Model Predictive Controller Design to Suppress Rate-Limiter-Based Pilot-Induced Oscillations

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Most severe pilot-induced oscillations (PIOs) are characterized as the presence of actuator rate limiting. Rate limiting causes phase lag and magnitude reduction between the limiter input and output when the input signal rate exceeds the rate limitation. This is the main cause of PIOs. Herein, two controllers, in which actuator rate limiting is redefined as input constraints, are designed based on the model predictive control (MPC) strategy and applied to the pilot-aircraft loop in both parallel and serial manners. In the parallel implementation, the MPC controller can suppress rate-limiter-based PIOs. In the serial implementation, the MPC controller can compensate the phase lag due to rate limiting, and hence, help the pilot prevent PIOs from occurring in advance.

Key Words: Pilot-Induced Oscillation, Rate Limiter, Model Predictive Control, Constraint

Nomenclature

x : system state vector, $x \in \mathbb{R}^n$
 x^* : augmented system state vector, $x^* \in \mathbb{R}^{n+m}$
 $\tilde{x}$: state offset between internal plant model and real plant model
 u : system input vector, $u \in \mathbb{R}^m$
 Δu : input increment
 y : system output vector, $y \in \mathbb{R}^l$
 x_k : measured state vector at current sampling instant k
 $x(i)$: predicted state vector at sampling instant i , $i = 0, 1, \dots, p - 1$
 p : predictive horizon
 A_d, B_d, C_d, D_d : discrete state space matrices
 A_a, B_a, C_a, D_a : discrete state space matrices of augmented system
 $x_{\min}, x_{\max}$: minimum values, maximum values of states
 $u_{\min}, u_{\max}$: minimum values, maximum values of inputs
 $x_{\min} \leq x \leq x_{\max}$: $x_{1\min} \leq x_1 \leq x_{1\max}, \dots, x_{n\min} \leq x_n \leq x_{n\max}$
 $u_{\min} \leq u \leq u_{\max}$: $u_{1\min} \leq u_1 \leq u_{1\max}, \dots, u_{m\min} \leq u_m \leq u_{m\max}$
 J : system cost function
 Q : positive semi-definite weighting matrix of states
 R : positive definite weighting matrix of inputs
 L : rate limitation
 $G(s)$: transfer function

Δt : sampling time

K_p : pilot gain

1. Introduction

Pilot-induced oscillations or pilot in-the-loop oscillations (PIOs) are sustained or uncontrollable oscillations resulting from the efforts of the pilot to control the aircraft (MIL-STD-1797A). PIOs are also described as “rare, unexpected, and unintended excursions in aircraft attitude and flight path caused by anomalous interactions between the aircraft and pilot.”¹⁾

Depending essentially on the degree of non-linearity in the events, PIOs may be classified into three categories:¹⁾

Category I Linear pilot-aircraft oscillations;

Category II Quasi-linear pilot-aircraft oscillations with nonlinear effects due to rate or position limiting;

Category III Nonlinear pilot-aircraft oscillations with transients.

Category I PIOs are well understood and there are many mature criteria (e.g., Neal-Smith criterion,²⁾ Smith-Geddes criterion,³⁾ Gibson criterion⁴⁾ and Bandwidth/Pitch Rate Overshoot criterion⁵⁾). They are easy to apply, and no further refinement work is justified. For Category III PIOs, most of them likely start out as Category II PIOs and happen rarely.⁶⁾ Therefore, this paper focuses on Category II PIOs.

A key feature of most of the well-recorded severe Category II PIOs (e.g., YF-22 in 1992,⁷⁾ JAS-39 in 1993⁸⁾) is the presence of actuator rate limiting. Rate limiting is used to protect the actuator from overload.⁹⁾ However, it may cause phase lag and magnitude reduction between the limiter input and output when the input signal rate exceeds the actuator rate limitation.^{10–13)} Phase lag implies a time delay in aircraft

dynamics. When the time delay becomes large, PIOs can occur. This is the primary cause of rate-limiter-based PIOs.

