration rates approach "0" by the end of the required control time.

For the second case, the controller Eq. (17) can also be modified into a form similar to Eq. (21) by introducing the boundary layer. Here, parameters $q_1 = q_2 = 15$ are selected through a considerable number of numerical simulations. The parameters N , Ω , Γ , ε , γ and α remain the same for a fair comparison. The same initial condition and disturbances as above are also used. The simulations are shown in Fig. 3. The maximum torque of the control torque is 8 Nm, which also does not exceed the saturated value, 10 Nm. The three-axis attitude response almost approaches "0" at the time of 100 s, and the steady error of each angle is also less than 0.001° as shown in the top plot of Fig. 3(b). Figure 3(b) also shows the responses of angular velocities of the flexible spacecraft with a steady error of less than $0.002^\circ/\text{s}$. It is clear that, from a comparison of Figs. 2(b) and 3(b), the attitude and rate responses under the exponential design case are faster than the asymptotical case, even if input dead-zone/nonlinearity is considered. In addition, the maximum induced vibration coordinate of the 1st mode is effectively suppressed below $0.1 \text{ kg}^{1/2}\text{m}$ as shown in Fig. 3(c).

5. Conclusions

In this paper, an approach to the vibration suppression of a three-axis stabilized flexible spacecraft is investigated during attitude maneuvering in the presence of bounded model uncertainty, external disturbances and control input nonlinearity/dead-zone. The approach of the current study is the construction of an adaptive variable structure output feedback controller to stabilize uncertain dynamics systems with explicitly considering nonlinearity/dead-zone in the control input. It is shown that the controller guarantees the global reaching condition of the sliding mode in uncertain systems. The significant advantages of this control include the following: (i) the presented adaptive

variable structure output feedback controller can derive the trajectories of an uncertain system with input nonlinearity/dead-zone onto the sliding mode, and (ii) the variable structure output feedback control can also be achieved through adaptive control without knowing the boundaries of the uncertainty and perturbation terms in advance.

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