Some methods or criteria for the analysis and prediction of rate-limiter-based PIOs have been proposed; for example, the describing function (DF) technique,^{14,15)} the open-loop onset point (OLOP) criterion,^{16,17)} the Pilot/Vehicle Dynamics Non-Linear (PVDNL) criterion¹⁸⁾ and so on. For the DF technique, the non-linearity elements are assumed to be independent. In other words, the describing functions are those obtained in the case of a single non-linearity. Moreover, the estimation of a possible limit cycle is difficult.¹⁵⁾ The OLOP criterion is applicable only to the prediction of rate-limiter-based PIOs.⁶⁾ In short, they provide methods for prediction and analysis of PIOs but do not provide active procedures for improving the flight control system to prevent PIOs or suppress PIOs.

There have been few efforts to design a controller that suppresses rate-limiter-based PIOs or prevents PIOs from occurring in advance. In this paper, an attempt is made to design such controllers using the model predictive control (MPC) strategy. By redefining rate limiting conditions as input constraints, the MPC controllers are designed in two ways: in the first design, the controller is put in parallel with the pilot model and is designed to circumvent the effects of rate limiting while suppressing PIOs; in the second design, the controller is put in series with the pilot model and is designed to compensate the phase lag due to rate limiting while helping the pilot prevent PIOs from occurring in advance. In both designs, a pilot is modeled as a constant gain based on the following well-known observation: when an aircraft exhibits a limit cycle oscillation, the pilot reacts just as a constant (high) gain.¹⁹⁾

This paper is organized as follows: in section 2, the MPC strategy is introduced; in section 3, the redefinition of rate limiting and the MPC controller design for the parallel system are proposed; in section 4, the effect of the MPC controller on compensation of phase lag is discussed and a new controller for the series system is designed; and conclusions are given in section 5.

2. Model Predictive Control Strategy

Model predictive control (MPC) strategy or receding horizon control strategy is motivated to satisfy increasingly stringent performance requirements with constraints on input and state. It allows taking the constraints on state and input into account while computing the input, which is the main difference from conventional control. In recent years, MPC has been applied over a wide range, from chemicals to transportation industries.^{20,21)}

MPC is a form of control in which the controller output is obtained by solving an open-loop optimal control problem on-line at each sampling instant using the current state of the plant as the initial state. Optimization yields an optimal control sequence and the first control in this sequence is applied to the plant.^{22–27)}

Consider a discrete, linear, time-invariant system:

$$x(k+i+1) = A_d x(k+i) + B_d u(k+i), \quad x(k) = x_k \quad (1)$$

with system constraints:

$$\begin{aligned} x_{\min} &\leq x(k+i) \leq x_{\max} \\ u_{\min} &\leq u(k+i) \leq u_{\max}. \end{aligned} \quad (2)$$

The MPC controller output at sampling instant k is given by solving the following optimal control problem:

$$\min_{u(k+i), i=0, \dots, p-1} J(x(k); u(k+i), i=0, \dots, p-1), \quad (3)$$

subject to

$$\begin{aligned} x(k+i+1) &= A_d x(k+i) + B_d u(k+i) \\ x_{\min} &\leq x(k+i) \leq x_{\max} \\ u_{\min} &\leq u(k+i) \leq u_{\max}. \end{aligned} \quad (4)$$

The cost function J is typically given by:

$$J = \sum_{i=0}^{p-1} x(k+i)^T Q x(k+i) + u(k+i)^T R u(k+i). \quad (5)$$

The optimal control problem, Eq. (3), can be reduced to a quadratic programming (QP) problem to calculate the optimal control sequence.²⁸⁾ The QP problem can be solved by using the MATLAB Optimization Toolbox.

The controller output (i.e., the first one of the optimal control sequence),

$$u_k = u(k) \quad (6)$$

is implemented to the plant between two sampling instants. At the next sampling instant $k+1$, the optimal control problem is recalculated based on a newly measured state.

Therefore, the optimal solution, Eq. (6), which stems from the optimal control problem, Eq. (3), can minimize the cost function and satisfy the system constraints, Eq. (4).

3. Parallel Connection of the MPC Controller and Pilot Model

3.1. System model

The block diagram of applying a MPC controller to the pilot-aircraft loop in parallel is shown in Fig. 1. A training jet T-33²⁹⁾ output feedback SISO system is chosen as an example. The system input is stick force and the output is pitch angle. A simplified transfer function of the aircraft is given as:

$$G(s) = \frac{\tau s + 1}{s \left(\frac{1}{\omega_{sp}^2} s^2 + 2 \frac{\zeta_{sp}}{\omega_{sp}} s + 1 \right)} \quad (7)$$

Fig. 1. Parallel connection of the MPC controller and pilot (K_p , pilot; A/F, aircraft model).

where $\omega_{sp} = 4.9$ [rad/s], $\zeta_{sp} = 0.7$ and $\tau = 0.8$ [s]. The rate limitation is given as $L = 10$ [deg/s], and the initial output value is chosen as $y_0 = 5$ [deg].

It is reported that when an aircraft is in a limit cycle oscillation, the pilot behaves only as a constant high gain.¹⁹⁾ Since our analysis and design are for an occurring PIO region, herein we assume that the pilot model can be approximated by a constant gain. For T-33 aircraft with a rate limitation of $L = 10$ [deg/s], the pilot gain is chosen as $K_p = 6$ where an occurring PIO is observed in the simulation results.

3.2. Redefinition of rate limiting

When the input rate of the rate limiter exceeds the rate limitation, a PIO may occur. In order to circumvent the effects of rate limiting, the input signal rate is required within the rate limitation:

$$-L \leq \frac{du}{dt} \leq L. \quad (8)$$

For the MPC controller, the inequality shown in Eq. (8) can be written as the following discrete form:

$$-L\Delta t \leq u(k+i) - u(k+i-1) \leq L\Delta t$$

or simplify as

$$-L\Delta t \leq \Delta u(k+i) \leq L\Delta t. \quad (9)$$

Herein, the sampling time Δt and the rate limitation L are assumed to be constant for all sampling instants.

3.3. MPC controller design

The inequality shown in Eq. (9) has the same form as system constraints on input, but it is on input increments $\Delta u(k+i)$. Then the system, Eq. (1), is rewritten in an augmented form:

$$\begin{aligned} x^*(k+i+1) &= A_a x^*(k+i) + B_a \Delta u(k+i) \\ y(k+i) &= C_a x^*(k+i) + D_a \Delta u(k+i) \end{aligned} \quad (10)$$

where $x^*(k+i) = \begin{bmatrix} x(k+i) \\ u(k+i-1) \end{bmatrix}$ and $A_a = \begin{bmatrix} A_d & B_d \\ 0 & I \end{bmatrix}$, $B_a = \begin{bmatrix} B_d \\ I \end{bmatrix}$, $C_a = \begin{bmatrix} C_d & D_d \end{bmatrix}$, $D_a = D_d$.

Using the augmented system shown in Eq. (10) as the controller model, the system cost function, Eq. (5), is modified as:

$$\begin{aligned} J_1 &= \sum_{i=0}^{p-1} x^*(k+i)^T Q_1 x^*(k+i) \\ &\quad + \Delta u(k+i)^T R_1 \Delta u(k+i), \end{aligned} \quad (11)$$

where $Q_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, $R_1 = 1$ are chosen for the

T-33 model, and the sampling time is chosen as $\Delta t = 0.1$ [s].

The MPC controller output is obtained by solving:

$$\begin{aligned} \min_{\Delta u(k+i), i=0, \dots, p-1} & J_1(x^*(k); \Delta u(k+i), i=0, \dots, p-1), \quad (12) \\ \text{subject to} & \end{aligned}$$

Fig. 2. System response of the parallel system with pilot operation: (a) system output, and (b) comparison of the pilot output and limiter output.

$$-L\Delta t \leq \Delta u(k+i) \leq L\Delta t.$$

The system input $u_k = \Delta u_k + u_{k-1}$ is implemented to the aircraft.

The simulations on the parallel system are carried out for two cases.

In the first case, the pilot controls the aircraft. Figure 2 shows the unstable results due to rate limiting. A severe limit cycle is observed in Fig. 2a, and a significant phase lag and a magnitude reduction between the pilot output and limiter output can be seen in Fig. 2b.

In the second case, the aircraft is controlled by the pilot together with the MPC controller under the same initial condition as that for the first case. Figure 3a shows that the system is stabilized quickly when the switch is changed over to the MPC controller at $t = 12$ [s]. In Fig. 3b, the phase lag and magnitude reduction due to rate limiting disappear after 12 [s]. This means that the MPC controller circumvents the

Fig. 3. System response of the parallel system with the pilot and MPC controller: (a) system output, and (b) comparison of the pilot/controller output and limiter output ($t < 12$ s by the pilot, $t \geq 12$ s by the controller).

Fig. 4. Series connection of the MPC controller and pilot (dot line box indicates the new controller, y_1 is a reference output, and y_2 is the real plant output).

effects of rate limiting and successfully suppresses rate-limiter-based PIOs.

4. Series Connection of the MPC Controller and Pilot Model

4.1. MPC controller redesign

In order for the MPC controller to be in synchro-control with the pilot rather than activated by a switch, they are connected in serial manner (Fig. 4).

The dot line box indicates a new controller, which has two inputs: the pilot output u_1 and the system state x_2 . Both u_1 and x_2 are assumed to be measurable. The new controller output u_2 is a combination of u_1 and the MPC controller output.

The internal A/F is an internal aircraft model in the new controller, the system equation is:

$$\begin{aligned} x_1(k+i+1) &= A_d x_1(k+i) + B_d u_1(k+i) \\ y_1(k+i) &= C_d x_1(k+i) + D_d u_1(k+i). \end{aligned} \quad (13)$$

The external A/F is the real plant model, and the system equation is:

$$\begin{aligned} x_2(k+i+1) &= A_d x_2(k+i) + B_d u_2(k+i) \\ y_2(k+i) &= C_d x_2(k+i) + D_d u_2(k+i). \end{aligned} \quad (14)$$

If the effects of rate limiting can be circumvented and the MPC controller does not affect the pilot's operation, y_1 may coincide with y_2 and x_1 coincides with x_2 . Therefore, the MPC controller is redesigned to minimize the offset between x_1 and x_2 .

The MPC controller model is then redefined as:

$$\begin{aligned} \tilde{x}(k+i+1) &= A_d \tilde{x}(k+i) + B_d \tilde{u}(k+i) \\ \tilde{y}(k+i) &= C_d \tilde{x}(k+i) + D_d \tilde{u}(k+i), \end{aligned} \quad (15)$$

where $\tilde{x} = x_2 - x_1$, $\tilde{u} = u_2 - u_1$, and $\tilde{y} = y_2 - y_1$. Correspondingly, the rate limiting condition, Eq. (8), is rewritten as:

$$\begin{aligned} -L\Delta t - \Delta u_1(k+i) &\leq \Delta \tilde{u}(k+i) \\ &\leq L\Delta t - \Delta u_1(k+i). \end{aligned} \quad (16)$$

Similar to Eq. (10), the augmented form of the system, Eq. (15), can be rewritten as:

$$\begin{aligned} \tilde{x}^*(k+i+1) &= A_a \tilde{x}^*(k+i) + B_a \Delta \tilde{u}(k+i) \\ \tilde{y}(k+i) &= C_a \tilde{x}^*(k+i) + D_a \Delta \tilde{u}(k+i), \end{aligned} \quad (17)$$

where $\tilde{x}^*(k+i) = \begin{bmatrix} \tilde{x}(k+i) \\ \tilde{u}(k+i-1) \end{bmatrix}$ and $A_a = \begin{bmatrix} A_d & B_d \\ 0 & I \end{bmatrix}$, $B_a = \begin{bmatrix} B_d \\ I \end{bmatrix}$, $C_a = \begin{bmatrix} C_d & D_d \end{bmatrix}$, $D_a = D_d$.

The cost function is rewritten as:

$$J_2 = \sum_{i=0}^{p-1} \tilde{x}^*(k+i)^T Q_2 \tilde{x}^*(k+i) + \Delta \tilde{u}(i)^T R_2 \Delta \tilde{u}(i), \quad (18)$$

where $Q_2 = Q_1$ and $R_2 = R_1$.

Then the MPC controller output is obtained by solving:

$$\min_{\Delta \tilde{u}(k+i), i=0, \dots, p-1} J_2(\tilde{x}^*(k); \Delta \tilde{u}(k+i), i=0, \dots, p-1), \quad (19)$$

subject to

$$-L\Delta t - \Delta u_1(k+i) \leq \Delta \tilde{u}(k+i) \leq L\Delta t - \Delta u_1(k+i).$$

The system input $u_{2k} = \Delta \tilde{u}_k + \tilde{u}_{k-1} + u_{1k}$ is implemented to the aircraft.

4.2. Effect of the MPC controller on compensating phase lag

In order to analyze the effect of the MPC controller on compensating the phase lag due to rate limiting, a sinusoid is chosen as an input signal. In Fig. 5, y_1 indicates one reference output of the loop without the rate limiter; y_2 is the output of the MPC controller loop, implying the real plant output; and y_3 is the other auxiliary reference output of the loop with the rate limiter, which is used to see the effects of the rate limiter and the MPC controller. The amplitude and frequency of the sinusoidal input signal are chosen as $a = 12$ [deg] and $\omega = 1$ [rad/s], respectively. The rate limitation is chosen as $L = 10$ [deg/s].

Figure 6 shows the comparison on phase among y_1 , y_2 and y_3 of Fig. 5. A significant phase lag is observed between y_1 and y_3 , and a phase lead is observed between y_1 and y_2 inversely. This suggests that the MPC controller can compensate the phase lag due to rate limiting. Furthermore, the difference in phase shift between y_2 and y_3 by comparing them with y_1 is shown in Fig. 7. The ordinate axis and abscissa axis are phase shift and frequency ω , respectively. $L = 10$ [deg/s] is fixed, and ω changes from 2 [rad/s] to 6 [rad/s] for every 0.5 [rad/s]. The simulation is carried out for $a \times \omega = 11$ [deg/s] and $a \times \omega = 12$ [deg/s]; in both cases, the rate limiters are active. Figure 7 confirms the phase lead due to the MPC controller.

Fig. 5. Analysis of the effects of the MPC controller on the compensation of phase lag with a sinusoid input (dot line box indicates the new controller, y_1 is a reference output, y_2 is the real plant output, and y_3 is the auxiliary reference output).

Fig. 6. Comparison of output phases y_1 , y_2 and y_3 with a sinusoid input $a \sin(\omega t)$. $a = 12$ [deg], $\omega = 1$ [rad/s]. (The lower one is a partially enlarged plot.)

Fig. 7. Difference in phase shifts between y_2 and y_3 by comparing them with y_1 for $a\omega = 11$ [deg/s] and $a\omega = 12$ [deg/s], respectively.

4.3. Cooperation of the MPC controller and the pilot

Based on the results of section 4.2, the MPC controller can compensate the phase lag due to rate limiting when it is put in series with the sinusoidal input signal. In order to analyze the pilot-controller-aircraft loop, the pilot replaces the sinusoidal input and the output feedback is added (Fig. 4). Moreover, the auxiliary reference output y_3 , which is the response of the pilot-aircraft output feedback system with the rate limiter, is also shown in the simulation results in order to see the effects of the rate limiter clearly.

Figure 8 shows the comparison results among y_1 , y_2 and y_3 for the pilot gain $K_p = 1$. Figures 8a and 8c illustrate that the outputs, y_1 and y_3 , are almost same. This implies that the input signal rate of the limiter does not exceed the rate limitation. Comparing Fig. 8b with the above two figures, it can be seen that the output y_2 , which is the real plant output, is well approximated by y_1 or y_3 . In other words, when the input rate is within the rate limitation, the pilot-controller-aircraft closed-loop seems to be operated mainly by the pilot and the series system is stable.

It is well-known that PIOs occur when pilots are involved

Fig. 8. Comparison of the outputs y_1 , y_2 and y_3 for $K_p = 1$: (a) system output by pilot without the limiter, (b) system output by controller and pilot with the limiter, and (c) system output by pilot with the limiter.

in severe tasks, such as landing and increasing their control amplitude. By increasing the pilot gain from $K_p = 1$ to $K_p = 6$, we try to realize the aircraft operational range in which PIOs are likely to occur.

Figure 9 shows the comparison results among y_1 , y_2 and y_3 for the pilot gain $K_p = 6$. Figure 9a shows a marginally stable result by a pilot without the rate limiter in the loop. A limit cycle due to rate limiting is observed in Fig. 9c. The results indicate that a PIO has occurred due to rate limiting though the pilot can stabilize the system without the rate limiter. Figure 9b shows a stable result due to the MPC controller even when the rate limiter exists in

Fig. 9. Comparison of the outputs y_1 , y_2 and y_3 for $K_p = 6$: (a) system output by pilot without the limiter, (b) system output by controller and pilot with the limiter, and (c) system output by pilot with the limiter.

the loop. In other words, the MPC controller stabilizes the pilot-controller-aircraft loop when the limiter input signal rate exceeds the rate limitation.

In summary, Figs. 8 and 9 show that the MPC controller as well as the pilot stabilizes the pilot-controller-aircraft closed-loop for both PIO and non-PIO regions. This means that the MPC controller may help the pilot prevent PIOs from occurring in advance without affecting the pilots' operation under moderate flight conditions.

5. Conclusion

Model predictive control strategy has the advantage of

taking system constraints into account while calculating the system input. Based on this advantage, two MPC controllers, in which the actuator rate limiting condition is redefined as the input constraints, have been designed and applied to the pilot-aircraft loop in both parallel and serial manners. The simulation results indicate that rate-limiter-based PIOs may be suppressed by using the MPC controller in parallel with the pilot. Additionally, the redesigned MPC controller can compensate the phase lag due to rate limiting, and moreover, help the pilot to prevent PIOs from occurring when it is put in series with the pilot. The approach taken in this paper using the MPC strategy may suppress occurring PIOs (Fig. 3) and/or prevent PIOs from occurring in advance (Fig. 9b), which are difficult to achieve using existing methods such as the DF technique or OLOP criterion.

References

- 1) National Research Council: *Aviation Safety and Pilot Control: Understanding and Preventing Unfavorable Pilot-Vehicle Interactions*, National Academy Press, Washington, D.C., 1997.
- 2) Neal, T. P. and Smith, R. E.: An In-Flight Investigation to Develop Control System Design Criteria for Fighter Airplanes, AFFDL-TR-70-74, Vol. 1, 1970.
- 3) Smith, R. H.: The Smith-Geddes Criteria, Presented at the SAE Aerospace Control and Guidance Systems Committee Meeting, Reno, NV, Mar. 1993.
- 4) Gibson, J. C.: The Prevention of PIO by Design, Active Control Technology: Applications and Lessons Learned, AGARD-CP-560, Jan., 1995, pp. 2-1-2-12.
- 5) Mitchell, D. G. and Hoh, R. H.: Development of Methods and Devices to Predict and Prevent Pilot-Induced Oscillations, AFRL-VA-WP-TR-2000-3046, Dec. 2000.
- 6) Mitchell, D. G. and Klyde, D. H.: Recommended Practices for Exposing Pilot-Induced Oscillations or Tendencies in the Development Process, USAF Development Test and Evaluation Summit, Nov. 2004, pp. 1-20.
- 7) Dornheim, M. A.: Report Pinpoints Factors Leading to YF-22 Crash, *Aviation Week and Space Technology*, **137** (1992), pp. 53-54.
- 8) Why the Gripen Crashed, *Aerospace Am.*, **32** (1994), p. 11.
- 9) Duda, H.: Prediction of Pilot-In-The-Loop Oscillations Due to Rate Saturation, *J. Guid. Control Dynam.*, **20** (1997), pp. 581-587.
- 10) Klyde, D. H., McRuer, D. T. and Myers, T. T.: Pilot-Induced Oscillation Analysis and Prediction with Actuator Rate Limiting, *J. Guid. Control Dynam.*, **20** (1997), pp. 81-89.
- 11) Duda, H.: Flight Control System Design Considering Rate Saturation, *Aerospace Sci. Technol.*, **4** (1998), pp. 265-275.
- 12) Hanley, J. G.: A Comparison of Algorithms to Prevent Pilot-Induced Oscillations Caused by Actuator Rate Limiting, Air University Master Thesis, U.S.A., 2003, pp. 1-84.
- 13) Klyde, D. H. and Mitchell, D. G.: Investigating the Role of Rate Limiting in Pilot-induced Oscillations, AIAA-2003-5463, AIAA Atmospheric Flight Mechanics Conference and Exhibit, Aug. 2003, pp. 1-12.
- 14) Graham, D. and McRuer, D. T.: *Analysis of Nonlinear Control Systems*, Wiley, New York, 1961.
- 15) Amato, F., Iervolino, R., Scala, S. and Verde, L.: Category II Pilot In-The-Loop Oscillations Analysis from Robust Stability Methods, *J. Guid. Control Dynam.*, **24** (2001), pp. 531-538.
- 16) Duda, H.: Effects of Rate Limiting Elements in Flight Control Systems—A New PIO Criterion, AIAA-95-3304, Aug. 1995.
- 17) Duda, H., Duus, G., Hovmark, G. and Forsell, L.: New Flight Simulator Experiments on Pilot-involved Oscillations due to Rate Saturation, *J. Guid. Control Dynam.*, **23** (2000), pp. 312-318.
- 18) Zeyada, Y., Hess, R. A. and Siwakosit, W.: Analysis of Aircraft Handling Qualities and Pilot-induced Oscillation Tendencies with Actuator Saturation, AIAA-98-4334, AIAA Atmospheric Flight

- Mechanics Conference and Exhibit, Aug. 1998, pp. 401–414.
- 19) McRuer, D. T.: Pilot-Induced Oscillations and Human Behavior. NASA CR-4683, Jul. 1995.
 - 20) Findeisen, R., Imsland, L., Allgöer, F. and Foss, B. A.: State and Output Feedback Nonlinear Model Predictive Control: An Overview, *Eur. J. Control*, **9** (2003), pp. 190–206.
 - 21) Henson, M. A.: Nonlinear Model Predictive Control: Current Status and Future Directions. *Comput. Chem. Eng.*, **23** (1998), pp. 187–202.
 - 22) Chen, C. C. and Shaw, L.: On Receding Horizon Feedback Control, *Automatica*, **18** (1982), pp. 349–352.
 - 23) Mayne, D. Q. and Michalska, H.: Receding Horizon Control of Nonlinear Systems, *IEEE Trans. Automat. Control*, **35** (1990), pp. 814–824.
 - 24) Chen, H. and Allgower, F.: A Quasi-Infinite Horizon Nonlinear Model Predictive Control Scheme with Guaranteed Stability, *Automatica*, **34** (1998), pp. 1205–1217.
 - 25) Mayne, D. Q., Rawlings, J. B., Rao, C. V. and Sokaert, P. O. M.: Constrained Model Predictive Control: Stability and Optimality, *Automatica*, **36** (2000), pp. 789–814.
 - 26) Chisci, L. and Mosca, E.: Stabilizing I-O Receding Horizon Control of CARMA Plants, *IEEE Trans. Automat. Control*, **39** (1994), pp. 614–618.
 - 27) Michalska, H. and Mayne, D. Q.: Robust Receding Horizon Control of Constrained Nonlinear Systems, *IEEE Trans. Automat. Control*, **38** (1993), pp. 1623–1632.
 - 28) Maciejowski, J. M.: *Predictive Control with Constraints*, Prentice Hall, Amsterdam, 2002, pp. 1–352.
 - 29) Neal, T. P. and Smith, R. E.: A Flying Qualities Criterion for the Design of Fighter Flight-Control Systems, *AIAA J. Aircraft*, **8** (1971), pp. 803–809.